

# 34 | INTERNAL RATE OF RETURN

Purpose: To illustrate how to compute the internal rate of return on an investment or series of future payments and costs. To show how to use the internal rate of return to make investment decisions.

Computer files: irr98.xls

Instructions and background information:

The first two spreadsheet columns show information on payments and costs associated with an asset in several years.

Column A lists the years payments are received or costs incurred. Year 0 is today, year 1 is one year from today, etc. Column B shows the dollar payments received or costs incurred on a hypothetical asset in respective years. On "wake up" this column has -9500, 500, 500, and 10500 in the topmost cells. Costs are entered as negative numbers, so the asset represented is one that costs \$9,500 today and pays \$500, \$500, and \$10,500 in each of the next 3 years, successively.

Cell D58 shows the internal rate of return (IRR) on the asset whose cash flow values are listed in the cells in column B. The IRR is the interest rate that makes the sum of the present values of the payments equal to zero. If  $i$  is a rate of interest, the IRR is the value of  $i$  that makes this equation true:

$$0 = R(0) + R(1)/(1+i) + R(2)/(1+i)^2 + \dots + R(T)/(1+i)^T,$$

where  $R(t)$  is the dollar return  $t$  years from now.

Another way to define the internal rate of return is as the interest rate that makes the net discounted present value of an asset equal to zero. Or, equivalently, as the interest rate that makes the discounted present value of the costs equal to the discounted present value of the returns. All of these ways of saying it come down to the same thing.

Of course, solving the equation for the internal rate of return may not be an easy task. In fact, for  $T$  larger than 5 there is, in general, no way to do it using the ordinary methods of algebra, so various methods of approximation are used. Excel finds the IRR by trial and error, and the way the computer you are working on is set up, it gives up if it doesn't get close to the answer after 20 tries.

To help Excel out, you can choose a value in cell F58, labeled BEST GUESS. It is the interest rate Excel starts with as it tries to find the IRR by trial and error. When the spreadsheet wakes up the BEST GUESS for the internal rate of return is 10%.

You can change BEST GUESS and all of the values in column B to answer the questions. All other cells in the sheet are locked.

The graph shows the net discounted present value of the asset you are working with at interest rates from 0 to 100 percent in 1 percent intervals. The internal rate of return can be estimated by seeing where the graph crosses zero on the vertical axis. You can use the graph to make a good BEST GUESS.

Experiment with some different values for returns and costs in different years and see what happens to the internal rate of return.

**NOTE:** The numbers in column B must contain at least one positive value and one negative value for the procedure to work. In addition, Excel has a quirk in the IRR computation that makes it treat blanks and zeros differently. If you want a zero payment or cost in an intermediate year of your asset, be sure to enter "0" in the cell, don't just leave it blank.

Here are some things to watch for and learn as you do the problems:

- 1) The answers to questions 1, 3, 4, and 8 are percentages -- rates of return. Enter your answer in the answer sheet either as a decimal, or with a percent sign (%). For example, if your answer is 7.52 percent, enter either .0752 or 7.52%.
- 2) The IRR is the average annual rate of return on an asset.
- 3) Generally, you can profit by buying assets that have an IRR greater than the rate of interest you have to pay to borrow money. The idea is that if IRR exceeds the rate of interest you can use the income from the asset to pay off both principal and interest from the loan and still have something left over.
- 4) Higher internal rates of return occur when costs go down or returns rise.

Here are some hints to help you get the answers quicker:

- 1) In question 3, remember to deduct the \$3,000 down payment from the cost of the car. Enter the loan amount as a negative number in cell B58. Then enter the 5 payments you must make in succeeding years.

- 2) Question 4 is very much like Question 7 in the problem set on discounted present value. Remember to add the scrap value of the machine to the last return that you enter in year 7.
- 3) In Question 8 on the lottery, you sell the winning ticket to someone else. That means the buyer's return in period zero is a negative amount that is the payment to you, plus the immediate payment when the ticket is cashed. The remaining 9 payments are entered as positive numbers.

Try this exercise to get an idea how economists use the IRR to measure the value of a college education.

Suppose your sister or brother is about to finish high school and is considering buying a college education (majoring in Economics?) which he/she expects to yield considerably higher incomes in future years than if he/she had just a high school education. Income will be \$18,000 per year higher with the college education. If your sibling goes to college, his/her working life will start 5 years from now, say, and last for 40 years. Suppose the costs in each of the 4 years of college are \$12000. The first year of costs occurs one year from now, so enter it next to Year 1. And remember to put a minus sign in front of the costs. Then enter the remaining costs and benefits next to the correct years. (The last income payment should be entered in year 44.)

What is the internal rate of return on a college education for your sibling?

In the computer exercise on discounted present value you gave an estimate of your real life estimated costs of your college education. Divide that total cost by 4 to get the estimated annual cost, and enter the costs in the spreadsheet. What is the expected internal rate of return on your investment if the annual income gain estimates of \$15,000 per year for 40 years are correct?

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### Questions

Note: The values of payments and costs you need to answer the questions are generated randomly by the spreadsheet when you open it. You will need to look at the questions in the spreadsheet to obtain the values.

- 1A) A government bond costs \$\_\_\_\_\_ today, and will pay you \$\_\_\_\_\_ in interest in each of the next 3 years. The first payment occurs one year from now. [Go on to 1B.]

At the time you receive the last interest payment you also receive the bond's principal or face value of \$10,000.

- 1B) What's the internal rate of return on the bond?

Continuing on from the last question:

- 2) If you can borrow money at 4% should you buy the bond? [Yes or No.]

- 3A) You are considering buying a car for \$\_\_\_\_\_, and need to borrow the entire amount less a down payment of \$3,000. You take out a loan that you will pay back in five annual installments of \$\_\_\_\_\_. [Go on to 3B.]

- 3B) The first loan payment is due one year from the time you get the car, the second two years hence, etc. The salesman tells you that the loan payments include both principal and interest. [Go on to 3C.]

You will own the car after you make the last payment.

- 3C) What rate of interest are you paying on the loan?

- 4A) You are in the pizza business and want to buy a new pizza making machine. The machine costs \$\_\_\_\_\_ today. At the end of each year you own the machine it will give you returns of \$\_\_\_\_\_ after paying for maintenance and repairs. [Go on to 4B.]

After you have owned the machine for 6 years you expect to sell it for scrap for \$1,000. (You scrap it at the same time you get the last \$\_\_\_\_\_ return.)

- 4B) What is the internal rate of return on the machine?

Consider again the pizza machine in the last question. Would you buy the machine if you could borrow money at the following rates? [Answer Yes or No.]

- 5) 5%  
6) 12%  
7) 20%

8A) You win a big prize in the Super Lottery. You will receive \$\_\_\_\_\_ today, and then \$\_\_\_\_\_ for each of the succeeding 9 years. [Go on to 8B.]

8B) There are private parties who will buy your winning ticket for a lump sum payable today. In fact, one such person offers to pay you one half of your total winnings (10 times \$\_\_\_\_\_). [Go on to 8C.]

Suppose you sell your ticket for the offered price.

8C) What rate of return is the buyer receiving on the investment?