

# 33 | DISCOUNTED PRESENT VALUE

Purpose: To illustrate the idea of discounted present value with computations of the value of payments to be received in the future at different rates of interest. To use discounted present value to determine the profitability of investments.

Computer file: **pv98.xls**

Instructions and background information:

The spreadsheet for this problem set is a kind of calculator that will help you compute the discounted present value of a series of payments and costs.

The first three spreadsheet columns show information on payments and costs associated with an asset in several years. The relationship upon which all the calculations are based is that the present value of  $P(t)$  dollars to be received  $t$  years hence is equal to  $[P(t)/(1+i)^t]$ , where  $i$  is the rate of interest expressed as a decimal.

Column A lists the years payments are to be received or costs incurred. Year 0 is today, year 1 is one year from today (one year hence, in economist's jargon), etc.

Column B shows the amounts of payments (in dollars) received by the owner of a hypothetical asset in respective years. On "wake up" this column has \$100 in rows 3 through 7. You can enter any numbers you wish in this column. Hitting the "Reset to Baseline" button will clear the values from this column.

Each entry in column C shows the discounted present value (DPV) of the payment just to the left in column B. E.g., the DPV of \$100.00 to be received 4 years in the future if the interest rate is 6% is \$79.21 [cell C7]. [ $\$79.21 = \$100/((1 + .06)^4)$ ] In economist's lingo this is called "The present value of \$100 four years hence at 6 percent."

Cell E3 shows the current interest rate. On "wake up"  $i = .06$  or 6%. To change the interest rate, select cell E3 and enter the rate of interest either as a percent or a decimal. For example, to change the interest rate to 10 percent, enter either .10 or 10%.

Many assets, and the one shown is an example, give the owner a series of payments in many years in the future, not just a single payment. Cell F3 shows the discounted present value of the series of payments in column B. That is, it is the sum of the values in column C. The value in cell F3 is the present value of the asset.

The graph is a bar chart showing the present value of each of the payments in the respective years. You should experiment by choosing some different values for the payments and/or inserting new ones, and seeing what happens to the present values in the graph. You should also experiment by choosing some different values for the rate of interest. Be sure you understand why an increase in the rate of interest must decrease the present value of a payment to be received in the future.

Here are some things to watch for and learn as you do the problems:

- 1) For a payment to be received in the future there is an inverse relationship between the rate of interest and the present value of the payment. Higher interest rates always lead to lower discounted values.
- 2) You should buy an asset if the discounted present value of the benefits (payments) is greater than the discounted present value of the costs.
- 3) The DPV of an asset is the amount of money you would have to put on deposit (lend) today at the current interest rate to be able to withdraw (receive) the payments in the future.

Here are some hints to help you get the answers quicker:

- 1) In these problems all future payments and costs are assumed to take place at the *end* of the period in question. For example, in question 1, enter the value of the payment in cell B6, next to year 3. In question 5, enter the first interest payment from the bond next to year 1. The payment for year 0 should be blank.
- 2) In question 5, enter the first interest payment from the bond next to year 1. The payment for year 0 should be blank.
- 3) In question 7, the first return from owning the machine occurs in year 1, that is, *after* one year. However, the cost of the machine occurs today, year 0. In all, you get 6 years of returns from owning the machine. Remember to add the scrap value of the machine to the returns in last year.
- 4) In question 17 you get the payoff from the first year of energy saving in year 1. (Year 0 should be blank or zero.) Then enter \$40 through year 10.

Try this exercise to get some insight into the value of your college education.

Suppose you are considering buying a college education (majoring in Economics?) which you expect to give you considerably higher incomes in future years than if you had just a high school education. Your income is expected to be

\$15,000 per year higher with the college education. Your working life will start 3 years from now, say, and last for 40 years.

What is the value of the *returns* to the college education if interest rates are 3%, 6%, and 10%?

What's your own personal estimate of the *cost* of your college education? Assume a 4 year college career. Add a) and b) together, discounting the future costs if you wish to get the total cost.

- a) Opportunity cost in lost wages.
- b) Out of pocket costs of tuition, books, supplies, etc. (but not housing or meals).

Is the investment going to be worth it?

## DISCOUNTED PRESENT VALUE

### Questions

Note: The values of payments and costs you need to answer the questions are generated randomly by the spreadsheet when you open it. You will need to look at the questions in the spreadsheet to obtain the values.

Your Uncle Harry says he will give you \$\_\_\_\_\_ three years from now to help you pay for a new car.

- 1) When the interest rate is 6%, what's the present value of his gift to you?
- 2) What's the present value if the rate of interest is 3%?
  
- 3A) Some people set up trusts or make other financial arrangements to pay for their kid's college education. Martha Klutz knows that when her daughter goes to college the expenses will be \$\_\_\_\_\_ per year for four years. [Go on to 3B.]

Her daughter is now a high school sophomore, so the first bill will arrive 3 years from now. The rate of interest is 8%.

- 3B) How large a trust must Klutz set up today so that her daughter's college education is just paid for in the future?
  
- 4) How large would the trust have to be if the rate of interest was 3% instead of 8%?
  
- 5A) A bond is a promise to pay its holder fixed amounts of money at specified times in the future. Suppose you have a chance to buy a bond that pays the following amounts in interest and principal: [Go on to 5B.]
  
- 5B) After one year you will receive a series of annual interest payments of \$\_\_\_\_\_ for 10 years. At the time you receive the last interest payment you will receive the principal value of the bond of \$20,000. [Go on to 5C.]
  
- 5C) If the rate of interest is 7%, what's the maximum amount you would be willing to pay for the bond?
- 6) If the rate of interest were 10%, what would be the price of the bond?
  
- 7A) You are in the pizza business and want to buy a new pizza making machine. The machine costs \$\_\_\_\_\_ today. At the end of each year you own the machine it will give you returns of \$\_\_\_\_\_ after paying for maintenance and repairs. [Go on to 7B.]

After you have owned the machine for 6 years you expect to sell it for scrap for \$1,000. (You scrap it at the same time you get the last \$\_\_\_\_\_ return.) What is the present value of the RETURNS at

- 7B) 3%?
- 8) 7%?
- 9) 11%?

What's the present value of the COST of the machine if the interest rate is:

- 10) 3%?
- 11) 7%?
- 12) 11%?

Should you buy the machine at each of the following interest rates? [Enter Yes or No.]

- 13) 3%?
- 14) 7%?
- 15) 11%?

16A) You win a big prize in the Super Lottery. You receive \$\_\_\_\_\_ today, and then \$\_\_\_\_\_ for each of the succeeding 9 years. [Go on to 16B.]

16B) How much would the state have to put on deposit today to just pay you off if the state's interest rate is 7%?

17A) You go to Big George's to buy a new refrigerator for your apartment. The store offers two models, regular (costs \$\_\_\_\_\_) and energy efficient (costs \$\_\_\_\_\_). Both will last for 10 years. [Go on to 17B.]

The energy efficient refrigerator will save you \$40 per year in electricity bills.

17B) What's the difference in the initial purchase price between the energy efficient model and the regular model?

Continuing on from the previous question assume the rate of interest is 12%.

- 18) What's the present value of the energy saving from the energy efficient model?
- 19) Should you buy the energy efficient model? [Yes or No]