

# 29 | UTILITY MAXIMIZATION

Purpose: To demonstrate the rule for utility maximization. To show the effects on consumer choice of changes in prices and income.

Computer files: **utilmax198.xls**.

Instructions and questions:

Sally Jones gets utility from consuming two goods, spaghetti and tacos. She receives a fixed money income each week, and can buy spaghetti and tacos at fixed known prices. The problem she faces is to choosing the amounts of spaghetti and tacos that will maximize her utility. In other words, she has to solve the standard problem in the theory of consumer behavior.

The utility maximizing amounts of the goods, the solution to Sally's problem, are those for which (i) she spends all of her income, and (ii) her marginal rate of substitution (MRS) of spaghetti for tacos equals the ratio of the price of spaghetti to the price of tacos. Another way to say this is that she should be on her budget constraint where an indifference curve is just tangent to the constraint.

This problem set is the first of three on this subject. Here you are given Sally's preferences as a set of indifference curves, and the income and prices she faces. The task is to find the utility maximizing amounts of the goods to consume. You accomplish this in Excel by using Goal Seek to find the amount of spaghetti that makes the *difference* between the MRS and price ratio equal to zero.

In the problem set after this one you are asked to solve the same problem on the assumption utility is cardinally measurable. One way to state the utility maximizing rule in this case is that the consumer should buy goods so that the marginal utility per dollar spent on all goods is the same. (Marginal utility per dollar spent on a good is simply its marginal utility divided by its price.)

In the third problem set you will examine the effects on a consumer's choices of changes in income and the prices of the goods, and derive the demand curve for a good.

Open the Excel file **utilmax198.xls**. What you see are a few of the Sally's indifference curves (in blue), her indifference curve for her current levels of consumption of tacos and spaghetti (in red), and her budget constraint for the current levels of income and prices (in black). You can change consumption by choosing different amounts of spaghetti. Taco consumption is automatically computed to exhaust income for each level of spaghetti you choose. Change money

income and the prices of the goods so you understand how the changes affect Sally's budget constraint.

Get a feel for how the model works by choosing some different values for spaghetti consumption. Notice that taco consumption adjusts so that all income is spent on these two goods. Notice also what happens when spaghetti consumption exceeds 37.5.

Set spaghetti consumption equal to 15. Make sure income is \$150, the prices of spaghetti and tacos are \$4.00 and \$1.50, respectively. You should be able to see that the consumer is not maximizing utility at the current consumption amounts.

The rule for utility maximization is that Sally should choose goods so that the marginal rate of substitution between them is equal to the ratio of their prices. In this example we want the MRS of spaghetti for tacos to equal  $P(S)/P(T)$ .

When you solve the problem of maximizing utility (use Goal Seek to do it), notice that at the solution to the consumption problem the budget constraint and the red indifference curve are tangent to each other. Notice also that the utility maximizing amounts of the goods are the amounts Sally *demands*. That is, they are the amounts she wants to buy, and is able to buy.

#### Hints and tips:

- 1) Use Goal Seek to find the utility maximizing amounts of S and T. Set the MRS equal to the price ratio of the goods by changing the amount of spaghetti.
- 2) Be sure you understand the different effects of changing income and prices on the budget constraint.
- 3) Remember that an important purpose of this model is to show that the demand for a good depends *only* on income, the good's own price, the prices of other goods, and preferences.

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**MATH MAVEN'S CORNER:** The utility function used to generate the graph in **utilmax98.xls** is given by

$$U = AS^aT^b$$

where  $S$  is the amount of spaghetti consumed and  $T$  is the amount of tacos consumed, and  $A$ ,  $a$ , and  $b$  are randomly chosen. The budget constraint is given by  $I = P_S S + P_T T$ , where  $I$  is income, and  $P_S$  and  $P_T$  are the prices of  $S$  and  $T$ , respectively. The math problem is to maximize utility subject to the budget constraint. It can be solved either of two ways: substitute the constraint into the utility function so only one of the goods is a variable and then maximize, or use the Lagrangian Theorem. Satisfy yourself, if you wish, that the marginal rate of substitution of  $S$  for  $T$  is  $(aT/bS)$  in this case. The  $MRS_{S \text{ for } T}$  in general is  $-dT/dS|_{dU=0} = (\partial U/\partial S)/(\partial U/\partial T)$ .

## UTILITY MAXIMIZATION

### Questions

- 1) With income and prices at their baseline values, what is the largest amount of spaghetti the consumer can buy?
- 2) Following on from the last question, what is the largest amount of tacos the consumer can buy?
- 3) Set income and prices to their baseline values. If the consumer buys 10 units of spaghetti, how many tacos can she buy?
- 4) Set income and prices to their baseline values. Set spaghetti consumption to 15 units. Beginning from this point, what's the utility maximizing amount of spaghetti?
- 5) When utility is maximized, what is total spending on spaghetti?
- 6) Continuing on from the last question, what's the utility maximizing amount of tacos?
- 7) What is spending on tacos?
- 8) Set income and prices to their baseline values. The consumer wins the lottery, which increases her income to \$220. What's the new demand (utility maximizing amount) for spaghetti?
- 9) Continuing on from the last question, what's the new demand for tacos?
- 10) Set income and prices to their baseline values, and set consumption at the utility maximizing values. The price of spaghetti now falls to \$2.50 per plate. What's the new best amount of spaghetti? (Compare to 4.)
- 11) Continuing on from the last question, what's the new utility maximizing amount of tacos? (Compare to 6.)