A man kicks the 150-g ball such that it leaves the ground at an angle of 60° and strikes the ground at the same elevation a distance of 12 m away. Determine the impulse of his foot on the ball at A. Neglect the impulse caused by the ball’s weight while it’s being kicked.

**SOLUTION**

**Kinematics.** Consider the vertical motion of the ball where

\( (s_0)_y = s_y = 0, (v_0)_y = v \sin 60° \uparrow \) and \( a_y = 9.81 \text{ m/s}^2 \downarrow \),

\[
\begin{align*}
  (\uparrow) \quad & s_y = (s_0)_y + (v_0)_yt + \frac{1}{2}a_yt^2; \quad 0 = 0 + v \sin 60°t + \frac{1}{2}(-9.81)t^2 \\
  & t(v \sin 60° - 4.905t) = 0
\end{align*}
\]

Since \( t \neq 0 \), then

\[
  v \sin 60° - 4.905t = 0
\]

\[
  t = \frac{0.1766}{v}
\]

Then, consider the horizontal motion where \( (v_0)_x = v \cos 60° \), and \( (s_0)_x = 0 \),

\[
\begin{align*}
  (\leftrightarrow) \quad & s_x = (s_0)_x + (v_0)_xt; \quad 12 = 0 + v \cos 60°t \\
  & t = \frac{24}{v}
\end{align*}
\]

Equating Eqs. (1) and (2)

\[
0.1766 v = \frac{24}{v}
\]

\[
v = 11.66 \text{ m/s}
\]

**Principle of Impulse and Momentum.**

\[
(mv_1 + \sum \int_{t_1}^{t_2} F dt) = mv_2
\]

\[
0 + I = 0.15 \times (11.66)
\]

\[
I = 1.749 \text{ N} \cdot \text{s} = 1.75 \text{ N} \cdot \text{s}
\]

**Ans.**

\[
v = 1.75 \text{ N} \cdot \text{s}
\]
A 20-lb block slides down a 30° inclined plane with an initial velocity of 2 ft/s. Determine the velocity of the block in 3 s if the coefficient of kinetic friction between the block and the plane is $\mu_k = 0.25$.

**SOLUTION**

\[
\begin{align*}
(+) & \quad m(v_x) + \sum F_y = m(v_y) \\
0 + N(3) - 20 \cos 30°(3) &= 0 \quad N = 17.32 \text{ lb} \\
(+) & \quad m(v_y) + \sum F_x = m(v_x) \\
\frac{20}{32.2}(2) + 20 \sin 30°(3) - 0.25(17.32)(3) &= \frac{20}{32.2} v \\
\Rightarrow v &= 29.4 \text{ ft/s} \quad \text{Ans.}
\end{align*}
\]
15–3.

The uniform beam has a weight of 5000 lb. Determine the average tension in each of the two cables $AB$ and $AC$ if the beam is given an upward speed of 8 ft/s in 1.5 s starting from rest. Neglect the mass of the cables.

SOLUTION

$$2 \left( \frac{1000}{3600} \right) = 6.944 \text{ m/s}$$

System:

$$mv_1 + \sum F \, dt = mv_2$$

$$[0 + 0] + F(35) = (50 + 75)(10^3)(6.944)$$

$$F = 24.8 \text{ kN}$$

Barge:

$$mv_1 + \sum F \, dt = mv_2$$

$$0 + T(35) = (75)(10^3)(6.944)$$

$$T = 14.881 = 14.9 \text{ kN}$$

Also, using this result for $T$,

Tugboat:

$$mv_1 + \sum F \, dt = mv_2$$


$$F = 24.8 \text{ kN}$$

Ans:

$$F = 24.8 \text{ kN}$$
15–4.

Each of the cables can sustain a maximum tension of 5000 lb. If the uniform beam has a weight of 5000 lb, determine the shortest time possible to lift the beam with a speed of 10 ft/s starting from rest.

**SOLUTION**

\[ + \uparrow \sum F_y = 0; \quad P_{\text{max}} - 2 \left( \frac{4}{5} \right) (5000) = 0 \]

\[ P_{\text{max}} = 8000 \text{ lb} \]

\[ (+ \uparrow) \quad mv_1 + \sum \int F \, dt = mv_2 \]

\[ 0 + 8000(t) - 5000(t) = \frac{5000}{32.2} \]

\[ t = 0.518 \text{ s} \]

**Ans:**

\[ t = 0.518 \text{ s} \]
15–5.

A hockey puck is traveling to the left with a velocity of \( v_1 = 10 \text{ m/s} \) when it is struck by a hockey stick and given a velocity of \( v_2 = 20 \text{ m/s} \) as shown. Determine the magnitude of the net impulse exerted by the hockey stick on the puck. The puck has a mass of 0.2 kg.

**SOLUTION**

\[ v_1 = -10 \text{ m/s} \]

\[ v_2 = [20 \cos 40^\circ \hat{i} + 20 \sin 40^\circ \hat{j}] \text{ m/s} \]

\[ I = m \Delta v = (0.2) \left[ 20 \cos 40^\circ - (-10) \right] \hat{i} + 20 \sin 40^\circ \hat{j} \]

\[ = [5.0642 \hat{i} + 2.5712 \hat{j}] \text{ kg} \cdot \text{m/s} \]

\[ I = \sqrt{(5.0642)^2 + (2.5712)^2} \]

\[ = 5.6795 = 5.68 \text{ kg} \cdot \text{m/s} \]

**Ans.**

\( I = 5.68 \text{ N} \cdot \text{s} \)
A train consists of a 50-Mg engine and three cars, each having a mass of 30 Mg. If it takes 80 s for the train to increase its speed uniformly to 40 km/h, starting from rest, determine the force $T$ developed at the coupling between the engine $E$ and the first car $A$. The wheels of the engine provide a resultant frictional tractive force $F$ which gives the train forward motion, whereas the car wheels roll freely. Also, determine $F$ acting on the engine wheels.

**SOLUTION**

$(v_x)_2 = 40 \text{ km/h} = 11.11 \text{ m/s}$

Entire train:

\[
\begin{align*}
\frac{1}{2}m(v_x)_1 + \int F_x \, dt &= m(v_x)_2 \\
0 + F(80) &= [50 + 3(30)](10^3)(11.11) \\
F &= 19.4 \text{ kN}
\end{align*}
\]

Three cars:

\[
\begin{align*}
\frac{1}{2}m(v_x)_1 + \int F_x \, dt &= m(v_x)_2 \\
0 + T(80) &= 3(30)(10^3)(11.11) \\
T &= 12.5 \text{ kN}
\end{align*}
\]
15–7.

Crates $A$ and $B$ weigh 100 lb and 50 lb, respectively. If they start from rest, determine their speed when $t = 5$ s. Also, find the force exerted by crate $A$ on crate $B$ during the motion. The coefficient of kinetic friction between the crates and the ground is $\mu_k = 0.25$.

**SOLUTION**

**Free-Body Diagram:** The free-body diagram of crates $A$ and $B$ are shown in Figs. $a$ and $b$, respectively. The frictional force acting on each crate is $(F_f)_A = \mu_k N_A = 0.25 N_A$ and $(F_f)_B = \mu_k N_B = 0.25 N_B$.

**Principle of Impulse and Momentum:** Referring to Fig. $a$,

$$ (+ \uparrow) \quad m(v_1)_x + \int_{t_1}^{t_2} F_x dt = m(v_2)_x $$

$$ \frac{100}{32.2}(0) + N_A(5) - 100(5) = \frac{100}{32.2} (0) $$

$N_A = 100$ lb

$$ (\downarrow) \quad m(v_1)_x + \int_{t_1}^{t_2} F_x dt = m(v_2)_x $$

$$ \frac{100}{32.2}(0) + 50(5) - 0.25(100)(5) - F_{AB}(5) = \frac{100}{32.2} v $$

$$ v = 40.25 - 1.61 F_{AB} \quad (1) $$

By considering Fig. $b$,

$$ (+ \uparrow) \quad m(v_1)_y + \int_{t_1}^{t_2} F_y dt = m(v_2)_y $$

$$ \frac{50}{32.2}(0) + N_B(5) - 50(5) = \frac{50}{32.2} (0) $$

$N_B = 50$ lb

$$ (\downarrow) \quad m(v_1)_y + \int_{t_1}^{t_2} F_y dt = m(v_2)_y $$

$$ \frac{50}{32.2}(0) + F_{AB}(5) - 0.25(50)(5) = \frac{50}{32.2} v $$

$$ v = 3.22 F_{AB} - 40.25 \quad (2) $$

Solving Eqs. (1) and (2) yields

$$ F_{AB} = 16.67 \text{ lb} = 16.7 \text{ lb} \quad v = 13.42 \text{ ft/s} = 13.4 \text{ ft/s} \quad \text{Ans.} $$

Ans:

$F_{AB} = 16.7 \text{ lb}$

$v = 13.4 \text{ ft/s}$
The automobile has a weight of 2700 lb and is traveling forward at 4 ft/s when it crashes into the wall. If the impact occurs in 0.06 s, determine the average impulsive force acting on the car. Assume the brakes are not applied. If the coefficient of kinetic friction between the wheels and the pavement is $\mu_k = 0.3$, calculate the impulsive force on the wall if the brakes were applied during the crash. The brakes are applied to all four wheels so that all the wheels slip.

**SOLUTION**

Impulse is area under curve for hole cavity.

$$I = \int F \, dt = 4(3) + \frac{1}{2} (8 + 4)(6 - 3) + \frac{1}{2} (8)(10 - 6)$$

$$= 46 \text{ lb} \cdot \text{s}$$

For starred cavity:

$$I = \int F \, dt = 6(8) + \frac{1}{2} (6)(10 - 8)$$

$$= 54 \text{ lb} \cdot \text{s}$$
The 200-kg crate rests on the ground for which the coefficients of static and kinetic friction are \( \mu_s = 0.5 \) and \( \mu_k = 0.4 \), respectively. The winch delivers a horizontal towing force \( T \) to its cable at \( A \) which varies as shown in the graph. Determine the speed of the crate when \( t = 4 \) s. Originally the tension in the cable is zero. \textit{Hint:} First determine the force needed to begin moving the crate.

**SOLUTION**

**Equilibrium.** The time required to move the crate can be determined by considering the equilibrium of the crate. Since the crate is required to be on the verge of sliding, \( F_f = \mu_s N = 0.5 \) N. Referring to the FBD of the crate, Fig. \( a \),

\[
\begin{align*}
\sum F_y &= 0; \\
N - 200(9.81) &= 0 \\
N &= 1962 \text{ N}
\end{align*}
\]

\[
\begin{align*}
\sum F_x &= 0; \\
2(400 t^2) - 0.5(1962) &= 0 \\
t &= 1.5037 \text{ s}
\end{align*}
\]

**Principle of Impulse and Momentum.** Since the crate is sliding, \( F_f = \mu_k N = 0.4(1962) = 784.8 \) N. Referring to the FBD of the crate, Fig. \( a \)

\[
\begin{align*}
\sum m(v_x) + \int_{t_1}^{t_2} F_x \, dt &= m(v_x) \\
0 + 2 \int_{1.5037}^{4} 400 t^2 \, dt - 784.8(4 - 1.5037) &= 200v \\
v &= 6.621 \text{ m/s} = 6.62 \text{ m/s}
\end{align*}
\]

**Ans:**

\( v = 6.62 \text{ m/s} \)
15–10.

The 50-kg crate is pulled by the constant force $P$. If the crate starts from rest and achieves a speed of 10 m/s in 5 s, determine the magnitude of $P$. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.2$.

**SOLUTION**

*Impulse and Momentum Diagram:* The frictional force acting on the crate is $F_f = \mu_k N = 0.2N$.

*Principle of Impulse and Momentum:*

\[
\begin{align*}
(+\uparrow) & \quad m(v_1)_y + \int_{t_1}^{t_2} F_y \, dt = m(v_2)_y \\
& \quad 0 + N(5) + P(5) \sin 30^\circ - 50(9.81)(5) = 0 \\
& \quad N = 490.5 - 0.5P \quad \text{(1)} \\
(-\downarrow) & \quad m(v_1)_x + \int_{t_1}^{t_2} F_x \, dt = m(v_2)_x \\
& \quad 50(0) + P(5) \cos 30^\circ - 0.2N(5) = 50(10) \\
& \quad 4.3301P - N = 500 \quad \text{(2)}
\end{align*}
\]

Solving Eqs. (1) and (2), yields

\[
\begin{align*}
N &= 387.97 \text{ N} \\
P &= 205 \text{ N} \\
\text{Ans.}
\end{align*}
\]

\[\text{Ans:} \quad P = 205 \text{ N}\]
15–11.

During operation the jack hammer strikes the concrete surface with a force which is indicated in the graph. To achieve this the 2-kg spike $S$ is fired into the surface at 90 m/s. Determine the speed of the spike just after rebounding.

**SOLUTION**

*Principle of Impulse and Momentum.* The impulse of the force $F$ is equal to the area under the $F$–$t$ graph. Referring to the FBD of the spike, Fig. $a$

\[
(+) \quad m(v_y)_1 + \sum_{t_1}^{t_2} F_y \, dt = m(v_y)_2
\]

\[
2(-90) + \frac{1}{2} [0.4(10^{-3})][1500(10^3)] = 2v
\]

\[
v = 60.0 \text{ m/s } \uparrow \quad \text{Ans.}
\]

Ans: $v = 60.0 \text{ m/s}$
For a short period of time, the frictional driving force acting on the wheels of the 2.5-Mg van is $F_D = (600t^2)$ N, where $t$ is in seconds. If the van has a speed of 20 km/h when $t = 0$, determine its speed when $t = 5$ s.

**SOLUTION**

*Principle of Impulse and Momentum:* The initial speed of the van is $v_1 = \left[ \frac{20(10^3) \text{m}}{\text{h}} \right]$.

Referring to the free-body diagram of the van shown in Fig. a,

\[ m(v_1)_x + \sum F_x dt = m(v_2)_x \]

\[ 2500(5.556) + \int_0^{5s} 600t^2 \, dt = 2500 \, v_2 \]

\[ v_2 = 15.6 \text{ m/s} \]

Ans: $v_2 = 15.6 \text{ m/s}$

The 2.5-Mg van is traveling with a speed of 100 km/h when the brakes are applied and all four wheels lock. If the speed decreases to 40 km/h in 5 s, determine the coefficient of kinetic friction between the tires and the road.

**SOLUTION**

**Free-Body Diagram:** The free-body diagram of the van is shown in Fig. a. The frictional force is $F_f = \mu_k N$ since all the wheels of the van are locked and will cause the van to slide.

**Principle of Impulse and Momentum:** The initial and final speeds of the van are $v_1 = \left[100 \left(\frac{10^3}{3600}\right) \frac{m}{h}\right] \frac{1 h}{3600 s} = 27.78 \text{ m/s}$ and $v_2 = \left[40 \left(\frac{10^3}{3600}\right) \frac{m}{h}\right] \frac{1 h}{3600 s} = 11.11 \text{ m/s}$. Referring to Fig. a,

\[
\begin{align*}
(+ \uparrow) & \quad m(v_1)_y + \sum \int_{v_1}^{v_2} F_x dt = m(v_2)_y \\
2500(0) + N(5) - 2500(9.81)(5) &= 2500(0) \\
N &= 24525 \text{ N}
\end{align*}
\]

\[
\begin{align*}
(- \downarrow) & \quad m(v_1)_x + \sum \int_{v_1}^{v_2} F_y dt = m(v_2)_x \\
2500(27.78) + [-\mu_k(24525)(5)] &= 2500(11.1) \\
\mu_k &= 0.340 \quad \text{Ans.}
\end{align*}
\]
A tankcar has a mass of 20 Mg and is freely rolling to the right with a speed of 0.75 m/s. If it strikes the barrier, determine the horizontal impulse needed to stop the car if the spring in the bumper $B$ has a stiffness (a) $k \rightarrow \infty$ (bumper is rigid), and (b) $k = 15$ kN/m.

**SOLUTION**

a) b) \( \hat{\rightarrow} \) \( mv_1 + \sum F \, dt = mv_2 \)

\[
20(10^3)(0.75) - \int F \, dt = 0
\]

\[
\int F \, dt = 15 \text{ kN} \cdot \text{s}
\]

Ans.

The impulse is the same for both cases. For the spring having a stiffness $k = 15$ kN/m, the impulse is applied over a longer period of time than for $k \rightarrow \infty$.

**Ans:**

\( I = 15 \text{ kN} \cdot \text{s} \) in both cases.
The motor, \( M \), pulls on the cable with a force \( F = (10t^2 + 300) \) N, where \( t \) is in seconds. If the 100 kg crate is originally at rest at \( t = 0 \), determine its speed when \( t = 4 \) s. Neglect the mass of the cable and pulleys. Hint: First find the time needed to begin lifting the crate.

**Solution**

**Principle of Impulse and Momentum.** The crate will only move when \( 3(10t^2 + 300) = 100(9.81) \). Thus, this instant is \( t = 1.6432 \) s. Referring to the FBD of the crate, Fig. a,

\[
(+\uparrow) \quad m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2
\]

\[
0 + \int_{1.6432}^{4} 3(10t^2 + 300) \, dt - 100(9.81)(4 - 1.6432) = 100v
\]

\[
3\left(\frac{10t^3}{3} + 300t\right)_{1.6432}^{4} - 2312.05 = 100v
\]

\[
v = 4.047 \, \text{m/s} = 4.05 \, \text{m/s} \uparrow
\]

Ans:

\( v = 4.05 \, \text{m/s} \)
15–16.

The choice of a seating material for moving vehicles depends upon its ability to resist shock and vibration. From the data shown in the graphs, determine the impulses created by a falling weight onto a sample of urethane foam and CONFOR foam.

**SOLUTION**

CONFOR foam:

\[
I_c = \int F \, dt = \left[ \frac{1}{2}(2)(0.5) + \frac{1}{2}(0.5 + 0.8)(7 - 2) + \frac{1}{2}(0.8)(14 - 7) \right] \times 10^{-3}
\]

\[
= 6.55 \text{ N} \cdot \text{ms}
\]

Urethane foam:

\[
I_u = \int F \, dt = \left[ \frac{1}{2}(4)(0.3) + \frac{1}{2}(1.2 + 0.3)(7 - 4) + \frac{1}{2}(1.2 + 0.4)(10 - 7) + \right.
\]

\[
\left. \frac{1}{2} (14 - 10)(0.4) \right] \times 10^{-3}
\]

\[
= 6.05 \text{ N} \cdot \text{ms}
\]

Ans:

\[
I_c = 6.55 \text{ N} \cdot \text{ms}
\]

\[
I_u = 6.05 \text{ N} \cdot \text{ms}
\]
15–17.

The towing force acting on the 400-kg safe varies as shown on the graph. Determine its speed, starting from rest, when \( t = 8 \text{ s} \). How far has it traveled during this time?

**SOLUTION**

*Principle of Impulse and Momentum.* The FBD of the safe is shown in Fig. a.

For \( 0 \leq t < 5 \text{ s} \), \( F = \frac{600}{5}t = 120t \).

\[
\begin{align*}
(\leftrightarrow) \quad & m(v_x)_1 + \sum F_x \ dt = m(v_x)_2 \\
& 0 + \int_0^t 120t \ dt = 400v \\
& v = \{0.15t^2\} \text{ m/s}
\end{align*}
\]

At \( t = 5 \text{ s} \),

\[ v = 0.15(5^2) = 3.75 \text{ m/s} \]

For \( 5 \text{ s} < t \leq 8 \text{ s} \), \( \frac{F - 600}{t - 5} = \frac{750 - 600}{8 - 5} = F = 50t + 350 \). Here,

\( (v_x)_1 = 3.75 \text{ m/s} \) and \( t_1 = 5 \text{ s} \).

\[
\begin{align*}
(\leftrightarrow) \quad & m(v_x)_1 + \sum F_x \ dt = m(v_x)_2 \\
& 400(3.75) + \int_{5}^{t} (50t + 350) \ dt = 400v \\
& v = \{0.0625t^2 + 0.875t - 2.1875\} \text{ m/s}
\end{align*}
\]

At \( t = 8 \text{ s} \),

\[ v = 0.0625(8^2) + 0.875(8) - 2.1875 = 8.8125 \text{ m/s} = 8.81 \text{ m/s} \quad \text{Ans.} \]
Kinematics. The displacement of the safe can be determined by integrating
$ds = v \, dt$. For $0 \leq t < 5$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t 0.15t^2 \, dt$$

At $t = 5$ s,

$$s = 0.05t^3 = 6.25 \text{ m}$$

For $5 < t \leq 8$ s, the initial condition is $s = 6.25$ m at $t = 5$ s.

$$\int_{6.25}^s ds = \int_{5}^t (0.0625t^2 + 0.875t - 2.1875) \, dt$$

At $t = 8$ s,

$$s = 0.02083(8^3) + 0.4375(8^2) - 2.1875(8) + 3.6458$$

$$= 24.8125 \text{ m} = 24.8 \text{ m} \quad \text{Ans.}$$

Ans:

$v = 8.81 \text{ m/s}$

$s = 24.8 \text{ m}$
The motor exerts a force $F$ on the 40-kg crate as shown in the graph. Determine the speed of the crate when $t = 3 \text{ s}$ and when $t = 6 \text{ s}$. When $t = 0$, the crate is moving downward at 10 m/s.

**SOLUTION**

*Principle of Impulse and Momentum.* The impulse of force $F$ is equal to the area under the $F$–$t$ graph. At $t = 3 \text{ s}$, $\frac{F - 150}{3 - 0} = \frac{450 - 150}{6 - 0} F = 300 \text{ N}$. Referring to the FBD of the crate, Fig. a

$$m(v_y)_1 + \sum F_y \ dt = m(v_y)_2$$

$$40(-10) + \frac{1}{2}(150 + 300)(3) - 40(9.81)(3) = 40v$$

$$v = -5.68 \text{ m/s} = 5.68 \text{ m/s ↓}$$

Ans.

At $t = 6 \text{ s}$,

$$m(v_y)_1 + \sum F_y \ dt = m(v_y)_2$$

$$40(-10) + \frac{1}{2}(450 + 150)(6) - 40(9.81)(6) = 40v$$

$$v = 21.14 \text{ m/s} = 21.1 \text{ m/s ↑}$$

Ans.

$v|_{t=3\text{ s}} = 5.68 \text{ m/s ↓}$

$v|_{t=6\text{ s}} = 21.1 \text{ m/s ↑}$
15–19.

The 30-kg slider block is moving to the left with a speed of 5 m/s when it is acted upon by the forces $F_1$ and $F_2$. If these loadings vary in the manner shown on the graph, determine the speed of the block at $t = 6$ s. Neglect friction and the mass of the pulleys and cords.

**SOLUTION**

*Principle of Impulse and Momentum.* The impulses produced by $F_1$ and $F_2$ are equal to the area under the respective $F$–$t$ graph. Referring to the FBD of the block Fig. a,

\[
\begin{align*}
(\pm) \quad & m(v_x)_1 + \sum F_x \, dx = m(v_x)_2 \\
& -30(5) + 4 \left[ 10(2) + \frac{1}{2}(10 + 30)(4 - 2) + 30(6 - 4) \right] \\
& + \left[ -40(4) - \frac{1}{2}(10 + 40)(6 - 4) \right] = 30v \\
& v = 4.00 \text{ m/s} \to \text{ Ans.}
\end{align*}
\]

Ans:

\[ v = 4.00 \text{ m/s} \]
*15–20.

The 200-lb cabinet is subjected to the force $F = 20(t + 1) \text{ lb}$ where $t$ is in seconds. If the cabinet is initially moving to the left with a velocity of 20 ft/s, determine its speed when $t = 5 \text{ s}$. Neglect the size of the rollers.

**SOLUTION**

**Principle of Impulse and Momentum.** Referring to the FBD of the cabinet, Fig. a

\[
(\sum) \quad m(v_x)_1 + \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2
\]

\[
\frac{200}{32.2}(-20) + 20 \cos 30^\circ \int_0^5 (t + 1) \, dt = \frac{200}{32.2} v
\]

\[
v = 28.80 \text{ ft/s} = 28.8 \text{ ft/s} \rightarrow \text{ Ans.}
\]
If it takes 35 s for the 50-Mg tugboat to increase its speed uniformly to 25 km/h, starting from rest, determine the force of the rope on the tugboat. The propeller provides the propulsion force \( F \) which gives the tugboat forward motion, whereas the barge moves freely. Also, determine \( F \) acting on the tugboat. The barge has a mass of 75 Mg.

**SOLUTION**

\[
25 \frac{1000}{3600} = 6.944 \text{ m/s}
\]

**System:**

\[
(\sum \vec{F}) \quad m v_1 + \sum F dt = m v_2
\]

\[
[0 + 0] + F(35) = (50 + 75)(10^3)(6.944)
\]

\[F = 24.8 \text{ kN}\]

**Barge:**

\[
(\sum \vec{F}) \quad m v_1 + \sum F dt = m v_2
\]

\[
0 + T(35) = (75)(10^3)(6.944)
\]

\[T = 14.881 = 14.9 \text{ kN}\]

Also, using this result for \( T \),

**Tugboat:**

\[
(\sum \vec{F}) \quad m v_1 + \sum F dt = m v_2
\]

\[
\]

\[F = 24.8 \text{ kN}\]
The thrust on the 4-Mg rocket sled is shown in the graph. Determine the sleds maximum velocity and the distance the sled travels when $t = 35$ s. Neglect friction.

**SOLUTION**

**Principle of Impulse And Momentum.** The FBD of the rocket sled is shown in Fig. a. For $0 \leq t < 25$ s,

\[
(\uparrow) \quad m(v_x)_1 + \sum \int_{0}^{t} F_x \, dt = m(v_x)_2
\]

\[
\begin{align*}
0 + \int_{0}^{25} 4(10^3) t \, dt &= 4(10^3) v \\
4(10^3) \left( \frac{2}{3} t^2 \right) \bigg|_{0}^{25} &= 4(10^3) v
\end{align*}
\]

\[v = \left\{ \frac{2}{3} t^2 \right\} \text{ m/s} \]

At $t = 25$ s,

\[v = \frac{2}{3} (25)^2 = 83.33 \text{ m/s} \]

For $25$ s $< t < 35$ s, \( \frac{T - 0}{t - 35} = \frac{20(10^3) - 0}{25 - 35} \) or $T = 2(10^3)(35 - t)$.

Here, $(v_x)_1 = 83.33$ m/s and $t_1 = 25$ s.

\[
(\uparrow) \quad m(v_x)_1 + \sum \int_{25}^{t} F_x \, dt = m(v_x)_2
\]

\[
\begin{align*}
4(10^3)(83.33) + \int_{25}^{t} 2(10^3)(35 - t) \, dt &= 4(10^3) v \\
v &= \left\{ -0.25 t^2 + 17.5 t - 197.9167 \right\} \text{ m/s}
\end{align*}
\]

The maximum velocity occurs at $t = 35$ s, Thus,

\[
v_{\text{max}} = -0.25(35^2) + 17.5(35) - 197.9167
\]

\[= 108.33 \text{ m/s} = 108 \text{ m/s} \]

**Ans:**

\[v_{\text{max}} = 108 \text{ m/s} \]
Kinematics. The displacement of the sled can be determined by integrating 
\[ ds = v dt. \]
For \(0 \leq t < 25\) s, the initial condition is \( s = 0 \) at \( t = 0\).

\[
\int_0^t ds = \int_0^t \frac{2}{3} t^2 \, dt \\
s \bigg|_0^t = \frac{2}{3} \left( \frac{t^3}{3} \right) \bigg|_0^t \\
s = \left\{ \frac{4}{15} t^3 \right\} \text{ m}
\]

At \( t = 25\) s,
\[
s = \frac{4}{15} (25)^3 = 833.33 \text{ m}
\]

For \(25 < t \leq 35\) s, the initial condition is \( s = 833.33\) at \( t = 25\) s.

\[
\int_{833.33\text{ m}}^s ds = \int_{25\text{ s}}^{35\text{ s}} (-0.25t^2 + 17.5t - 197.9167) \, dt \\
s \bigg|_{833.33\text{ m}}^s = (-0.08333t^3 + 8.75t^2 - 197.9167t) \bigg|_{25\text{ s}}^{35\text{ s}} \\
s = \left\{ -0.08333t^3 + 8.75t^2 - 197.9167t + 1614.58 \right\} \text{ m}
\]

At \( t = 35\) s,
\[
s = -0.08333(35^3) + 8.75(35^2) - 197.9167(35) + 1614.58 \\
   = 1833.33 \text{ m} = 1833 \text{ m} \quad \text{Ans.}
\]
15–23.

The motor pulls on the cable at $A$ with a force $F = (30 + t^2)$ lb, where $t$ is in seconds. If the 34-lb crate is originally on the ground at $t = 0$, determine its speed in $t = 4$ s. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.

**SOLUTION**

\[30 + t^2 = 34\]

$t = 2$ s for crate to start moving

\[
(+) \quad mv_1 + \sum F dt = mv_2
\]

\[
0 + \int_2^4 (30 + t^2) dt - 34(4 - 2) = \frac{34}{32.2}v_2
\]

\[
\left[30t + \frac{1}{3}t^3\right]_2^4 - 68 = \frac{34}{32.2}v_2
\]

\[v_2 = 10.1 \text{ ft/s} \quad \text{Ans.}\]
*15–24.

The motor pulls on the cable at A with a force $F = (e^{2t})$ lb, where $t$ is in seconds. If the 34-lb crate is originally at rest on the ground at $t = 0$, determine the crate's velocity when $t = 2$ s. Neglect the mass of the cable and pulleys. Hint: First find the time needed to begin lifting the crate.

**SOLUTION**

$$F = e^{2t} = 34$$

$t = 1.7632$ s for crate to start moving

$$mv_1 + \sum F dt = mv_2$$

$$0 \cdot \int_{1.7632}^{2} e^{2t} dt - 34(2 - 1.7632) = \frac{34}{32.2}v_2$$

$$\frac{1}{2} e^{2t} \int_{1.7632}^{2} - 8.0519 = 1.0559 \ v_2$$

$$v_2 = 2.13 \text{ m/s}$$

**Ans:**

$$v_2 = 2.13 \text{ m/s}$$
The balloon has a total mass of 400 kg including the passengers and ballast. The balloon is rising at a constant velocity of 18 km/h when $h = 10$ m. If the man drops the 40-kg sand bag, determine the velocity of the balloon when the bag strikes the ground. Neglect air resistance.

**Solution**

**Kinematic.** When the sand bag is dropped, it will have an upward velocity of $v_0 = \left( \frac{18 \text{ km}}{h} \right) \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) = 5 \text{ m/s} \uparrow$. When the sand bag strikes the ground $s = 10$ m. The time taken for the sand bag to strike the ground can be determined from

$$s = s_0 + v_0 t + \frac{1}{2} a t^2;$$

$$-10 = 0 + 5t + \frac{1}{2}(-9.81) t^2$$

$$4.905t^2 - 5t - 10 = 0$$

Solve for the positive root,

$$t = 2.0258 \text{ s}$$

**Principle of Impulse and Momentum.** The FBD of the ballon when the ballon is rising with the constant velocity of 5 m/s is shown in Fig. a

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

$$400(5) + T(t) - 400(9.81)t = 400(5)$$

$$T = 3924 \text{ N}$$

When the sand bag is dropped, the thrust $T = 3924 \text{ N}$ is still maintained as shown in the FBD, Fig. b.

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

$$360(5) + 3924(2.0258) - 360(9.81)(2.0258) = 360v$$

$$v = 7.208 \text{ m/s} = 7.21 \text{ m/s} \uparrow$$

Ans.

$$v = 7.21 \text{ m/s} \uparrow$$
As indicated by the derivation, the principle of impulse and momentum is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which slides along the smooth surface and is subjected to a horizontal force of 6 N. If observer $A$ is in a *fixed* frame $x$, determine the final speed of the block in 4 s if it has an initial speed of $v_1 = 5 \text{ m/s}$ measured from the fixed frame. Compare the result with that obtained by an observer $B$, attached to the $x'$ axis that moves at a constant velocity of $2 \text{ m/s}$ relative to $A$.

**SOLUTION**

**Observer $A$:**

\[
(\downarrow) \quad m v_1 + \sum F \, dt = m v_2
\]

\[
10(5) + 6(4) = 10v
\]

\[
v = 7.40 \text{ m/s}
\]

**Observer $B$:**

\[
(\downarrow) \quad m v_1 + \sum F \, dt = m v_2
\]

\[
10(3) + 6(4) = 10v
\]

\[
v = 5.40 \text{ m/s}
\]
15–27.

The 20-kg crate is lifted by a force of \( F = (100 + 5t^2) \) N, where \( t \) is in seconds. Determine the speed of the crate when \( t = 3 \) s, starting from rest.

SOLUTION

Principle of Impulse and Momentum. At \( t = 0 \), \( F = 100 \) N. Since at this instant, \( 2F = 200 \) N > \( W = 20(9.81) = 196.2 \) N, the crate will move the instant force \( F \) is applied. Referring to the FBD of the crate, Fig. a,

\[
\begin{align*}
(\uparrow) \quad m(v_y)_1 + \sum F_y dt &= m(v_y)_2 \\
0 + 2 \int_0^{3s} (100 + 5t^2)dt - 20(9.81)(3) &= 20v \\
2 \left(100t + \frac{5}{3} t^3\right)_{0}^{3} - 588.6 &= 20v \\
v &= 5.07 \text{ m/s} \quad \text{Ans.}
\end{align*}
\]
The 20-kg crate is lifted by a force of $F = (100 + 5t^2)$ N, where $t$ is in seconds. Determine how high the crate has moved upward when $t = 3$ s, starting from rest.

**SOLUTION**

**Principle of Impulse and Momentum.** At $t = 0$, $F = 100$ N. Since at this instant, $2F = 200$ N $> W = 20(9.81) = 196.2$ N, the crate will move the instant force $F$ is applied. Referring to the FBD of the crate, Fig. a

\[
(+\uparrow) \quad m(v_y)_i + \int_{t_1}^{t_2} F_y dt = m(v_y)_2
\]

\[
0 + 2 \int_0^t (100 + 5t^2) dt - 20(9.81)t = 20v
\]

\[
2 \left( 100t + \frac{5t^3}{3} \right) \bigg|_0^t - 196.2t = 20v
\]

\[
v = \{0.1667t^3 + 0.19t\} \text{ m/s}
\]

**Kinematics.** The displacement of the crate can be determined by integrating $ds = v \, dt$ with the initial condition $s = 0$ at $t = 0$.

\[
\int_0^s ds = \int_0^t (0.1667t^3 + 0.19t) \, dt
\]

\[
s = \{0.04167t^4 + 0.095t^2\} \text{ m}
\]

At $t = 3$ s,

\[
s = 0.04167(3^4) + 0.095(3^2) = 4.23 \text{ m}
\]

Ans.

Ans: $s = 4.23$ m
In case of emergency, the gas actuator is used to move a 75-kg block \( B \) by exploding a charge \( C \) near a pressurized cylinder of negligible mass. As a result of the explosion, the cylinder fractures and the released gas forces the front part of the cylinder, \( A \), to move \( B \) forward, giving it a speed of 200 mm/s in 0.4 s. If the coefficient of kinetic friction between \( B \) and the floor is \( \mu_k = 0.5 \), determine the impulse that the actuator imparts to \( B \).

**SOLUTION**

**Principle of Linear Impulse and Momentum:** In order for the package to rest on top of the belt, it has to travel at the same speed as the belt. Applying Eq. 15–4, we have

\[
m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2
\]

\[+(\uparrow)\]
\[6(0) + Nt - 6(9.81)t = 6(0)\]
\[N = 58.86 \text{ N}\]

\[
m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2
\]

\[\begin{align*}
6(3) + [-0.2(58.86)t] & = 6(1) \\
\text{Ans.} & = 6(1) \\
& = 6(1)
\end{align*}\]

\[
t = 1.02 \text{ s}
\]

\[
m(v_x)_1 + \Sigma \int F_x \, dt = m(v_x)_2
\]

\[
0 + \int F \, dt = (0.5)(9.81)(75)(0.4) = 75(0.2)
\]

\[
\int F \, dt = 162 \text{ N} \cdot \text{s}
\]

**Ans:**

\[t = 1.02 \text{ s}\]

\[I = 162 \text{ N} \cdot \text{s}\]
15–30.

A jet plane having a mass of 7 Mg takes off from an aircraft carrier such that the engine thrust varies as shown by the graph. If the carrier is traveling forward with a speed of 40 km/h, determine the plane’s airspeed after 5 s.

**SOLUTION**

The impulse exerted on the plane is equal to the area under the graph.

\[ \nu_1 = 40 \text{ km/h} = 11.11 \text{ m/s} \]

\[
(\therefore) \quad m(\nu_1) + \sum F_x \, dt = m(\nu_2)
\]

\[
(7 \times 10^3)(11.11) - \frac{1}{2}(2)(5)(10^3) + \frac{1}{2}(15 + 5)(5 - 2)(10^3) = 7(10^3)\nu_2
\]

\[ \nu_2 = 16.1 \text{ m/s} \]

**Ans:**

\[ v = 16.1 \text{ m/s} \]

Block \( A \) weighs 10 lb and block \( B \) weighs 3 lb. If \( B \) is moving downward with a velocity \( (v_B)_1 = 3 \text{ ft/s} \) at \( t = 0 \), determine the velocity of \( A \) when \( t = 1 \text{ s} \). Assume that the horizontal plane is smooth. Neglect the mass of the pulleys and cords.

**SOLUTION**

\[ s_A + 2s_B = l \]

\[ v_A = -2v_B \]

\[ \begin{align*}
(\uparrow) & \quad mv_1 + \sum F \int dt = mv_2 \\
& \quad - \frac{10}{32.2}(2)(3) - T(1) = \frac{10}{32.2}(v_A)_2 \\
(\downarrow) & \quad mv_1 + \sum F \int dt = mv_2 \\
& \quad \frac{3}{32.2}(3) + 3(1) - 2T(1) = \frac{3}{32.2}\left(\frac{(v_A)_2}{2}\right) \\
& \quad - 32.2T - 10(v_A)_2 = 60 \\
& \quad - 64.4T + 1.5(v_A)_2 = -105.6 \\
& \quad T = 1.40 \text{ lb} \\
& \quad (v_A)_2 = -10.5 \text{ ft/s} = 10.5 \text{ ft/s} \rightarrow \text{Ans.}
\]
Block A weighs 10 lb and block B weighs 3 lb. If B is moving downward with a velocity \((v_B)_1 = 3 \text{ ft/s}\) at \(t = 0\), determine the velocity of A when \(t = 1 \text{ s}\). The coefficient of kinetic friction between the horizontal plane and block A is \(\mu_A = 0.15\).

\[ \text{SOLUTION} \]

\[ s_A + 2s_B = l \]

\[ v_A = -2v_B \]

\[ (\downarrow) \quad mv_1 + \sum F \, dt = mv_2 \]

\[ - \frac{10}{32.2} (2) (3) - T (1) + 0.15 (10) = \frac{10}{32.2} (v_A)_2 \]

\[ (\downarrow) \quad mv_1 + \sum F \, dt = mv_2 \]

\[ \frac{3}{32.2} (3) - 3 (1) - 2T (1) = \frac{3}{32.2} \left( \frac{(v_A)_2}{2} \right) \]

\[ - 32.2 T - 10(v_A)_2 = 11.70 \]

\[ - 64.4 T + 1.5(v_A)_2 = -105.6 \]

\[ T = 1.50 \text{ lb} \]

\[ (v_A)_2 = -6.00 \text{ ft/s} = 6.00 \text{ ft/s} \rightarrow \quad \text{Ans.} \]

\[ \text{Ans:} \quad (v_A)_2 = 6.00 \text{ ft/s} \rightarrow \]
15–33.

The log has a mass of 500 kg and rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force $T$ to its cable at $A$ which varies as shown in the graph. Determine the speed of the log when $t = 5$ s. Originally the tension in the cable is zero. *Hint: First determine the force needed to begin moving the log.*

**SOLUTION**

\[ \sum F = 0; \quad F - 0.5(500)(9.81) = 0 \]

\[ F = 2452.5 \text{ N} \]

Thus,

\[ 2T = F \]

\[ 2(200t^2) = 2452.5 \]

\[ t = 2.476 \text{ s} \] to start log moving

\[ m \overset{\rightarrow}{v}_1 + \sum \int F \, dt = m \overset{\rightarrow}{v}_2 \]

\[ 0 + 2 \int_{2.476}^{3} 200t^2 \, dt + 2(1800)(5 - 3) - 0.4(500)(9.81)(5 - 2.476) = 500v_2 \]

\[ 400(t^3 \bigg|_{2.476}^{3}) + 2247.91 = 500v_2 \]

\[ v_2 = 7.65 \text{ m/s} \]  

**Ans:**

\[ v = 7.65 \text{ m/s} \]
15–34.

The 0.15-kg baseball has a speed of \( v = 30 \text{ m/s} \) just before it is struck by the bat. It then travels along the trajectory shown before the outfielder catches it. Determine the magnitude of the average impulsive force imparted to the ball if it is in contact with the bat for 0.75 ms.

**SOLUTION**

\[
\text{(\hspace{1em} \downarrow \hspace{1em}) \quad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2}
\]

\[
\frac{4500}{32.2} \quad \left(3\right) - \frac{3000}{32.2} \quad \left(6\right) = \frac{7500}{32.2} \quad v_2
\]

\[
v_2 = -0.600 \text{ ft/s} = 0.600 \text{ ft/s} \quad \leftarrow \quad \text{Ans.}
\]
SOLUTION

Conservation of Linear Momentum.

\[
(\pm) \quad m_A v_A + m_B v_B = (m_A + m_B) v
\]

\[
[5\times 10^3](20) + [2\times 10^3](15) = [5(10^3) + 2(10^3)] v
\]

\[
v = 18.57 \text{ m/s} = 18.6 \text{ m/s} \rightarrow \text{ Ans.}
\]
The 50-kg boy jumps on the 5-kg skateboard with a horizontal velocity of 5 m/s. Determine the distance $s$ the boy reaches up the inclined plane before momentarily coming to rest. Neglect the skateboard’s rolling resistance.

**SOLUTION**

**Free-Body Diagram:** The free-body diagram of the boy and skateboard system is shown in Fig. a. Here, $W_b$, $W_{sb}$, and $N$ are nonimpulsive forces. The pair of impulsive forces $F$ resulting from the impact during landing cancel each other out since they are internal to the system.

**Conservation of Linear Momentum:** Since the resultant of the impulsive force along the $x$ axis is zero, the linear momentum of the system is conserved along the $x$ axis.

\[
m_b(v_{b1}) + m_{sb}(v_{sb1}) = (m_b + m_{sb})v
\]

\[
50(5) + 5(0) = (50 + 5)v
\]

\[
v = 4.545 \text{ m/s}
\]

**Conservation of Energy:** With reference to the datum set in Fig. b, the gravitational potential energy of the boy and skateboard at positions $A$ and $B$ are

\[
(V_g)_A = (m_b + m_{sb})gh_A = 0 \quad \text{and} \quad (V_g)_B = (m_b + m_{sb})gh_B = (50 + 5)(9.81)(s \sin 30°) = 269.775s.
\]

\[
T_A + V_A = T_B + V_B
\]

\[
\frac{1}{2}(m_b + m_{sb})v_A^2 + (V_g)_A = \frac{1}{2}(m_b + m_{sb})v_B^2 + (V_g)_B
\]

\[
\frac{1}{2}(50 + 5)(4.545^2) + 0 = 0 + 269.775s
\]

\[
s = 2.11 \text{ m} \quad \text{Ans.}
\]

Ans:

\[
s = 2.11 \text{ m}
\]
The 2.5-Mg pickup truck is towing the 1.5-Mg car using a cable as shown. If the car is initially at rest and the truck is coasting with a velocity of 30 km/h when the cable is slack, determine the common velocity of the truck and the car just after the cable becomes taut. Also, find the loss of energy.

**SOLUTION**

**Free-Body Diagram:** The free-body diagram of the truck and car system is shown in Fig. a. Here, $W_r$, $W_c$, $N_r$, and $N_c$ are nonimpulsive forces. The pair of impulsive forces $F$ generated at the instant the cable becomes taut are internal to the system and thus cancel each other out.

**Conservation of Linear Momentum:** Since the resultant of the impulsive force is zero, the linear momentum of the system is conserved along the $x$ axis. The initial speed of the truck is

$$v_t = \left( \frac{30 \text{ m}}{3600 \text{ s}} \right) = 8.333 \text{ m/s}.$$ 

$$m_t(v_t)_1 = m_t(v_t)_2 = (m_t + m_c)v_2$$

$$2500(8.333) + 0 = (2500 + 1500)v_2$$

$$v_2 = 5.208 \text{ m/s} = 5.21 \text{ m/s} \quad \text{Ans.}$$

**Kinetic Energy:** The initial and final kinetic energy of the system is

$$\frac{1}{2} m_t(v_t)_1^2 = \frac{1}{2} m_c(v_c)_1^2$$

$$\frac{1}{2} m_t(v_t)_1^2 = \frac{1}{2} (2500)(8.333^2) + 0$$

$$= 86 805.56 \text{ J}$$

and

$$T_2 = (m_t + m_c)v_2^2$$

$$= \frac{1}{2} (2500 + 1500)(5.208^2)$$

$$= 54 253.47 \text{ J}$$

Thus, the loss of energy during the impact is

$$\Delta T = T_1 - T_2 = 86 805.56 - 54 253.47 = 32.55(10^3) \text{ J} = 32.6 \text{ kJ} \quad \text{Ans.}$$

**Ans:**

$v = 5.21 \text{ m/s}$

$\Delta T = -32.6 \text{ kJ}$
A railroad car having a mass of 15 Mg is coasting at 1.5 m/s on a horizontal track. At the same time another car having a mass of 12 Mg is coasting at 0.75 m/s in the opposite direction. If the cars meet and couple together, determine the speed of both cars just after the coupling. Find the difference between the total kinetic energy before and after coupling has occurred, and explain qualitatively what happened to this energy.

**SOLUTION**

\[ \sum m v_1 = \sum m v_2 \]

\[ 15000(1.5) - 12000(0.75) = 27000(v_2) \]

\[ v_2 = 0.5 \text{ m/s} \quad \text{Ans.} \]

\[ T_1 = \frac{1}{2}(15000)(1.5)^2 + \frac{1}{2}(12000)(0.75)^2 = 20.25 \text{ kJ} \]

\[ T_2 = \frac{1}{2}(27000)(0.5)^2 = 3.375 \text{ kJ} \]

\[ \Delta T = T_2 - T_1 \]

\[ = 3.375 - 20.25 = -16.9 \text{ kJ} \quad \text{Ans.} \]

This energy is dissipated as noise, shock, and heat during the coupling.
15–39.

A ballistic pendulum consists of a 4-kg wooden block originally at rest, $\theta = 0^\circ$. When a 2-g bullet strikes and becomes embedded in it, it is observed that the block swings upward to a maximum angle of $\theta = 6^\circ$. Estimate the speed of the bullet.

**SOLUTION**

Just after impact:
Datum at lowest point.

\[ T_2 + V_2 = T_3 + V_3 \]

\[
\frac{1}{2} (4 + 0.002)(v_B)^2 + 0 = 0 + (4 + 0.002)(9.81)(1.25)(1 - \cos 6^\circ)
\]

\[(v_B)^2 = 0.3665 \text{ m/s} \]

For the system of bullet and block:
\[
(\Sigma m) v_1 = \Sigma m v_2 \]
\[0.002(v_B)_1 = (4 + 0.002)(0.3665)\]

\[(v_B)_1 = 733 \text{ m/s} \quad \text{Ans.} \]
*15–40.

The boy jumps off the flat car at $A$ with a velocity of $v = 4 \text{ ft/s}$ relative to the car as shown. If he lands on the second flat car $B$, determine the final speed of both cars after the motion. Each car has a weight of 80 lb. The boy’s weight is 60 lb. Both cars are originally at rest. Neglect the mass of the car’s wheels.

**SOLUTION**

\[ \Sigma m(v_1) = \Sigma m(v_2) \]

\[ 0 + 0 = \frac{80}{32.2} v_A + \frac{60}{32.2} v_B \]

\[ v_A = 0.75(v_B) \]

\[ v_b = v_A + v_{b/A} \]

\[ (v_b)_x = v_A + 4 \left( \frac{12}{13} \right) \]

\[ (v_b)_x = 2.110 \text{ ft/s} \]

\[ v_A = 1.58 \text{ ft/s} \rightarrow \text{ Ans.} \]

\[ \Sigma m(v_1) = \Sigma m(v_2) \]

\[ \frac{60}{32.2} (2.110) = \left( \frac{80}{32.2} + \frac{60}{32.2} \right) v \]

\[ v = 0.904 \text{ ft/s} \rightarrow \text{ Ans.} \]

**Ans:**

\[ v_A = 1.58 \text{ ft/s} \]

\[ v = 0.904 \text{ ft/s} \]
A 0.03-lb bullet traveling at 1300 ft/s strikes the 10-lb wooden block and exits the other side at 50 ft/s as shown. Determine the speed of the block just after the bullet exits the block, and also determine how far the block slides before it stops. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.5$.

**SOLUTION**

$$\sum m_i v_i = \sum m_j v_j$$

$$\left( \frac{0.03}{32.2} \right) (1300) \left( \frac{12}{13} \right) + 0 = \left( \frac{10}{32.2} \right) v_B + \left( \frac{0.03}{32.2} \right) (50) \left( \frac{4}{5} \right)$$

$v_B = 3.48 \text{ ft/s}$

$T_i + \sum U_{i-2} = T_2$

$$\frac{1}{2} \left( \frac{10}{32.2} \right) (3.48)^2 = 5(d) = 0$$

$$d = 0.376 \text{ ft}$$

*Ans:*

$v_B = 3.48 \text{ ft/s}$

$d = 0.376 \text{ ft}$
A 0.03-lb bullet traveling at 1300 ft/s strikes the 10-lb wooden block and exits the other side at 50 ft/s as shown. Determine the speed of the block just after the bullet exits the block. Also, determine the average normal force on the block if the bullet passes through it in 1 ms, and the time the block slides before it stops. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.5$.

**SOLUTION**

\[ \Sigma m_1 v_1 = \Sigma m_2 v_2 \]

\[ \left( \frac{0.03}{32.2} \right) (1300) \left( \frac{12}{13} \right) + 0 = \left( \frac{10}{32.2} \right) v_B + \left( \frac{0.03}{32.2} \right) (50) \left( \frac{4}{5} \right) \]

\[ v_B = 3.48 \text{ ft/s} \quad \text{Ans.} \]

\[ m v_1 + \Sigma \int F \, dt = m v_2 \]

\[ - \left( \frac{0.03}{32.2} \right) (1300) \left( \frac{5}{13} \right) - 10(1)(10^{-3}) + N(1)(10^{-3}) = \left( \frac{0.03}{32.2} \right) (50) \left( \frac{3}{5} \right) \]

\[ N = 504 \text{ lb} \quad \text{Ans.} \]

\[ m v_1 + \Sigma \int F \, dt = m v_2 \]

\[ \left( \frac{10}{32.2} \right) (3.48) - 5(t) = 0 \]

\[ t = 0.216 \text{ s} \quad \text{Ans.} \]

<table>
<thead>
<tr>
<th>Ans:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_B = 3.48 \text{ ft/s}$</td>
</tr>
<tr>
<td>$N_{\text{avg}} = 504 \text{ lb}$</td>
</tr>
<tr>
<td>$t = 0.216 \text{ s}$</td>
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</table>
15–43.

The 20-g bullet is traveling at 400 m/s when it becomes embedded in the 2-kg stationary block. Determine the distance the block will slide before it stops. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.2$.

**SOLUTION**

**Conservation of Momentum.**

\[
\begin{align*}
(\pm) & \quad m_bv_b + m_Bv_B = (m_b + m_B)v \\
0.02(400) + 0 &= (0.02 + 2)v \\
v &= 3.9604 \text{ m/s}
\end{align*}
\]

**Principle of Impulse and Momentum.** Here, friction $F_f = \mu_kN = 0.2 \text{ N}$. Referring to the FBD of the blocks, Fig. a,

\[
\begin{align*}
(\uparrow) & \quad m(v_y) + \sum \int_{t_1}^{t_2} F_y dt = m(v_y)_2 \\
0 + N(t) - 2.02(9.81)(t) &= 0 \\
N &= 19.8162 \text{ N} \\
(\downarrow) & \quad m(v_x) + \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2 \\
2.02(3.9604) + [-0.2(19.8162) t] &= 2.02v \\
v &= \{3.9604 - 1.962 t\} \text{ m/s}
\end{align*}
\]

Thus, the stopping time can be determined from

\[
\begin{align*}
0 &= 3.9604 - 1.962 t \\
t &= 2.0186 \text{ s}
\end{align*}
\]

**Kinematics.** The displacement of the block can be determined by integrating $ds = v dt$ with the initial condition $s = 0$ at $t = 0$.

\[
\int_0^t ds = \int_0^t (3.9604 - 1.962 t) dt \\
s = \{3.9604t - 0.981t^2\} \text{ m}
\]

The block stopped at $t = 2.0186 \text{ s}$. Thus

\[
\begin{align*}
s &= 3.9604(2.0186) - 0.981(2.0186^2) \\
&= 3.9971 \text{ m} = 4.00 \text{ m}
\end{align*}
\]

Ans: $s = 4.00 \text{ m}$
*15–44.

A toboggan having a mass of 10 kg starts from rest at A and carries a girl and boy having a mass of 40 kg and 45 kg, respectively. When the toboggan reaches the bottom of the slope at B, the boy is pushed off from the back with a horizontal velocity of \( v_{b/t} = 2 \text{ m/s} \), measured relative to the toboggan. Determine the velocity of the toboggan afterwards. Neglect friction in the calculation.

**SOLUTION**

**Conservation of Energy:** The datum is set at the lowest point B. When the toboggan and its rider is at A, their position is 3 m above the datum and their gravitational potential energy is 
\[
(10 + 40 + 45)(9.81)(3) = 2795.85 \text{ N} \cdot \text{m}.
\]
Applying Eq. 14–21, we have
\[
T_1 + V_1 = T_2 + V_2
\]
\[
0 + 2795.85 = \frac{1}{2}(10 + 40 + 45) v_b^2 + 0
\]
\[
v_B = 7.672 \text{ m/s}
\]

**Relative Velocity:** The relative velocity of the falling boy with respect to the toboggan is \( v_{b/t} = 2 \text{ m/s} \). Thus, the velocity of the boy falling off the toboggan is
\[
v_b = v_t + v_{b/t}
\]
\[
(\downarrow) \quad v_b = v_t - 2 \quad \text{[1]}
\]

**Conservation of Linear Momentum:** If we consider the toboggan and the riders as a system, then the impulsive force caused by the push is internal to the system. Therefore, it will cancel out. As the result, the linear momentum is conserved along the x axis.
\[
m_f v_B = m_b v_b + \left( m_t + m_k \right) v_t
\]
\[
(\downarrow) \quad (10 + 40 + 45)(7.672) = 45v_b + (10 + 40) v_t \quad \text{[2]}
\]

Solving Eqs. [1] and [2] yields
\[
v_t = 8.62 \text{ m/s}
\]
\[
v_b = 6.619 \text{ m/s}
\]

Ans:

\[
v_t = 8.62 \text{ m/s}
\]
15–45.

The block of mass \( m \) is traveling at \( v_1 \) in the direction \( \theta_1 \) shown at the top of the smooth slope. Determine its speed \( v_2 \) and its direction \( \theta_2 \) when it reaches the bottom.

**SOLUTION**

**There are no impulses in the \( v \) direction:**

\[
m v_1 \sin \theta_1 = m v_2 \sin \theta_2
\]

\[
T_1 + V_1 = T_2 + V_2
\]

\[
\frac{1}{2} m v_1^2 + mgh = \frac{1}{2} m v_2^2 + 0
\]

\[
v_2 = \sqrt{v_1^2 + 2gh}
\]

\[
\sin \theta_2 = \frac{v_1 \sin \theta_1}{\sqrt{v_1^2 + 2gh}}
\]

\[
\theta_2 = \sin^{-1}\left(\frac{v_1 \sin \theta_1}{\sqrt{v_1^2 + 2gh}}\right)
\]

**Ans:**

\[
v_2 = \sqrt{v_1^2 + 2gh}
\]

\[
\theta_2 = \sin^{-1}\left(\frac{v_1 \sin \theta}{\sqrt{v_1^2 + 2gh}}\right)
\]
The two blocks \( A \) and \( B \) each have a mass of 5 kg and are suspended from parallel cords. A spring, having a stiffness of \( k = 60 \text{ N/m} \), is attached to \( B \) and is compressed 0.3 m against \( A \) and \( B \) as shown. Determine the maximum angles \( \theta \) and \( \phi \) of the cords when the blocks are released from rest and the spring becomes unstretched.

**SOLUTION**

\[
\Sigma m \mathbf{v}_1 = \Sigma m \mathbf{v}_2
\]

\[
0 + 0 = -5v_A + 5v_B
\]

\[
v_A = v_B = v
\]

Just before the blocks begin to rise:

\[
T_1 + V_1 = T_2 + V_2
\]

\[
(0 + 0) + \frac{1}{2}(60)(0.3)^2 = \frac{1}{2}(5)(v)^2 + \frac{1}{2}(5)(v)^2 + 0
\]

\[
v = 0.7348 \text{ m/s}
\]

For \( A \) or \( B \):

Datum at lowest point.

\[
T_1 + V_1 = T_2 + V_2
\]

\[
\frac{1}{2}(5)(0.7348)^2 + 0 = 0 + 5(9.81)(2)(1 - \cos \theta)
\]

\[
\theta = \phi = 9.52^\circ
\]

**Ans:**

\[
\theta = \phi = 9.52^\circ
\]
15–47.

The 30-Mg freight car $A$ and 15-Mg freight car $B$ are moving towards each other with the velocities shown. Determine the maximum compression of the spring mounted on car $A$. Neglect rolling resistance.

SOLUTION

Conservation of Linear Momentum: Referring to the free-body diagram of the freight cars $A$ and $B$ shown in Fig. $a$, notice that the linear momentum of the system is conserved along the $x$ axis. The initial speed of freight cars $A$ and $B$ are $v_A = \frac{20(10^3) \text{ m}}{\text{h}} = \frac{1 \text{ h}}{3600 \text{ s}} = 5.556 \text{ m/s}$ and $v_B = \frac{10(10^3) \text{ m}}{\text{h}} = \frac{1 \text{ h}}{3600 \text{ s}} = 2.778 \text{ m/s}$. At this instant, the spring is compressed to its maximum, and no relative motion occurs between freight cars $A$ and $B$ and they move with a common speed.

\[ (\pm) \quad m_A v_A + m_B v_B = (m_A + m_B)v 
\]

\[ 30(10^3)(5.556) + 15(10^3)(2.778) = 30(10^3) + 15(10^3) v 
\]

\[ v = 2.778 \text{ m/s} \rightarrow \]

Conservation of Energy: The initial and final elastic potential energy of the spring is $(V_e)_1 = \frac{1}{2} k s_1^2 = 0$ and $(V_e)_2 = \frac{1}{2} k s_2^2 = \frac{1}{2} (3)(10^6)s_{\text{max}}^2 = 1.5(10^6)s_{\text{max}}^2$.

\[ \Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2 
\]

\[ \left[ \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \right] + (V_e)_1 = \left[ \frac{1}{2} (m_A + m_B)v^2 + (V_e)_2 
\]

\[ = \frac{1}{2} \left[ 30(10^3) + 15(10^3) \right] (2.778^2) + 1.5(10^6)s_{\text{max}}^2 
\]

\[ s_{\text{max}} = 0.4811 \text{ m} = 481 \text{ mm} \]

Ans: $s_{\text{max}} = 481 \text{ mm}$
Blocks $A$ and $B$ have masses of 40 kg and 60 kg, respectively. They are placed on a smooth surface and the spring connected between them is stretched 2 m. If they are released from rest, determine the speeds of both blocks the instant the spring becomes unstretched.

**SOLUTION**

\[
\begin{align*}
\sum m v_1 &= \sum m v_2 \\
0 + 0 &= 40v_A - 60v_B \\
T_1 + V_1 &= T_2 + V_2 \\
0 + \frac{1}{2}(180)(2)^2 &= \frac{1}{2}(40)(v_A)^2 + \frac{1}{2}(60)(v_B)^2
\end{align*}
\]

\[v_A = 3.29 \text{ m/s} \quad \text{Ans.}\]

\[v_B = 2.19 \text{ m/s} \quad \text{Ans.}\]
A boy $A$ having a weight of 80 lb and a girl $B$ having a weight of 65 lb stand motionless at the ends of the toboggan, which has a weight of 20 lb. If they exchange positions, $A$ going to $B$ and then $B$ going to $A$‘s original position, determine the final position of the toboggan just after the motion. Neglect friction between the toboggan and the snow.

**SOLUTION**

$A$ goes to $B$,

\[
\sum m v_1 = \sum m v_2
\]

\[
0 = m_A v_A - (m_t + m_B) v_B
\]

\[
0 = m_A s_A - (m_t + m_B) s_B
\]

Assume $B$ moves $x$ to the left, then $A$ moves $(4 - x)$ to the right

\[
x = \frac{4m_A}{m_A + m_B + m_t}
\]

\[
= \frac{4(80)}{80 + 65 + 20} = 1.939 \text{ ft} \leftarrow
\]

$B$ goes to other end.

\[
\sum m v_1 = \sum m v_2
\]

\[
0 = -m_B v_B + (m_t + m_A) v_A
\]

\[
0 = -m_B s_B + (m_t + m_A) s_A
\]

Assume $B$ moves $x'$ to the right, then $A$ moves $(4 - x')$ to the left

\[
x' = \frac{4m_B}{m_A + m_B + m_t}
\]

\[
= \frac{4(65)}{80 + 65 + 20} = 1.576 \text{ ft} \rightarrow
\]

Net movement of sled is

\[
x = 1.939 - 1.576 = 0.364 \text{ ft} \leftarrow
\]

**Ans:**

\[
x = 0.364 \text{ ft} \leftarrow
\]
A boy $A$ having a weight of 80 lb and a girl $B$ having a weight of 65 lb stand motionless at the ends of the toboggan, which has a weight of 20 lb. If $A$ walks to $B$ and stops, and both walk back together to the original position of $A$, determine the final position of the toboggan just after the motion stops. Neglect friction between the toboggan and the snow.

**SOLUTION**

$A$ goes to $B$,

\[
\sum m v_1 = \sum m v_2
\]

\[
0 = m_A v_A - (m_t + m_B)v_B
\]

\[
0 = m_A s_A - (m_t + m_B)s_B
\]

Assume $B$ moves $x$ to the left, then $A$ moves $(4 - x)$ to the right

\[
0 = m_A (4 - x) - (m_t + m_B)x
\]

\[
x = \frac{4 m_A}{m_A + m_B + m_t}
\]

\[
= \frac{4(80)}{80 + 65 + 20} = 1.939 \text{ ft} \quad \leftarrow
\]

$A$ and $B$ go to other end.

\[
\sum m v_1 = \sum m v_2
\]

\[
0 = -m_B v - m_A v + m_t v_y
\]

\[
0 = -m_B s - m_A s + m_t s
\]

Assume the toboggan moves $x'$ to the right, then $A$ and $B$ move $(4 - x')$ to the left

\[
0 = -m_B (4 - x') - m_A (4 - x') + m_t x'
\]

\[
x' = \frac{4 (m_B + m_A)}{m_A + m_B + m_t}
\]

\[
= \frac{4(65 + 80)}{80 + 65 + 20} = 3.515 \text{ ft} \quad \rightarrow
\]

Net movement of sled is

\[
\sum m v_1 = \sum m v_2
\]

\[
x = 3.515 - 1.939 = 1.58 \text{ ft \quad \rightarrow}
\]

\[\text{Ans:} \quad x = 1.58 \text{ ft} \rightarrow\]
The 10-Mg barge \( B \) supports a 2-Mg automobile \( A \). If someone drives the automobile to the other side of the barge, determine how far the barge moves. Neglect the resistance of the water.

SOLUTION

**Conservation of Momentum.** Assuming that \( V_B \) is to the left,

\[
(\downarrow) \quad m_A v_A + m_B v_B = 0
\]

\[
2 \left( 10^3 \right) v_A + 10 \left( 10^3 \right) v_B = 0
\]

Integrate this equation,

\[
2s_A + 10s_B = 0
\]  \( \text{(1)} \)

**Kinematics.** Here, \( s_{A/B} = 40 \text{ m} \), using the relative displacement equation by assuming that \( s_B \) is to the left,

\[
(\downarrow) \quad s_A = s_B + s_{A/B}
\]

\[
s_A = s_B + 40 \quad \text{(2)}
\]

Solving Eq. (1) and (2),

\[
s_B = -6.6667 \text{ m} = 6.67 \text{ m} \rightarrow \quad \text{Ans.}
\]

\[
s_A = 33.33 \text{ m} \leftarrow
\]

The negative sign indicates that \( s_B \) is directed to the right which is opposite to that of the assumed.

\[s_B = 6.67 \text{ m} \rightarrow\]
*15–52.

The free-rolling ramp has a mass of 40 kg. A 10-kg crate is released from rest at A and slides down 3.5 m to point B. If the surface of the ramp is smooth, determine the ramp’s speed when the crate reaches B. Also, what is the velocity of the crate?

**SOLUTION**

**Conservation of Energy:** The datum is set at lowest point B. When the crate is at point A, it is 3.5 sin 30° = 1.75 m above the datum. Its gravitational potential energy is 10(9.81)(1.75) = 171.675 N⋅m. Applying Eq. 14–21, we have

\[ T_i + V_i = T_f + V_f \]

\[ 0 + 171.675 = \frac{1}{2}(10)v_C^2 + \frac{1}{2}(40)v_R^2 \]

\[ 171.675 = 5v_C^2 + 20v_R^2 \]  

(1)

**Relative Velocity:** The velocity of the crate is given by

\[ \mathbf{v}_C = \mathbf{v}_R + \mathbf{v}_{C/R} \]

\[ = -v_R \mathbf{i} + (v_{C/R} \cos 30° \mathbf{i} - v_{C/R} \sin 30° \mathbf{j}) \]

\[ = (0.8660v_{C/R} - v_R)\mathbf{i} - 0.5v_{C/R}\mathbf{j} \]  

(2)

The magnitude of \( v_C \) is

\[ v_C = \sqrt{(0.8660v_{C/R} - v_R)^2 + (-0.5v_{C/R})^2} \]

\[ = \sqrt{v_{C/R}^2 + v_R^2 - 1.732v_Rv_{C/R}} \]  

(3)

**Conservation of Linear Momentum:** If we consider the crate and the ramp as a system, from the FBD, one realizes that the normal reaction \( N_C \) (impulsive force) is internal to the system and will cancel each other. As the result, the linear momentum is conserved along the x axis.

\[ 0 = m_C(v_C)_x + m_Rv_R \]

\[ (\downarrow) \]

\[ 0 = 10(0.8660v_{C/R} - v_R) + 40(-v_R) \]

\[ 0 = 8.660v_{C/R} - 50v_R \]  

(4)

Solving Eqs. (1), (3), and (4) yields

\[ v_R = 1.101 \text{ m/s} = 1.10 \text{ m/s} \quad v_C = 5.43 \text{ m/s} \]

\[ v_{C/R} = 6.356 \text{ m/s} \]

Ans.

From Eq. (2)

\[ v_C = [0.8660(6.356) - 1.101]\mathbf{i} - 0.5(6.356)\mathbf{j} = \{4.403\mathbf{i} - 3.178\mathbf{j}\} \text{ m/s} \]

Thus, the directional angle \( \phi \) of \( v_C \) is

\[ \phi = \tan^{-1} \frac{3.178}{4.403} = 35.8° \]

Ans.

\[ v_{C/R} = 6.356 \text{ m/s} \]

\[ \phi = 35.8° \]
15–53.

Block A has a mass of 5 kg and is placed on the smooth triangular block B having a mass of 30 kg. If the system is released from rest, determine the distance B moves from point O when A reaches the bottom. Neglect the size of block A.

**SOLUTION**

\[
\Sigma m v_1 = \Sigma m v_2
\]

\[
0 = 30v_B - 5(v_A)_x
\]

\[
(v_A)_x = 6v_B
\]

\[
v_B = v_A + v_{B/A}
\]

\[
\Sigma v_B = -(v_A)_x + (v_{B/A})_x
\]

\[
v_B = -6v_B + (v_{B/A})_x
\]

\[
(v_{B/A})_x = 7v_B
\]

Integrate

\[
(s_{B/A})_x = 7s_B
\]

\[
(s_{B/A})_x = 0.5 \text{ m}
\]

Thus,

\[
s_B = \frac{0.5}{7} = 0.0714 \text{ m} = 71.4 \text{ mm} \rightarrow \text{ Ans.}
\]
15–54.
Solve Prob. 15–53 if the coefficient of kinetic friction between A and B is $\mu_k = 0.3$. Neglect friction between block B and the horizontal plane.

**SOLUTION**

\[ + \sum F_y = 0; \quad N_A - 5(9.81) \cos 30^\circ = 0 \]

\[ N_A = 42.4785 \text{ N} \]

\[ + \sum F_x = 0; \quad F_A - 5(9.81) \sin 30^\circ = 0 \]

\[ F_A = 24.525 \text{ N} \]

\[ F_{\text{max}} = \mu N_A = 0.3(42.4785) = 12.74 \text{ N} < 24.525 \text{ N} \]

Block indeed slides.

Solution is the same as in Prob. 15–53. Since $F_A$ is internal to the system.

\[ s_B = 71.4 \text{ mm} \rightarrow \text{ Ans.} \]
The cart has a mass of 3 kg and rolls freely down the slope. When it reaches the bottom, a spring loaded gun fires a 0.5-kg ball out the back with a horizontal velocity of $v_{b/c} = 0.6 \text{ m/s}$, measured relative to the cart. Determine the final velocity of the cart.

**SOLUTION**

Datum at B:

$$T_A + V_A = T_B + V_B$$

$$0 + (3 + 0.5)(9.81)(1.25) = \frac{1}{2}(3 + 0.5)v_B^2 + 0$$

$v_B = 4.952 \text{ m/s}$

(+) $\Sigma m v_1 = \Sigma m v_2$

$$3 + 0.5)(4.952) = (3)v_c - (0.5)v_b$$

$$v_c = 5.04 \text{ m/s} \leftarrow \text{Ans.}$$

$$v_b = v_c - 0.6$$

Solving Eqs. (1) and (2),

$v_c = 5.04 \text{ m/s} \leftarrow \text{Ans.}$

$v_b = -4.44 \text{ m/s} = 4.44 \text{ m/s} \leftarrow$
Two boxes $A$ and $B$, each having a weight of 160 lb, sit on the 500-lb conveyor which is free to roll on the ground. If the belt starts from rest and begins to run with a speed of 3 ft/s, determine the final speed of the conveyor if (a) the boxes are not stacked and $A$ falls off then $B$ falls off, and (b) $A$ is stacked on top of $B$ and both fall off together.

**SOLUTION**

a) Let $v_b$ be the velocity of $A$ and $B$.

\[
\begin{align*}
\Sigma m v_1 &= \Sigma m v_2 \\
0 &= \left(\frac{320}{32.2}\right)(v_b) - \left(\frac{500}{32.2}\right)(v_c)
\end{align*}
\]

\[
\begin{align*}
&v_b = v_c + v_{b/c} \\
v_b = -v_c + 3
\end{align*}
\]

Thus, $v_b = 1.83$ ft/s $\Rightarrow$ $v_c = 1.17$ ft/s $\Leftarrow$

When a box falls off, it exerts no impulse on the conveyor, and so does not alter the momentum of the conveyor. Thus,

a) $v_c = 1.17$ ft/s $\Leftarrow$ \hspace{1cm} Ans.

b) $v_c = 1.17$ ft/s $\Leftarrow$ \hspace{1cm} Ans.
The 10-kg block is held at rest on the smooth inclined plane by the stop block at $A$. If the 10-g bullet is traveling at 300 m/s when it becomes embedded in the 10-kg block, determine the distance the block will slide up along the plane before momentarily stopping.

**SOLUTION**

*Conservation of Linear Momentum:* If we consider the block and the bullet as a system, then from the FBD, the impulsive force $F$ caused by the impact is internal to the system. Therefore, it will cancel out. Also, the weight of the bullet and the block are nonimpulsive forces. As the result, linear momentum is conserved along the $x'$ axis.

\[ m_b(v_b') = (m_b + m_B) v_x' \]

\[ 0.01(300 \cos 30°) = (0.01 + 10) v \]

\[ v = 0.2595 \text{ m/s} \]

*Conservation of Energy:* The datum is set at the blocks initial position. When the block and the embedded bullet is at their highest point they are $h$ above the datum. Their gravitational potential energy is \((10 + 0.01)(9.81)h = 98.1981h\). Applying Eq. 14–21, we have

\[ T_1 + V_1 = T_2 + V_2 \]

\[ 0 + \frac{1}{2}(10 + 0.01)(0.2595^2) = 0 + 98.1981h \]

\[ h = 0.003433 \text{ m} = 3.43 \text{ mm} \]

\[ d = 3.43 / \sin 30° = 6.87 \text{ mm} \]

Ans.
Disk A has a mass of 250 g and is sliding on a smooth horizontal surface with an initial velocity \((v_A)_1 = 2 \text{ m/s}\). It makes a direct collision with disk B, which has a mass of 175 g and is originally at rest. If both disks are of the same size and the collision is perfectly elastic \((e = 1)\), determine the velocity of each disk just after collision. Show that the kinetic energy of the disks before and after collision is the same.

**SOLUTION**

\[
(\pm) \quad (0.250)(2) + 0 = (0.250)(v_A)_2 + (0.175)(v_B)_2
\]

\[
(\pm) \quad e = 1 = \frac{(v_B)_2 - (v_A)_2}{2 - 0}
\]

Solving

\[(v_A)_2 = 0.353 \text{ m/s} \quad \text{Ans.}\]
\[(v_B)_2 = 2.35 \text{ m/s} \quad \text{Ans.}\]

\[T_1 = \frac{1}{2} (0.25)(2)^2 = 0.5 \text{ J}\]

\[T_2 = \frac{1}{2} (0.25)(0.353)^2 + \frac{1}{2} (0.175)(2.35)^2 = 0.5 \text{ J}\]

\[T_1 = T_2 \quad \text{QED}\]

**Ans:**

\[(v_A)_2 = 0.353 \text{ m/s}\]
\[(v_B)_2 = 2.35 \text{ m/s}\]
15–59.

The 5-Mg truck and 2-Mg car are traveling with the free-rolling velocities shown just before they collide. After the collision, the car moves with a velocity of 15 km/h to the right relative to the truck. Determine the coefficient of restitution between the truck and car and the loss of energy due to the collision.

**SOLUTION**

*Conservation of Linear Momentum:* The linear momentum of the system is conserved along the x axis (line of impact).

The initial speeds of the truck and car are \((v_t)_1 = \left[30(10^3) \frac{m}{h}\right] \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 8.333 \text{ m/s}\) and \((v_c)_1 = \left[10(10^3) \frac{m}{h}\right] \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 2.778 \text{ m/s}\).

By referring to Fig. a,

\[
5(v_t)_2 + 2(v_c)_2 = 5000(8.333) + 2000(2.778) = 5000(v_t)_2 + 2000(v_c)_2
\]

\[5(v_t)_2 + 2(v_c)_2 = 47.22 \tag{1}\]

*Coefficient of Restitution:* Here, \((v_{cr}) = \left[15(10^3) \frac{m}{h}\right] \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 4.167 \text{ m/s} \rightarrow\).

Applying the relative velocity equation,

\[(v_c)_2 = (v_t)_2 + (v_{cr})\]

\[(v_c)_2 = (v_t)_2 + 4.167\]

\[(v_c)_2 - (v_t)_2 = 4.167 \tag{2}\]

Applying the coefficient of restitution equation,

\[e = \frac{(v_c)_2 - (v_t)_2}{(v_t)_2 - (v_c)_1}\]

\[e = \frac{(v_c)_2 - (v_t)_2}{8.333 - 2.778} \tag{3}\]
Substituting Eq. (2) into Eq. (3),

\[ e = \frac{4.167}{8.333 - 2.778} = 0.75 \quad \text{Ans.} \]

Solving Eqs. (1) and (2) yields

\[ (v_i)_2 = 5.556 \text{ m/s} \]
\[ (v_i)_2 = 9.722 \text{ m/s} \]

**Kinetic Energy:** The kinetic energy of the system just before and just after the collision are

\[ T_1 = \frac{1}{2} m(v_i)_1^2 + \frac{1}{2} m(v_i)_1^2 \]

\[ = \frac{1}{2} (5000)(8.333^2) + \frac{1}{2} (2000)(2.778^2) \]
\[ = 181.33 \times 10^3 \text{ J} \]

\[ T_2 = \frac{1}{2} m(v_i)_2^2 + \frac{1}{2} m(v_i)_2^2 \]

\[ = \frac{1}{2} (5000)(5.556^2) + \frac{1}{2} (2000)(9.722^2) \]
\[ = 171.68 \times 10^3 \text{ J} \]

Thus,

\[ \Delta T = T_1 - T_2 = 181.33 \times 10^3 - 171.68 \times 10^3 \]
\[ = 9.645 \times 10^3 \text{ J} \]
\[ = 9.65 \text{ kJ} \quad \text{Ans.} \]
Disk $A$ has a mass of 2 kg and is sliding forward on the smooth surface with a velocity $(v_A)_1 = 5 \text{ m/s}$ when it strikes the 4-kg disk $B$, which is sliding towards $A$ at $(v_B)_1 = 2 \text{ m/s}$, with direct central impact. If the coefficient of restitution between the disks is $e = 0.4$, compute the velocities of $A$ and $B$ just after collision.

**SOLUTION**

Conservation of Momentum:

\[
m_A (v_A)_1 + m_B (v_B)_1 = m_A (v_A)_2 + m_B (v_B)_2
\]

\[
(\downarrow) \quad 2(5) + 4(-2) = 2(v_A)_2 + 4(v_B)_2 \tag{1}
\]

Coefficient of Restitution:

\[
e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}
\]

\[
(\downarrow) \quad 0.4 = \frac{(v_B)_2 - (v_A)_2}{5 - (-2)} \tag{2}
\]

Solving Eqs. (1) and (2) yields

\[
(v_A)_2 = -1.53 \text{ m/s} = 1.53 \text{ m/s} \quad \leftrightarrow \quad (v_B)_2 = 1.27 \text{ m/s} \quad \rightarrow \quad \text{Ans.}
\]

**Ans:**

\[
(v_A)_2 = 1.53 \text{ m/s} \quad \leftrightarrow \quad (v_B)_2 = 1.27 \text{ m/s}
\]
15–61.

The 15-kg block A slides on the surface for which \( m_k = 0.3 \). The block has a velocity \( v = 10 \text{ m/s} \) when it is \( s = 4 \text{ m} \) from the 10-kg block B. If the unstretched spring has a stiffness \( k = 1000 \text{ N/m} \), determine the maximum compression of the spring due to the collision. Take \( e = 0.6 \).

**SOLUTION**

*Principle of Work and Energy.* Referring to the FBD of block A, Fig. a, motion along the \( y \) axis gives \( N_A = 15(9.81) = 147.15 \text{ N} \). Thus the friction is

\[
F_f = m_k N_A = 0.3(147.15) = 44.145 \text{ N}
\]

\[
T_1 + \sum U_{1-2} = T_2
\]

\[
\frac{1}{2} (15)(10^2) + (-44.145)(4) = \frac{1}{2} (15)(v_A)^2
\]

\[
(v_A)_1 = 8.7439 \text{ m/s} \leftarrow
\]

*Conservation of Momentum.*

\[
m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2
\]

\[
15(8.7439) + 0 = 15(v_A)_2 + 10(v_B)_2
\]

\[
3(v_A)_2 + 2(v_B)_2 = 26.2317
\]

(1)

*Coefficient of Restitution.*

\[
e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \quad 0.6 = \frac{(v_B)_2 - (v_A)_2}{8.7439 - 0}
\]

\[
(v_B)_2 - (v_A)_2 = 5.2463
\]

(2)

Solving Eqs. (1) and (2)

\[
(v_B)_2 = 8.3942 \text{ m/s} \leftarrow (v_A)_2 = 3.1478 \text{ m/s} \leftarrow
\]

*Conservation of Energy.* When block B stops momentarily, the compression of the spring is maximum. Thus, \( T_2 = 0 \).

\[
T_1 + V_1 = T_2 + V_2
\]

\[
\frac{1}{2} (10)(8.394^2) + 0 = 0 + \frac{1}{2} (1000)x_{\text{max}}^2
\]

\[
x_{\text{max}} = 0.8394 \text{ m} = 0.839 \text{ m}
\]

Ans.

\[
(\text{Ans:} \quad x_{\text{max}} = 0.839 \text{ m})
\]
The four smooth balls each have the same mass $m$. If $A$ and $B$ are rolling forward with velocity $v$ and strike $C$, explain why after collision $C$ and $D$ each move off with velocity $v$. Why doesn’t $D$ move off with velocity $2v$? The collision is elastic, $e = 1$. Neglect the size of each ball.

**SOLUTION**

Collision will occur in the following sequence;

$B$ strikes $C$

\[
\begin{ aligned}
\text{( } & m_v = -mv_B + mv_C \\
& v = -v_B + v_C \\
\text{( } & e = 1 = \frac{v_C + v_B}{v} \\
& v_C = v, \quad v_B = 0
\end{aligned}
\]

$C$ strikes $D$

\[
\begin{ aligned}
\text{( } & m_v = -mv_C + mv_D \\
\text{( } & e = 1 = \frac{v_D + v_C}{v} \\
& v_C = 0, \quad v_D = v
\end{aligned}
\]

Ans.

$A$ strikes $B$

\[
\begin{ aligned}
\text{( } & m_v = -mv_A + mv_B \\
\text{( } & e = 1 = \frac{v_B + v_A}{v} \\
& v_B = v, \quad v_A = 0
\end{aligned}
\]

Ans.

Finally, $B$ strikes $C$

\[
\begin{ aligned}
\text{( } & m_v = -mv_B + mv_C \\
\text{( } & e = 1 = \frac{v_C + v_B}{v} \\
& v_C = v, \quad v_B = 0
\end{aligned}
\]

Ans.

Note: If $D$ rolled off with twice the velocity, its kinetic energy would be twice the energy available from the original two $A$ and $B$:

\[
\frac{1}{2}mv^2 + \frac{1}{2}mv^2 \neq \frac{1}{2}(2v)^2
\]

Ans:

\[
\begin{aligned}
v_C = 0, v_D = v \\
v_B = v, v_A = 0 \\
v_C = v, v_B = 0
\end{aligned}
\]
The four balls each have the same mass \( m \). If \( A \) and \( B \) are rolling forward with velocity \( v \) and strike \( C \), determine the velocity of each ball after the first three collisions. Take \( e = 0.5 \) between each ball.

**SOLUTION**

Collision will occur in the following sequence;

**B strikes C**

\[
\begin{align*}
\text{(} & \leftrightarrow \text{) } m v = m v_B + m v_C \\
& v = v_B + v_C \\
\text{(} & \leftrightarrow \text{) } e = 0.5 = \frac{v_C - v_B}{v} \\
& v_C = 0.75v \rightarrow, \quad v_B = 0.25v \rightarrow
\end{align*}
\]

**C strikes D**

\[
\begin{align*}
\text{(} & \leftrightarrow \text{) } m(0.75v) = m v_C + m v_D \\
\text{(} & \leftrightarrow \text{) } e = 0.5 = \frac{v_D - v_C}{0.75v} \\
& v_C = 0.1875v \rightarrow \quad \text{Ans.} \\
& v_D = 0.5625v \rightarrow \quad \text{Ans.}
\end{align*}
\]

**A strikes B**

\[
\begin{align*}
\text{(} & \leftrightarrow \text{) } m v + m(0.25v) = m v_A + m v_B \\
\text{(} & \leftrightarrow \text{) } e = 0.5 = \frac{v_B - v_A}{v - 0.25v} \\
& v_B = 0.8125v \rightarrow, \quad v_A = 0.4375v \rightarrow \quad \text{Ans.}
\end{align*}
\]

\[\text{Ans: } v_C = 0.1875v \rightarrow, \quad v_D = 0.5625v \rightarrow, \quad v_B = 0.8125v \rightarrow, \quad v_A = 0.4375v \rightarrow\]
Ball $A$ has a mass of 3 kg and is moving with a velocity of 8 m/s when it makes a direct collision with ball $B$, which has a mass of 2 kg and is moving with a velocity of 4 m/s. If $e = 0.7$, determine the velocity of each ball just after the collision. Neglect the size of the balls.

**SOLUTION**

*Conservation of Momentum.* The velocity of balls $A$ and $B$ before and after impact are shown in Fig. $a$

\[
m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2
\]

\[
3(8) + 2(-4) = 3v_A + 2v_B
\]

\[3v_A + 2v_B = 16 \tag{1}\]

*Coefficient of Restitution.*

\[
e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \quad 0.7 = \frac{v_B - v_A}{8 - (-4)}
\]

\[v_B - v_A = 8.4 \tag{2}\]

Solving Eqs. (1) and (2),

\[v_B = 8.24 \text{ m/s} \rightarrow \text{ Ans.}\]

\[v_A = -0.16 \text{ m/s} = 0.160 \text{ m/s} \leftarrow \text{ Ans.}\]
A 1-lb ball $A$ is traveling horizontally at 20 ft/s when it strikes a 10-lb block $B$ that is at rest. If the coefficient of restitution between $A$ and $B$ is $e = 0.6$, and the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the time for the block $B$ to stop sliding.

**SOLUTION**

\[
\pm \Sigma m_1 v_1 = \Sigma m_2 v_2
\]

\[
\left(\frac{1}{32.2}\right)(20) + 0 = \left(\frac{1}{32.2}\right)(v_A)_2 + \left(\frac{10}{32.2}\right)(v_B)_2
\]

\[
(v_A)_2 + 10(v_B)_2 = 20
\]

\[
\pm e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}
\]

\[
0.6 = \frac{(v_B)_2 - (v_A)_2}{20 - 0}
\]

\[
(v_B)_2 - (v_A)_2 = 12
\]

Thus,

\[
(v_B)_2 = 2.909 \text{ ft/s} \rightarrow
\]

\[
(v_A)_2 = -9.091 \text{ ft/s} = 9.091 \text{ ft/s} \leftarrow
\]

Block $B$:

\[
\pm m v_1 + \Sigma \int F \, dt = m v_2
\]

\[
\left(\frac{10}{32.2}\right)(2.909) - 4t = 0
\]

\[
t = 0.226 \text{ s}
\]

**Ans:**

\[
t = 0.226 \text{ s}
\]
### SOLUTION

Just before impact, the velocity of $A$ is

$$ T_1 + V_1 = T_2 + V_2 $$

$$ 0 + 0 = \frac{1}{2}mv_A^2 - mgh $$

$$ v_A = \sqrt{2gh} $$

\(+\downarrow\) \hspace{1cm} e = \frac{(v_B)_2 - (v_A)_2}{\sqrt{2gh}}

$$ e \sqrt{2gh} = (v_B)_2 - (v_A)_2 $$ \hspace{1cm} (1)

\(+\downarrow\) \hspace{1cm} \Sigma mv_1 = \Sigma mv_2

$$ m(v_A) + 0 = m(v_A)_2 + 2m(v_B)_2 $$ \hspace{1cm} (2)

Solving Eqs. (1) and (2) for $(v_B)_2$ yields;

$$(v_B)_2 = \frac{1}{3} \sqrt{2gh}(1 + e)$$

Ans: $$ (v_B)_2 = \frac{1}{3} \sqrt{2gh}(1 + e) $$
15–67.

The three balls each weigh 0.5 lb and have a coefficient of restitution of \( e = 0.85 \). If ball \( A \) is released from rest and strikes ball \( B \) and then ball \( B \) strikes ball \( C \), determine the velocity of each ball after the second collision has occurred. The balls slide without friction.

**SOLUTION**

**Ball \( A \):**

Datum at lowest point.

\[
T_1 + V_1 = T_2 + V_2
\]

\[
0 + (0.5)(3) = \frac{1}{2}(0.5)(v_A)^2 + 0
\]

\( (v_A)_1 = 13.90 \text{ ft/s} \)

**Balls \( A \) and \( B \):**

\[
\sum mv_1 = \sum mv_2
\]

\[
\left( \frac{0.5}{32.2} \right)(13.90) + 0 = \left( \frac{0.5}{32.2} \right)(v_A)_2 + \left( \frac{0.5}{32.2} \right)(v_B)_2
\]

\[
e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}
\]

\[
0.85 = \frac{(v_B)_2 - (v_A)_2}{13.90 - 0}
\]

Solving:

\( (v_A)_2 = 1.04 \text{ ft/s} \)  \hspace{1cm}  \text{Ans.}

\( (v_B)_2 = 12.86 \text{ ft/s} \)

**Balls \( B \) and \( C \):**

\[
\sum mv_2 = \sum mv_3
\]

\[
\left( \frac{0.5}{32.2} \right)(12.86) + 0 = \left( \frac{0.5}{32.2} \right)(v_B)_3 + \left( \frac{0.5}{32.2} \right)(v_C)_3
\]

\[
e = \frac{(v_C)_3 - (v_B)_3}{(v_B)_2 - (v_C)_2}
\]

\[
0.85 = \frac{(v_C)_3 - (v_B)_3}{12.86 - 0}
\]

Solving:

\( (v_B)_3 = 0.964 \text{ ft/s} \)  \hspace{1cm}  \text{Ans.}

\( (v_C)_3 = 11.9 \text{ ft/s} \)  \hspace{1cm}  \text{Ans.}

\textbf{Ans:}

\[
(v_A)_2 = 1.04 \text{ ft/s}
\]

\[
(v_B)_3 = 0.964 \text{ ft/s}
\]

\[
(v_C)_3 = 11.9 \text{ ft/s}
\]
A pitching machine throws the 0.5-kg ball toward the wall with an initial velocity \( v_A = 10 \text{ m/s} \) as shown. Determine (a) the velocity at which it strikes the wall at \( B \), (b) the velocity at which it rebounds from the wall if \( e = 0.5 \), and (c) the distance \( s \) from the wall to where it strikes the ground at \( C \).

**SOLUTION**

(a)

\[ (v_B)_1 = 10 \cos 30^\circ = 8.660 \text{ m/s} \]

\[ s = s_0 + v_0 t \]

\[ 3 = 0 + 10 \cos 30^\circ t \]

\[ t = 0.3464 \text{ s} \]

\[ v = v_0 + a_\ell t \]

\[ (v_B)_1 = 10 \sin 30^\circ - 9.81(0.3464) = 1.602 \text{ m/s} \]

\[ s = s_0 + v_0 t + \frac{1}{2} a_\ell t^2 \]

\[ h = 1.5 + 10 \sin 30^\circ(0.3464) - \frac{1}{2}(9.81)(0.3464)^2 = 2.643 \text{ m} \]

\[ (v_B)_1 = \sqrt{(1.602)^2 + (8.660)^2} = 8.81 \text{ m/s} \]

\[ \theta_1 = \tan^{-1}\left(\frac{1.602}{8.660}\right) = 10.5^\circ \]

(b)

\[ e = \frac{(v_B)_2 - (v_A)_1}{(v_A)_1 - (v_B)_1} = 0.5 = \frac{(v_{Bx})_2 - 0}{0 - (8.660)} \]

\[ (v_{Bx})_2 = 4.330 \text{ m/s} \]

\[ (v_{By})_2 = (v_{By})_1 = 1.602 \text{ m/s} \]

\[ (v_B)_2 = \sqrt{(4.330)^2 + (1.602)^2} = 4.62 \text{ m/s} \]

\[ \theta_2 = \tan^{-1}\left(\frac{1.602}{4.330}\right) = 20.3^\circ \]

(c)

\[ s = s_0 + v_{Bx} + \frac{1}{2} a_\ell t^2 \]

\[ -2.643 = 0 + 1.602(t) - \frac{1}{2}(9.81)(t)^2 \]

\[ t = 0.9153 \text{ s} \]

\[ s = s_0 + v_0 t \]

\[ s = 0 + 4.330(0.9153) = 3.96 \text{ m} \]
A 300-g ball is kicked with a velocity of $v_A = 25 \text{ m/s}$ at point $A$ as shown. If the coefficient of restitution between the ball and the field is $e = 0.4$, determine the magnitude and direction $\theta$ of the velocity of the rebounding ball at $B$.

**SOLUTION**

**Kinematics:** The parabolic trajectory of the football is shown in Fig. a. Due to the symmetrical properties of the trajectory, $v_B = v_A = 25 \text{ m/s}$ and $\phi = 30^\circ$.

**Conservation of Linear Momentum:** Since no impulsive force acts on the football along the $x$ axis, the linear momentum of the football is conserved along the $x$ axis.

\[ \left[ \begin{array}{c} \downarrow \end{array} \right] \begin{array}{c} m(v_B)_x = m(v'_B)_x \\ 0.3(25 \cos 30^\circ) = 0.3(v'_B)_x \\ (v'_B)_x = 21.65 \text{ m/s} \end{array} \]

**Coefficient of Restitution:** Since the ground does not move during the impact, the coefficient of restitution can be written as

\[ e = \frac{0 - (v'_B)_y}{(v_B)_y - 0} = \frac{-25 \sin 30^\circ}{0} = 0.4 \]

\[ (v'_B)_y = 5 \text{ m/s} \uparrow \]

Thus, the magnitude of $v'_B$ is

\[ v'_B = \sqrt{(v'_B)_x^2 + (v'_B)_y^2} = \sqrt{21.65^2 + 5^2} = 22.2 \text{ m/s} \quad \text{Ans.} \]

and the angle of $v'_B$ is

\[ \theta = \tan^{-1}\left[ \frac{(v'_B)_y}{(v'_B)_x} \right] = \tan^{-1}\left( \frac{5}{21.65} \right) = 13.0^\circ \quad \text{Ans.} \]

**Ans:**

\[ v'_B = 22.2 \text{ m/s} \]

\[ \theta = 13.0^\circ \]
15–70.

Two smooth spheres $A$ and $B$ each have a mass $m$. If $A$ is given a velocity of $v_0$, while sphere $B$ is at rest, determine the velocity of $B$ just after it strikes the wall. The coefficient of restitution for any collision is $e$.

**SOLUTION**

**Impact:** The first impact occurs when sphere $A$ strikes sphere $B$. When this occurs, the linear momentum of the system is conserved along the $x$ axis (line of impact). Referring to Fig. $a$,

$$m_Av_A + m_Bv_B = m_A(v_A)_1 + m_B(v_B)_1$$

$$m_0 + 0 = m(v_A)_1 + m(v_B)_1$$

$$(v_A)_1 + (v_B)_1 = v_0$$

(1)

$$e = \frac{(v_B)_1 - (v_A)_1}{v_A - v_B}$$

$$e = \frac{(v_B)_1 - (v_A)_1}{v_0 - 0}$$

$$(v_B)_1 - (v_A)_1 = ev_0$$

(2)

Solving Eqs. (1) and (2) yields

$$(v_B)_1 = \left(\frac{1 + e}{2}\right)v_0 \rightarrow (v_A)_1 = \left(\frac{1 - e}{2}\right)v_0 \rightarrow$$

The second impact occurs when sphere $B$ strikes the wall, Fig. $b$. Since the wall does not move during the impact, the coefficient of restitution can be written as

$$e = \frac{0 - (- (v_B)_2)}{(v_B)_1 - 0}$$

$$e = \frac{0 + (v_B)_2}{\left[\frac{1 + e}{2}\right]v_0 - 0}$$

$$(v_B)_2 = \frac{e(1 + e)}{2}v_0$$

Ans.

$$(v_B)_2 = \left(\frac{He}{2}\right)v_0$$

Ans:

$$(v_B)_2 = \frac{e(1 + e)}{2}v_0$$
15–71.

It was observed that a tennis ball when served horizontally 7.5 ft above the ground strikes the smooth ground at B 20 ft away. Determine the initial velocity $v_A$ of the ball and the velocity $v_B$ (and $\theta$) of the ball just after it strikes the court at B. Take $e = 0.7$.

**SOLUTION**

\[(\vec{v}) \quad s = s_0 + v_d t
\]
\[20 = 0 + v_A t \]
\[(+\downarrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2 \]

\[7.5 = 0 + 0 + \frac{1}{2} (32.2) t^2 \]
\[t = 0.682524 \]
\[v_A = 29.303 = 29.3 \text{ ft/s} \]
\[v_{Bx1} = 29.303 \text{ ft/s} \]
\[(+\downarrow) \quad v = v_0 + a_c t \]

\[v_{By1} = 0 + 32.2(0.68252) = 21.977 \text{ ft/s} \]

\[(\vec{v}) \quad m v_1 = m v_2 \]

\[v_{Bx2} = v_{B1x} = 29.303 \text{ ft/s} \rightarrow \]
\[e = \frac{v_{By2}}{v_{By1}} \]
\[0.7 = \frac{v_{By2}}{21.977}, \quad v_{By2} = 15.384 \text{ ft/s} \uparrow \]

\[v_B = \sqrt{(29.303)^2 + (15.384)^2} = 33.1 \text{ ft/s} \]
\[\theta = \tan^{-1} \frac{15.384}{29.303} = 27.7^\circ \]

**Ans:**

\[v_A = 29.3 \text{ ft/s} \]
\[v_B = 33.1 \text{ ft/s} \]
\[\theta = 27.7^\circ \]
*15–72.

The tennis ball is struck with a horizontal velocity \( \mathbf{v}_A \), strikes the smooth ground at \( B \), and bounces upward at \( \theta = 30^\circ \). Determine the initial velocity \( \mathbf{v}_A \), the final velocity \( \mathbf{v}_B \), and the coefficient of restitution between the ball and the ground.

**SOLUTION**

\[ (+\downarrow) \quad v^2 = v_0^2 + 2a_c(s - s_0) \]

\[ (v_{B1})^2 = 0 + 2(32.2)(7.5 - 0) \]

\[ v_{B1} = 21.9773 \text{ m/s} \]

\[ (+\downarrow) \quad v = v_0 + a_c t \]

\[ 21.9773 = 0 + 32.2 \, t \]

\[ t = 0.68252 \text{ s} \]

\[ (+\downarrow) \quad s = s_0 + v_0 t \]

\[ 20 = 0 + v_A (0.68252) \]

\[ v_A = 29.303 = 29.3 \text{ ft/s} \]

\[ (\downarrow\to) \quad m\mathbf{v}_1 = m\mathbf{v}_2 \]

\[ v_{Bx2} = v_{Bx1} = v_A = 29.303 \]

\[ v_{By1} = 29.303\cos 30^\circ = 33.8 \text{ ft/s} \]

\[ v_{By2} = 29.303\tan 30^\circ = 16.918 \text{ ft/s} \]

\[ e = \frac{v_{By2}}{v_{By1}} = \frac{16.918}{21.9773} = 0.770 \]

**Ans:**

\[ v_A = 29.3 \text{ ft/s} \]

\[ v_{Bx2} = 33.8 \text{ ft/s} \]

\[ e = 0.770 \]
15–73.

Two smooth disks $A$ and $B$ each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine their final velocities just after collision. The coefficient of restitution is $e = 0.75$.

**SOLUTION**

\[ \sum m v_1 = \sum m v_2 \]
\[ 0.5(4)(\frac{3}{5}) - 0.5(6) = 0.5(v_B)_{2x} + 0.5(v_A)_{2x} \]

\[ e = \frac{(v_A)_{2x} - (v_B)_{2x}}{(v_B)_{1x} - (v_A)_{1x}} \]
\[ 0.75 = \frac{(v_A)_{2x} - (v_B)_{2x}}{4(\frac{3}{5}) - (-6)} \]
\[ (v_A)_{2x} = 1.35 \text{ m/s} \rightarrow \]
\[ (v_B)_{2x} = 4.95 \text{ m/s} \leftarrow \]

\[ m v_1 = m v_2 \]
\[ 0.5(\frac{4}{5})(4) = 0.5(v_B)_{3y} \]
\[ (v_B)_{3y} = 3.20 \text{ m/s} \uparrow \]
\[ v_A = 1.35 \text{ m/s} \rightarrow \text{ Ans.} \]
\[ v_B = \sqrt{(4.59)^2 + (3.20)^2} = 5.89 \text{ m/s} \rightarrow \text{ Ans.} \]
\[ \theta = \tan^{-1} \frac{3.20}{4.95} = 32.9^\circ \leftarrow \text{ Ans.} \]
15–74.

Two smooth disks \( A \) and \( B \) each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine the coefficient of restitution between the disks if after collision \( B \) travels along a line, 30° counterclockwise from the \( y \) axis.

**SOLUTION**

\[ \Sigma mv_1 = \Sigma mv_2 \]

\[
\begin{align*}
\text{( ) } & \quad 0.5(4)(\frac{3}{5}) - 0.5(6) = -0.5(v_B)_{2x} + 0.5(v_A)_{2x} \\
& \quad -3.60 = -(v_B)_{2x} + (v_A)_{2x} \\
\text{() } & \quad 0.5(4)(\frac{4}{5}) = 0.5(v_B)_{2y} \\
& \quad (v_B)_{2y} = 3.20 \text{ m/s \uparrow} \\
& \quad (v_B)_{2x} = 3.20 \tan 30^\circ = 1.8475 \text{ m/s \leftarrow} \\
& \quad (v_A)_{2x} = -1.752 \text{ m/s} = 1.752 \text{ m/s \leftarrow} \\
\text{( ) } & \quad e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1} \\
& \quad e = \frac{-1.752-(-1.8475)}{4(\frac{3}{5})-(-6)} = 0.0113 \quad \text{Ans.}
\]

\[ e = 0.0113 \]

\( \text{Ans:} \)
The 0.5-kg ball is fired from the tube at A with a velocity of \( v = 6 \text{ m/s} \). If the coefficient of restitution between the ball and the surface is \( e = 0.8 \), determine the height \( h \) after it bounces off the surface.

**SOLUTION**

*Kinematics.* Consider the vertical motion from A to B.

\[
(v_B)_y^2 = (v_A)_y^2 + 2a_y[(s_B)_y - (s_A)_y] ; \\
(v_B)_y^2 = (6 \sin 30^\circ)^2 + 2(-9.81)(-2 - 0) \\
(v_B)_y = 6.9455 \text{ m/s} \downarrow
\]

*Coefficient of Restitution.* The \( y \)-component of the rebounding velocity at B is \( (v_B')_y \) and the ground does not move. Then

\[
\left (+\right ) \quad e = \frac{(v_B)_y^2 - (v_B')_y^2}{(v_B)_y - (v_B')_y} ; \\
0.8 = \frac{0 - (v_B')_y}{-6.9455 - 0} \\
(v_B')_y = 5.5564 \text{ m/s} \uparrow
\]

*Kinematics.* When the ball reach the maximum height \( h \) at C, \( (v_c)_y = 0 \).

\[
\left (+\right ) \quad (v_c)_y^2 = (v_B')_y^2 + 2a_y[(s_c)_y - (s_B)_y] ; \\
0^2 = 5.5564^2 + 2(-9.81)(h - 0) \\
h = 1.574 \text{ m} = 1.57 \text{ m} \quad \text{Ans.}
\]
A ball of mass $m$ is dropped vertically from a height $h_0$ above the ground. If it rebounds to a height of $h_1$, determine the coefficient of restitution between the ball and the ground.

**SOLUTION**

**Conservation of Energy:** First, consider the ball’s fall from position $A$ to position $B$. Referring to Fig. $a$,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^2 + (V_y)_A = \frac{1}{2}mv_B^2 + (V_y)_B$$

$$0 + mg(h_0) = \frac{1}{2}m(v_B)_1^2 + 0$$

Subsequently, the ball’s return from position $B$ to position $C$ will be considered.

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_B^2 + (V_y)_B = \frac{1}{2}mv_C^2 + (V_y)_C$$

$$\frac{1}{2}m(v_B)_2^2 + 0 = 0 + mgh_1$$

$$(v_B)_2 = \sqrt{2gh_1}$$

**Coefficient of Restitution:** Since the ground does not move,

$$e = -\frac{(v_B)_2}{(v_B)_1}$$

$$e = -\frac{\sqrt{2gh_1}}{-\sqrt{2gh_0}} = \sqrt{\frac{h_1}{h_0}}$$

Ans:

$$e = \sqrt{\frac{h_1}{h_0}}$$
15–77.

The cue ball $A$ is given an initial velocity $(v_A)_1 = 5 \text{ m/s}$. If it makes a direct collision with ball $B (e = 0.8)$, determine the velocity of $B$ and the angle $\theta$ just after it rebounds from the cushion at $C (e' = 0.6)$. Each ball has a mass of 0.4 kg. Neglect their size.

**SOLUTION**

*Conservation of Momentum:* When ball $A$ strikes ball $B$, we have

\[
m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2
\]

\[
0.4(5) + 0 = 0.4(v_A)_2 + 0.4(v_B)_2
\]

(1)

*Coefficient of Restitution:*

\[
e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}
\]

\[
(\pm) \quad 0.8 = \frac{(v_B)_2 - (v_A)_2}{5 - 0}
\]

(2)

Solving Eqs. (1) and (2) yields

$(v_A)_2 = 0.500 \text{ m/s} \quad (v_B)_2 = 4.50 \text{ m/s}$

*Conservation of “y” Momentum:* When ball $B$ strikes the cushion at $C$, we have

\[
m_B(v_B)_2 = m_B(v_B)_3
\]

\[
(\downarrow) \quad 0.4(4.50 \sin 30^\circ) = 0.4(v_B)_3 \sin \theta
\]

\[
(v_B)_3 \sin \theta = 2.25
\]

(3)

*Coefficient of Restitution (x):*

\[
e = \frac{(v_C)_2 - (v_B)_3}{(v_B)_2 - (v_C)_1}
\]

\[
(\pm) \quad 0.6 = \frac{0 - [- (v_B)_3 \cos \theta]}{4.50 \cos 30^\circ - 0}
\]

(4)

Solving Eqs. (1) and (2) yields

$(v_B)_3 = 3.24 \text{ m/s} \quad \theta = 43.9^\circ$

**Ans.**
15–78.

Using a slingshot, the boy fires the 0.2-lb marble at the concrete wall, striking it at B. If the coefficient of restitution between the marble and the wall is \( e = 0.5 \), determine the speed of the marble after it rebounds from the wall.

**SOLUTION**

**Kinematics:** By considering the \( x \) and \( y \) motion of the marble from \( A \) to \( B \), Fig. a,

\[
\begin{align*}
\left( \pm \right) \quad (s_B)_x &= (s_A)_x + (v_A)_x t \\
100 &= 0 + 75 \cos 45^\circ \ t \\
t &= 1.886 \text{ s}
\end{align*}
\]

and

\[
\begin{align*}
\left( \pm \right) \quad (s_B)_y &= (s_A)_y + (v_A)_y t + \frac{1}{2} a_y t^2 \\
(s_B)_y &= 0 + 75 \sin 45^\circ (1.886) + \frac{1}{2} (-32.2)(1.886^2) \\
&= 42.76 \text{ ft}
\end{align*}
\]

and

\[
\begin{align*}
\left( \pm \right) \quad (v_B)_y &= (v_A)_y + a_y t \\
(v_B)_y &= 75 \sin 45^\circ + (-32.2)(1.886) = -7.684 \text{ ft/s} = 7.684 \text{ ft/s} \downarrow
\end{align*}
\]

Since \((v_B)_y = (v_A)_y = 75 \cos 45^\circ = 53.03 \text{ ft/s}\), the magnitude of \( v_B \) is

\[
v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{53.03^2 + 7.684^2} = 53.59 \text{ ft/s}
\]

and the direction angle of \( v_B \) is

\[
\theta = \tan^{-1} \left( \frac{(v_B)_y}{(v_B)_x} \right) = \tan^{-1} \left( \frac{7.684}{53.03} \right) = 8.244^\circ
\]

**Conservation of Linear Momentum:** Since no impulsive force acts on the marble along the inclined surface of the concrete wall (\( x' \) axis) during the impact, the linear momentum of the marble is conserved along the \( x' \) axis. Referring to Fig. b,

\[
\begin{align*}
\left( + \right) \quad m_B(v'_B)_{x'} &= m_B(v_B)_{x'} \\
\frac{0.2}{32.2} (53.59 \sin 21.756^\circ) &= \frac{0.2}{32.2} (v_B \cos \phi) \\
v_B \cos \phi &= 19.862
\end{align*}
\]
15–78. Continued

**Coefficient of Restitution:** Since the concrete wall does not move during the impact, the coefficient of restitution can be written as

\[
\left( + \right) e = \frac{0 - (v_B')/\gamma}{(v_B')/\gamma - 0}
\]

\[
0.5 = \frac{-v_B' \sin \phi}{-53.59 \cos 21.756^\circ}
\]

\[
v_B' \sin \phi = 24.885
\]

(2)

Solving Eqs. (1) and (2) yields

\[
v_B' = 31.8 \text{ ft/s}
\]

Ans.
15–79.

The two disks $A$ and $B$ have a mass of 3 kg and 5 kg, respectively. If they collide with the initial velocities shown, determine their velocities just after impact. The coefficient of restitution is $e = 0.65$.

SOLUTION

\[
\begin{align*}
(v_{A1}) & = 6 \text{ m/s} \quad (v_{A1})_1 = 0 \\
(v_{B1}) & = -7 \cos 60^\circ = -3.5 \text{ m/s} \quad (v_{B1})_1 = -7 \cos 60^\circ = -6.062 \text{ m/s} \\
\left(\begin{array}{c}
\text{Ans:} \\
\end{array}\right) \\
& m_A(v_{A1})_1 + m_B(v_{B1})_1 = m_A(v_{A2}) + m_B(v_{B2}) \\
& 3(6) - 5(3.5) = 3(v_{A2})_1 + 5(v_{B2})_1 \\
\left(\begin{array}{c}
\text{Ans:} \\
\end{array}\right) \\
& e = \frac{(v_{B2})_1 - (v_{A2})_1}{(v_{A1})_1 - (v_{B1})_1} \\
& 0.65 = \frac{(v_{B2})_1 - (v_{A2})_1}{6(-3.5)} \\
& (v_{B2})_1 - (v_{A2})_1 = 6.175 \\
\end{align*}
\]

Solving,

\[
\begin{align*}
(v_{A2}) & = -3.80 \text{ m/s} \quad (v_{B2})_2 = 2.378 \text{ m/s} \\
(+) \quad & m_A(v_{A2})_1 + m_A(v_{A2})_2 \\
& (v_{A2})_2 = 0 \\
(+) \quad & m_B(v_{B2})_1 + m_B(v_{A2})_2 \\
& (v_{B2})_1 = -6.062 \text{ m/s} \\
& (v_{A2}) = \sqrt{(3.80)^2 + (0)^2} = 3.80 \text{ m/s} \leftarrow \text{ Ans.} \\
& (v_{B2}) = \sqrt{(2.378)^2 + (-6.062)^2} = 6.51 \text{ m/s} \leftarrow \text{ Ans.} \\
& (\theta_{B2}) = \tan^{-1}\left(\frac{6.062}{2.378}\right) = 68.6^\circ \leftarrow \text{ Ans.} \\
\end{align*}
\]
A ball of negligible size and mass \( m \) is given a velocity of \( v_0 \) on the center of the cart which has a mass \( M \) and is originally at rest. If the coefficient of restitution between the ball and walls \( A \) and \( B \) is \( e \), determine the velocity of the ball and the cart just after the ball strikes \( A \). Also, determine the total time needed for the ball to strike \( A \), rebound, then strike \( B \), and rebound and then return to the center of the cart. Neglect friction.

**SOLUTION**

After the first collision:

\[
\dot{\Sigma} m v_1 = \Sigma' m v_2
\]

\[
0 + m v_0 = m v_b + M v_c
\]

\[
e = \frac{v_c - v_b}{v_0}
\]

\[
m v_0 = m v_b + \frac{M}{m} v_c
\]

\[
e v_0 = v_c - v_b
\]

\[
v_b(1 + e) = \left(1 + \frac{M}{m}\right) v_c
\]

\[
v_c = \frac{v_b(1 + e) m}{(m + M)}
\]

\[
v_b = \frac{v_b(1 + e) m}{(m + M)} - e v_0
\]

\[
v_b \left( \frac{m + m e - e m - e M}{m + M} \right)
\]

\[
v_b \left( \frac{m - e M}{m + M} \right)
\]

The relative velocity on the cart after the first collision is

\[
e = \frac{v_{\text{rel}}}{v_0}
\]

\[
v_{\text{rel}} = e v_0
\]

Similarly, the relative velocity after the second collision is

\[
e = \frac{v_{\text{rel}}}{e v_0}
\]

\[
v_{\text{rel}} = e^2 v_0
\]

Total time is

\[
t = \frac{d}{v_0} + \frac{2d}{e v_0} + \frac{d}{e^2 v_0}
\]

\[
= \frac{d}{v_0} \left(1 + \frac{1}{e}\right)^2
\]

\[
v_c = \frac{v_b(1 + e) m}{(m + M)}
\]

\[
v_b = \frac{v_b(m - e M)}{m + M}
\]

\[
t = \frac{d}{v_0} \left(1 + \frac{1}{e}\right)^2
\]
15–81.

The girl throws the 0.5-kg ball toward the wall with an initial velocity \( v_A = 10 \text{ m/s} \). Determine (a) the velocity at which it strikes the wall at \( B \), (b) the velocity at which it rebounds from the wall if the coefficient of restitution \( e = 0.5 \), and (c) the distance \( s \) from the wall to where it strikes the ground at \( C \).

\[ 558 \]

**SOLUTION**

**Kinematics:** By considering the horizontal motion of the ball before the impact, we have

\[
\begin{align*}
\downarrow & \\
(\downarrow) & s_x = (s_0)_x + v_x t \\
& 3 = 0 + 10 \cos 30^\circ t \\
& t = 0.3464 \text{ s}
\end{align*}
\]

By considering the vertical motion of the ball before the impact, we have

\[
\begin{align*}
(\uparrow) & \\
& v_y = (v_0)_y + (a_c)_y t \\
& = 10 \sin 30^\circ + (-9.81)(0.3464) \\
& = 1.602 \text{ m/s}
\end{align*}
\]

The vertical position of point \( B \) above the ground is given by

\[
\begin{align*}
(\uparrow) & \\
& s_y = (s_0)_y + (v_0)_y t + \frac{1}{2} (a_c)_y t^2 \\
& (s_B)_y = 1.5 + 10 \sin 30^\circ (0.3464) + \frac{1}{2} (-9.81)(0.3464^2) = 2.643 \text{ m}
\end{align*}
\]

Thus, the magnitude of the velocity and its directional angle are

\[
\begin{align*}
(v_B)_1 &= \sqrt{(10 \cos 30^\circ)^2 + 1.602^2} = 8.807 \text{ m/s} = 8.81 \text{ m/s} \\
\theta &= \tan^{-1} \frac{1.602}{10 \cos 30^\circ} = 10.48^\circ = 10.5^\circ
\end{align*}
\]

**Conservation of “y” Momentum:** When the ball strikes the wall with a speed of \((v_B)_1 = 8.807 \text{ m/s}\), it rebounds with a speed of \((v_B)_2\).

\[
\begin{align*}
& m_b (v_B)_1 = m_b (v_B)_2 \\
\downarrow & \\
m_b (1.602) = m_b [(v_B)_2 \sin \phi] \\
(v_B)_2 \sin \phi &= 1.602
\end{align*}
\]

(1)

**Coefficient of Restitution \((x)\):**

\[
\begin{align*}
e &= \frac{(v_w)_2 - (v_B)_2}{(v_B)_1 - (v_w)_1} \\
& = 0 - \frac{[-(v_B)_2 \cos \phi]}{10 \cos 30^\circ - 0}
\end{align*}
\]

(2)
Solving Eqs. (1) and (2) yields

\[ \phi = 20.30^\circ = 20.3^\circ \quad (v_B)_2 = 4.617 \text{ m/s} = 4.62 \text{ m/s} \quad \text{Ans.} \]

**Kinematics**: By considering the vertical motion of the ball after the impact, we have

\[ \begin{align*}
\uparrow \quad s_y &= (s_0)_y + (v_0)_y t + \frac{1}{2} (a_y) t^2 \\
-2.643 &= 0 + 4.617 \sin 20.3^\circ t_1 + \frac{1}{2} (-9.81)t_1^2 \\
\end{align*} \]

\[ t_1 = 0.9153 \text{ s} \]

By considering the horizontal motion of the ball after the impact, we have

\[ \begin{align*}
\downarrow \quad s_x &= (s_0)_x + (v_x) t \\
s &= 0 + 4.617 \cos 20.3^\circ (0.9153) = 3.96 \text{ m} \quad \text{Ans.} \]

**Ans:**

(a) \( (v_B)_1 = 8.81 \text{ m/s}, \theta = 10.5^\circ \uparrow \)

(b) \( (v_B)_2 = 4.62 \text{ m/s}, \phi = 20.3^\circ \downarrow \)

(c) \( s = 3.96 \text{ m} \)
15–82.

The 20-lb box slides on the surface for which \( \mu_k = 0.3 \). The box has a velocity \( v = 15 \) ft/s when it is 2 ft from the plate. If it strikes the smooth plate, which has a weight of 10 lb and is held in position by an unstretched spring of stiffness \( k = 400 \) lb/ft, determine the maximum compression imparted to the spring. Take \( e = 0.8 \) between the box and the plate. Assume that the plate slides smoothly.

**SOLUTION**

\[ T_1 + \sum U_{k-2} = T_2 \]

\[
\frac{1}{2} \left( \frac{20}{32.2} \right) (15)^2 - (0.3)(20)(2) = \frac{1}{2} \left( \frac{20}{32.2} \right)(v_2)^2
\]

\( v_2 = 13.65 \) ft/s

\[
\sum mv_1 = \sum mv_2
\]

\[
\left( \frac{20}{32.2} \right)(13.65) = \left( \frac{20}{32.2} \right) v_A + 10 \frac{32.2}{v_B}
\]

\( e = \frac{(v_B)^2 - (v_A)^2}{(v_A)^2 - (v_B)^2} \)

\( 0.8 = \frac{v_p - v_A}{13.65} \)

Solving,

\( v_p = 16.38 \) ft/s, \( v_A = 5.46 \) ft/s

\[
T_1 + V_1 = T_2 + V_2
\]

\[
\frac{1}{2} \left( \frac{10}{32.2} \right) (16.38)^2 + 0 = 0 + \frac{1}{2} (400)(s)^2
\]

\( s = 0.456 \) ft

Ans.
15–83.

The 10-lb collar $B$ is at rest, and when it is in the position shown the spring is unstretched. If another 1-lb collar $A$ strikes it so that $B$ slides 4 ft on the smooth rod before momentarily stopping, determine the velocity of $A$ just after impact, and the average force exerted between $A$ and $B$ during the impact if the impact occurs in 0.002 s. The coefficient of restitution between $A$ and $B$ is $e = 0.5$.

**SOLUTION**

Collar $B$ after impact:

\[ T_2 + V_2 = T_3 + V_3 \]

\[ \frac{1}{2} \left( \frac{10}{32.2} \right) (v_B)^2 + 0 = 0 + \frac{1}{2} (20)(5 - 3)^2 \]

\[ (v_B)_2 = 16.05 \text{ ft/s} \]

System:

(1) $\Sigma m_1 v_1 = \Sigma m_1 v_2$

\[ \frac{1}{32.2} (v_A)_1 + 0 = \frac{1}{32.2} (v_A)_2 + \frac{10}{32.2} (16.05) \]

\[ (v_A)_1 - (v_A)_2 = 160.5 \]

(2) $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

\[ 0.5 = \frac{16.05 - (v_A)_2}{(v_A)_1 - 0} \]

\[ 0.5(v_A)_1 + (v_A)_2 = 16.05 \]

Solving:

\[ (v_A)_1 = 117.7 \text{ ft/s} = 118 \text{ ft/s} \rightarrow \]

\[ (v_A)_2 = -42.8 \text{ ft/s} = 42.8 \text{ ft/s} \leftarrow \text{ Ans.} \]

Collar $A$:

(3) $mv_1 + \sum \int F \, dt = mv_2$

\[ \left( \frac{1}{32.2} \right) (117.7) - F(0.002) = \left( \frac{1}{32.2} \right)(-42.8) \]

\[ F = 2492.2 \text{ lb} = 2.49 \text{ kip} \text{ Ans.} \]
A ball is thrown onto a rough floor at an angle $\theta$. If it rebounds at an angle $\phi$ and the coefficient of kinetic friction is $\mu$, determine the coefficient of restitution $e$. Neglect the size of the ball. *Hint:* Show that during impact, the average impulses in the $x$ and $y$ directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.

**SOLUTION**

\[ (+\downarrow) \quad e = \frac{0 - [-v_2 \sin \phi]}{v_1 \sin \theta - 0} = \frac{v_2 \sin \phi}{v_1 \sin \theta} \quad \text{(1)} \]

\[ (-\downarrow) \quad m(v_y)_1 + \int_{v_1}^{v_2} F_y \, dx = m(v_y)_2 \]

\[ mv_1 \cos \theta - F_x \Delta t = mv_2 \cos \phi \]

\[ F_x = \frac{mv_1 \cos \theta - mv_2 \cos \phi}{\Delta t} \quad \text{(2)} \]

\[ (+\downarrow) \quad m(v_x)_1 + \int_{v_1}^{v_2} F_y \, dx = m(v_x)_2 \]

\[ mv_1 \sin \theta - F_y \Delta t = -mv_2 \sin \phi \]

\[ F_y = \frac{mv_1 \sin \theta + mv_2 \sin \phi}{\Delta t} \quad \text{(3)} \]

Since $F_x = \mu F_y$, from Eqs. (2) and (3)

\[ \frac{mv_1 \cos \theta - mv_2 \cos \phi}{\Delta t} = \mu \left( \frac{mv_1 \sin \theta + mv_2 \sin \phi}{\Delta t} \right) \]

\[ \frac{v_2}{v_1} = \cos \theta - \mu \frac{\sin \theta}{\sin \phi + \cos \phi} \quad \text{(4)} \]

Substituting Eq. (4) into (1) yields:

\[ e = \frac{\sin \phi}{\sin \theta} \left( \frac{\cos \theta - \mu \sin \theta}{\mu \sin \phi + \cos \phi} \right) \quad \text{Ans.} \]
A ball is thrown onto a rough floor at an angle of $\theta = 45^\circ$. If it rebounds at the same angle $\phi = 45^\circ$, determine the coefficient of kinetic friction between the floor and the ball. The coefficient of restitution is $\epsilon = 0.6$. Hint: Show that during impact, the average impulses in the $x$ and $y$ directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.

**SOLUTION**

\[ e = \frac{0 - [-v_2 \sin \phi]}{v_1 \sin \theta - 0} = \frac{v_2 \sin \phi}{v_1 \sin \theta} \quad (1) \]

\[ m(v_x)_1 + \int_{t_1}^{t_2} F_x dx = m(v_x)_2 \]

\[ mv_1 \cos \theta - F_x \Delta t = mv_2 \cos \phi \]

\[ F_x = \frac{mv_1 \cos \theta - mv_2 \cos \phi}{\Delta t} \quad (2) \]

\[ m(v_y)_1 + \int_{t_1}^{t_2} F_y dx = m(v_y)_2 \]

\[ mv_1 \sin \theta - F_y \Delta t = -mv_2 \sin \phi \]

\[ F_y = \frac{mv_1 \sin \theta + mv_2 \sin \phi}{\Delta t} \quad (3) \]

Since $F_x = \mu F_y$, from Eqs. (2) and (3)

\[ \frac{mv_1 \cos \theta - mv_2 \cos \phi}{\Delta t} = \mu \left( \frac{mv_1 \sin \theta + mv_2 \sin \phi}{\Delta t} \right) \]

\[ \frac{v_2}{v_1} = \frac{\cos \theta - \mu \sin \theta}{\mu \sin \phi + \cos \phi} \quad (4) \]

Substituting Eq. (4) into (1) yields:

\[ e = \frac{\cos \theta - \mu \sin \theta}{\mu \sin \phi + \cos \phi} \]

\[ 0.6 = \frac{\sin 45^\circ \left( \cos 45^\circ - \mu \sin 45^\circ \right)}{\sin 45^\circ \left( \mu \sin 45^\circ + \cos 45^\circ \right)} \]

\[ 0.6 = \frac{1 - \mu}{1 + \mu} \quad \mu = 0.25 \quad \text{Ans.} \]

**Ans:**

$\mu_k = 0.25$
15–86.

Two smooth billiard balls $A$ and $B$ each have a mass of 200 g. If $A$ strikes $B$ with a velocity $(v_A)_1 = 1.5 \text{ m/s}$ as shown, determine their final velocities just after collision. Ball $B$ is originally at rest and the coefficient of restitution is $e = 0.85$. Neglect the size of each ball.

**SOLUTION**

$$(v_A)_1 = -1.5 \cos 40^\circ = -1.1491 \text{ m/s}$$

$$(v_A)_1 = -1.5 \sin 40^\circ = -0.9642 \text{ m/s}$$

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$-0.2(1.1491) + 0 = 0.2(v_A)_2 + 0.2(v_B)_2$$

$$e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}; \quad 0.85 = \frac{(v_A)_2 - (v_B)_2}{1.1491}$$

Solving,

$$(v_A)_2 = -0.08618 \text{ m/s}$$

$$(v_B)_2 = -1.0629 \text{ m/s}$$

For $A$:

$$(v_A)_2 = 0.9642 \text{ m/s}$$

For $B$:

$$(v_B)_2 = 0$$

Hence.

$$(v_B)_2 = (v_B)_2 = 1.06 \text{ m/s} \leftarrow \quad \text{Ans.}$$

$$(v_A)_2 = \sqrt{(-0.08618)^2 + (0.9642)^2} = 0.968 \text{ m/s} \quad \text{Ans.}$$

$$(\theta_A)_2 = \tan^{-1}\left(\frac{0.08618}{0.9642}\right) = 5.11^\circ \quad \text{Ans.}$$

**Ans:**

$$(v_B)_2 = 1.06 \text{ m/s} \leftarrow$$

$$(v_A)_2 = 0.968 \text{ m/s}$$

$$(\theta_A)_2 = 5.11^\circ \quad \text{Ans.}$$
The “stone” $A$ used in the sport of curling slides over the ice track and strikes another “stone” $B$ as shown. If each “stone” is smooth and has a weight of 47 lb, and the coefficient of restitution between the “stones” is $e = 0.8$, determine their speeds just after collision. Initially $A$ has a velocity of 8 ft/s and $B$ is at rest. Neglect friction.

**SOLUTION**

Line of impact ($x$-axis):

$\Sigma m v_1 = \Sigma m v_2$

$0 + \frac{47}{32.2} (8) \cos 30^\circ = \frac{47}{32.2} (v_B)_2x + \frac{47}{32.2} (v_A)_2x$

$e = 0.8 = \frac{(v_B)_2x - (v_A)_2x}{8 \cos 30^\circ - 0}$

Solving:

$(v_A)_2x = 0.6928 \text{ ft/s}$

$(v_B)_2x = 6.235 \text{ ft/s}$

Plane of impact ($y$-axis):

Stone $A$:

$mv_1 = mv_2$

$(\rho+) \ 0 = \frac{47}{32.2} (v_A)_2y$

$(v_A)_2y = 4$

Stone $B$:

$mv_1 = mv_2$

$(\rho+) \ 0 = \frac{47}{32.2} (v_B)_2y$

$(v_B)_2y = 0$

$(v_A)_2 = \sqrt{(0.6928)^2 + (4)^2} = 4.06 \text{ ft/s}$ \hspace{1cm} \text{Ans.}$

$(v_B)_2 = \sqrt{(0)^2 + (6.235)^2} = 6.235 = 6.24 \text{ ft/s}$ \hspace{1cm} \text{Ans.}$

Ans:

$(v_A)_2 = 4.06 \text{ ft/s}$

$(v_B)_2 = 6.24 \text{ ft/s}$
*15–88.

The “stone” $A$ used in the sport of curling slides over the ice track and strikes another “stone” $B$ as shown. If each “stone” is smooth and has a weight of 47 lb, and the coefficient of restitution between the “stone” is $e = 0.8$, determine the time required just after collision for $B$ to slide off the runway. This requires the horizontal component of displacement to be 3 ft.

**SOLUTION**

See solution to Prob. 15–87.

\[
(v_B)_2 = 6.235 \text{ ft/s} \\
3 = 0 + (6.235 \cos 60^\circ)t \\
t = 0.962 \text{ s}
\]

Ans.
15–89.

Two smooth disks $A$ and $B$ have the initial velocities shown just before they collide. If they have masses $m_A = 4 \text{ kg}$ and $m_B = 2 \text{ kg}$, determine their speeds just after impact. The coefficient of restitution is $e = 0.8$.

**SOLUTION**

**Impact.** The line of impact is along the line joining the centers of disks $A$ and $B$ represented by $y$ axis in Fig. $a$. Thus

$$ [(v_A)_1]_y = 15 \left( \frac{3}{5} \right) = 9 \text{ m/s} \uparrow \quad [(v_A)_1]_x = 15 \left( \frac{4}{5} \right) = 12 \text{ m/s} \downarrow$$

$$ [(v_B)_1]_y = 8 \text{ m/s} \uparrow \quad [(v_B)_1]_x = 0$$

**Coefficient of Restitution.** Along the line of impact ($y$ axis),

$$ +e = \frac{[(v_B)_2]_y - [(v_A)_2]_y}{[(v_A)_1]_y - [(v_B)_1]_y}; \quad 0.8 = \frac{[(v_B)_2]_y - [(v_A)_2]_y}{-9 - 8}$$

$$ [(v_A)_2]_y - [(v_B)_2]_y = 13.6 \quad (1)$$

**Conservation of ‘$y$’ Momentum.**

$$ +e \quad m_A[(v_A)_1]_y + m_B[(v_B)_1]_y = m_A[(v_A)_2]_y + m_B[(v_B)_2]_y$$

$$ 4(-9) + 2(8) = 4[(v_A)_2]_y + 2[(v_B)_2]_y$$

$$ 2[(v_A)_2]_y + [(v_B)_2]_y = -10 \quad (2)$$

Solving Eqs. (1) and (2)

$$ [(v_A)_2]_y = 1.20 \text{ m/s} \uparrow \quad [(v_B)_2]_y = -12.4 \text{ m/s} = 12.4 \text{ m/s} \downarrow$$

**Conservation of ‘$x$’ Momentum.** Since no impact occurs along the $x$ axis, the component of velocity of each disk remain constant before and after the impact. Thus

$$ [(v_A)_2]_x = [(v_A)_1]_x = 12 \text{ m/s} \downarrow \quad [(v_B)_2]_x = [(v_B)_1]_x = 0$$

Thus, the magnitude of the velocity of disks $A$ and $B$ just after the impact is

$$(v_A)_2 = \sqrt{[(v_A)_2]_x^2 + [(v_A)_2]_y^2} = \sqrt{12^2 + 1.20^2} = 12.06 \text{ m/s} = 12.1 \text{ m/s} \quad \text{Ans.}$$

$$(v_B)_2 = \sqrt{[(v_B)_2]_x^2 + [(v_B)_2]_y^2} = \sqrt{0^2 + 12.4^2} = 12.4 \text{ m/s} \quad \text{Ans.}$$

Ans:

$$(v_A)_2 = 12.1 \text{ m/s}$$

$$(v_B)_2 = 12.4 \text{ m/s}$$
Before a cranberry can make it to your dinner plate, it must pass a bouncing test which rates its quality. If cranberries having an are to be accepted, determine the dimensions $d$ and $h$ for the barrier so that when a cranberry falls from rest at $A$ it strikes the incline at $B$ and bounces over the barrier at $C$.

**SOLUTION**

**Conservation of Energy:** The datum is set at point $B$. When the cranberry falls from a height of 3.5 ft above the datum, its initial gravitational potential energy is $W(3.5) = 3.5W$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 3.5W = \frac{1}{2} \left( \frac{W}{32.2} \right) \left( v_c \right)_1^2 + 0$$

$$\left( v_c \right)_1 = 15.01 \text{ ft/s}$$

**Conservation of "x" Momentum:** When the cranberry strikes the plate with a speed of $(v_c)_1 = 15.01 \text{ ft/s}$, it rebounds with a speed of $(v_c)_2$.

$$m \left( v_c \right)_1 = m \left( v_c \right)_2$$

$$m \left( 15.01 \right) \left( \frac{3}{5} \right) = m \left[ (v_c)_2 \cos \phi \right]$$

$$\left( v_c \right)_2 \cos \phi = 9.008$$

**Coefficient of Restitution (y):**

$$e = \frac{(v_P)_2 - (v_c)_2}{\left( v_c \right)_1 - (v_P)_1}$$

$$\left( v_c \right)_2 = 0 - (v_c)_2 \sin \phi$$

$$0.8 = \frac{-15.01 \left( \frac{4}{5} \right)}{15.01 \left( \frac{3}{5} \right)}$$

Solving Eqs. (1) and (2) yields

$$\phi = 46.85^\circ$$

$$(v_c)_2 = 13.17 \text{ ft/s}$$

**Kinematics:** By considering the vertical motion of the cranberry after the impact, we have

$$v_y = (v_0)_y + a_y t$$

$$0 = 13.17 \sin 9.978^\circ + (-32.2) t$$

$$t = 0.07087 \text{ s}$$

$$s_y = (s_0)_y + (v_0)_y t + \frac{1}{2} (a_y) t^2$$

$$= 0 + 13.17 \sin 9.978^\circ (0.07087) + \frac{1}{2} (-32.2) \left( 0.07087^2 \right)$$

$$= 0.080864 \text{ ft}$$
By considering the horizontal motion of the cranberry after the impact, we have

\[
\begin{align*}
\frac{4}{5}d &= 0 + 13.17 \cos 9.978^\circ \times (0.07087) \\
d &= 1.149 \text{ ft} = 1.15 \text{ ft} \quad \text{Ans.}
\end{align*}
\]

Thus,

\[
\begin{align*}
h &= s_y + \frac{3}{5}d = 0.080864 + \frac{3}{5}(1.149) = 0.770 \text{ ft} \quad \text{Ans.}
\end{align*}
\]

\[
\text{Ans:} \\
\begin{align*}
d &= 1.15 \text{ ft} \\
h &= 0.770 \text{ ft}
\end{align*}
\]
15–91.

The 200-g billiard ball is moving with a speed of 2.5 m/s when it strikes the side of the pool table at A. If the coefficient of restitution between the ball and the side of the table is \( e = 0.6 \), determine the speed of the ball just after striking the table twice, i.e., at A, then at B. Neglect the size of the ball.

**SOLUTION**

At A:

\[
(v_A)_1 = 2.5 \sin 45° = 1.7678 \text{ m/s} \rightarrow
\]

\[
e = \frac{(v_A)_2}{(v_A)_1}; \quad 0.6 = \frac{(v_A)_2}{1.7678}
\]

\[(v_A)_2 = 1.061 \text{ m/s} \leftarrow
\]

\[(v_A)_2 = (v_A)_1 = 2.5 \cos 45° = 1.7678 \text{ m/s} \downarrow
\]

At B:

\[
e = \frac{(v_B)_3}{(v_B)_2}; \quad 0.6 = \frac{(v_B)_3}{1.7678}
\]

\[(v_B)_3 = 1.061 \text{ m/s}
\]

\[(v_B)_3 = (v_A)_2 = 1.061 \text{ m/s}
\]

Hence,

\[
(v_B)_3 = \sqrt{(1.061)^2 + (1.061)^2} = 1.50 \text{ m/s}
\]

**Ans:**

\[(v_B)_3 = 1.50 \text{ m/s}\]
*15–92.

The two billiard balls $A$ and $B$ are originally in contact with one another when a third ball $C$ strikes each of them at the same time as shown. If ball $C$ remains at rest after the collision, determine the coefficient of restitution. All the balls have the same mass. Neglect the size of each ball.

**SOLUTION**

Conservation of “$x$” momentum:

$$mv = 2mv' \cos 30^\circ$$

$$v = 2v' \cos 30^\circ$$

(1)

Coefficient of restitution:

$$e = \frac{v'}{v \cos 30^\circ}$$

(2)

Substituting Eq. (1) into Eq. (2) yields:

$$e = \frac{v'}{2v' \cos^2 30^\circ} = \frac{2}{3}$$

Ans.
15–93.

Disks A and B have a mass of 15 kg and 10 kg, respectively. If they are sliding on a smooth horizontal plane with the velocities shown, determine their speeds just after impact. The coefficient of restitution between them is $e = 0.8$.

**SOLUTION**

*Conservation of Linear Momentum:* By referring to the impulse and momentum of the system of disks shown in Fig. a, notice that the linear momentum of the system is conserved along the $n$ axis (line of impact). Thus,

$$+ \mathbf{m}_A (v_A)_n + m_B (v_B)_n = m_A (v'_A)_n + m_B (v'_B)_n$$

$$15(10) \left( \frac{3}{5} \right) - 10(8) \left( \frac{3}{5} \right) = 15v'_A \cos \phi_A + 10v'_B \cos \phi_B$$

$$15v'_A \cos \phi_A + 10v'_B \cos \phi_B = 42 \quad (1)$$

Also, we notice that the linear momentum of disks A and B are conserved along the $t$ axis (tangent to plane of impact). Thus,

$$+ \mathbf{m}_A (v_A)_t = m_A (v_A)_t$$

$$15(10) \left( \frac{4}{5} \right) = 15v_A \sin \phi_A$$

$$v_A \sin \phi_A = 8$$

and

$$+ \mathbf{m}_B (v_B)_t = m_B (v_B)_t$$

$$10(8) \left( \frac{4}{5} \right) = 10v_B \sin \phi_B$$

$$v_B \sin \phi_B = 6.4 \quad (2)$$

*Coefficient of Restitution:* The coefficient of restitution equation written along the $n$ axis (line of impact) gives

$$+ \mathbf{e} = \frac{(v'_B)_n - (v'_A)_n}{(v_A)_n - (v_B)_n}$$

$$0.8 = \frac{v'_B \cos \phi_B - v'_A \cos \phi_A}{10 \left( \frac{3}{5} \right) - \left[ -8 \left( \frac{3}{5} \right) \right]}$$

$$v'_B \cos \phi_B - v'_A \cos \phi_A = 8.64 \quad (4)$$

Solving Eqs. (1), (2), (3), and (4), yeilds

$$v'_A = 8.19 \text{ m/s}$$

$$\phi_A = 102.52^\circ$$

$$v'_B = 9.38 \text{ m/s}$$

$$\phi_B = 42.99^\circ$$

Ans:

$$(v_A)_2 = 8.19 \text{ m/s}$$

$$(v_B)_2 = 9.38 \text{ m/s}$$
15–94.

Determine the angular momentum $H_O$ of the 6-lb particle about point $O$.

**SOLUTION**

*Position and Velocity Vector.* The coordinates of points $A$ and $B$ are $A(-8, 8, 12)$ ft and $B(0, 18, 0)$ ft. Then

$$r_{OB} = \{18j\} \text{ ft} \quad r_{OA} = \{-8i + 8j + 12k\} \text{ ft}$$

$$V_A = v_A \left(\frac{r_{AB}}{r_{AB}}\right) = 4 \left\{\frac{[0 - (-8)]i + (18 - 8)j + (0 - 12)k}{\sqrt{[0 - (-8)]^2 + (18 - 8)^2 + (0 - 12)^2}}\right\}$$

$$= \left\{\frac{32i}{\sqrt{308}} + \frac{40j}{\sqrt{308}} - \frac{48k}{\sqrt{308}}\right\} \text{ ft/s}$$

*Angular Momentum about Point $O$.*

$$H_O = r_{OB} \times mV_A$$

$$= \begin{vmatrix} i & j & k \\ 6 & 32 \frac{32}{\sqrt{308}} & 0 \\ 32.2 & \frac{40}{\sqrt{308}} & 6 \frac{-48}{\sqrt{308}} \end{vmatrix}$$

$$= \{-9.1735i - 6.1156k\} \text{ slug} \cdot \text{ft}^2/\text{s}$$

$$= \{-9.17i - 6.12k\} \text{ slug} \cdot \text{ft}^2/\text{s} \quad \text{Ans.}$$

Also,

$$H_O = r_{OA} \times mV_A$$

$$= \begin{vmatrix} i & j & k \\ 6 & -8 \frac{32}{\sqrt{308}} & 8 \frac{40}{\sqrt{308}} \\ 32.2 & 6 \frac{-48}{\sqrt{308}} & 6 \frac{12}{\sqrt{308}} \end{vmatrix}$$

$$= \{-9.1735i - 6.1156k\} \text{ slug} \cdot \text{ft}^2/\text{s}$$

$$= \{-9.17i - 6.12k\} \text{ slug} \cdot \text{ft}^2/\text{s} \quad \text{Ans.}$$

Ans:

$$\{-9.17i - 6.12k\} \text{ slug} \cdot \text{ft}^2/\text{s}$$
15–95.
Determine the angular momentum $H_p$ of the 6-lb particle about point $P$.

**SOLUTION**

*Position and Velocity Vector.* The coordinates of points $A$, $B$ and $P$ are $A(-8, 8, 12)$ ft, $B(0, 18, 0)$ ft and $P(-8, 0, 0)$. Then

$$ r_{PB} = [0 - (-8)]i + (18 - 0)j = \{8i + 18j\} \text{ ft} $$

$$ r_{PA} = [8 - (-8)]i + (8 - 0)j + (12 - 0)k = \{8i + 12k\} \text{ ft} $$

$$ V_A = v_A \left(\frac{r_{AB}}{r_{AB}}\right) = 4 \left\{ \frac{[0 - (-8)]i + (18 - 8)j + (0 - 12)k}{[0 - (-8)]^2 + (18 - 8)^2 + (0 - 12)^2} \right\} $$

$$ = \left\{ \frac{32i}{\sqrt{308}} + \frac{40j}{\sqrt{308}} - \frac{48k}{\sqrt{308}} \right\} \text{ ft/s} $$

**Angular Momentum about Point $P$.*

$$ H_p = r_{PA} \times mV_A $$

$$ = \begin{vmatrix} i & j & k \\ 0 & 8 & 12 \\ \frac{32}{32.2} \sqrt{308} & \frac{40}{32.2} \sqrt{308} & \frac{-48}{32.2} \sqrt{308} \end{vmatrix} $$

$$ = \{ -9.1735i + 4.0771j - 2.7181k \} \text{ slug} \cdot \text{ft}^2/\text{s} $$

$$ = \{ -9.17i + 4.08j - 2.72k \} \text{ slug} \cdot \text{ft}^2/\text{s} $$

**Ans.**

Also,

$$ H_p = r_{PB} \times mV_A $$

$$ = \begin{vmatrix} i & j & k \\ 8 & 18 & 0 \\ \frac{32}{32.2} \sqrt{308} & \frac{40}{32.2} \sqrt{308} & \frac{-48}{32.2} \sqrt{308} \end{vmatrix} $$

$$ = \{ -9.1735i + 4.0771j - 2.7181k \} \text{ slug} \cdot \text{ft}^2/\text{s} $$

$$ = \{ -9.17i + 4.08j - 2.72k \} \text{ slug} \cdot \text{ft}^2/\text{s} $$

**Ans:**

$$ \{ -9.17i + 4.08j - 2.72k \} \text{ slug} \cdot \text{ft}^2/\text{s} $$
*15–96.

Determine the angular momentum \( \mathbf{H}_p \) of each of the two particles about point \( O \).

**SOLUTION**

\[
\zeta + (\mathbf{H}_A)_O = (-1.5) \left[ 3(8) \left( \frac{4}{5} \right) \right] - (2) \left[ 3(8) \left( \frac{3}{5} \right) \right] = -57.6 \text{ kg} \cdot \text{m}^2/\text{s}
\]

\[
\zeta + (\mathbf{H}_B)_O = (-1) \left[ 4(6 \sin 30^\circ) \right] - (4) \left[ 4 (6 \cos 30^\circ) \right] = -95.14 \text{ kg} \cdot \text{m}^2/\text{s}
\]

Thus

\[
(\mathbf{H}_A)_O = \{-57.6 \, k\} \text{ kg} \cdot \text{m}^2/\text{s} \quad \text{Ans.}
\]

\[
(\mathbf{H}_B)_O = \{-95.1 \, k\} \text{ kg} \cdot \text{m}^2/\text{s} \quad \text{Ans.}
\]

**Ans:**

\[
(\mathbf{H}_A)_O = \{-57.6 \, k\} \text{ kg} \cdot \text{m}^2/\text{s}
\]

\[
(\mathbf{H}_B)_O = \{-95.1 \, k\} \text{ kg} \cdot \text{m}^2/\text{s}
\]
15–97.

Determine the angular momentum \( \mathbf{H}_p \) of each of the two particles about point \( P \).

\[ \zeta + (\mathbf{H}_A)_P = (2.5) \left[ 3(8)\left(\frac{4}{5}\right) \right] - (7) \left[ 3(8)\left(\frac{3}{5}\right) \right] = -52.8 \text{ kg} \cdot \text{m}^2/\text{s} \]

\[ \zeta + (\mathbf{H}_B)_P = (4)[4(6 \sin 30^\circ)] - 8[4(6 \cos 30^\circ)] = -118.28 \text{ kg} \cdot \text{m}^2/\text{s} \]

Thus,

\( (\mathbf{H}_A)_P = \{ -52.8 \text{k} \} \text{ kg} \cdot \text{m}^2/\text{s} \)  

\( (\mathbf{H}_B)_P = \{ -118 \text{k} \} \text{ kg} \cdot \text{m}^2/\text{s} \)  

Ans:

\[ (\mathbf{H}_A)_P = \{ -52.8 \text{k} \} \text{ kg} \cdot \text{m}^2/\text{s} \]

\[ (\mathbf{H}_B)_P = \{ -118 \text{k} \} \text{ kg} \cdot \text{m}^2/\text{s} \]
15–98.

Determine the angular momentum $H_O$ of the 3-kg particle about point $O$.

**SOLUTION**

**Position and Velocity Vectors.** The coordinates of points $A$ and $B$ are $A(2, -1.5, 2)$ m and $B(3, 3, 0)$.

$$r_{OB} = (3i + 3j) \text{ m} \quad r_{OA} = (2i - 1.5j + 2k) \text{ m}$$

$$V_A = v_A \left( \frac{r_{AB}}{r_{AB}} \right) = (6) \left[ \frac{(3 - 2)i + [3 - (-1.5)]j + (0.2)k}{\sqrt{(3 - 2)^2 + [3 - (-1.5)]^2 + (0 - 2)^2}} \right]$$

$$= \left\{ \frac{6}{\sqrt{25.25}}i + \frac{27}{\sqrt{25.25}}j - \frac{12}{\sqrt{25.25}}k \right\} \text{ m/s}$$

**Angular Momentum about Point O.** Applying Eq. 15

$$H_O = r_{OB} \times mV_A$$

$$= \left\{ \begin{array}{ccc} i & j & k \\ 3 & 3 & 0 \\ \frac{6}{\sqrt{25.25}} & \frac{27}{\sqrt{25.25}} & \frac{-12}{\sqrt{25.25}} \end{array} \right\}$$

$$= \{ -21.4928i + 21.4928j + 37.6124k \} \text{ kg} \cdot \text{m}^2/\text{s}$$

$$= \{ -21.5i + 21.5j + 37.6 \} \text{ kg} \cdot \text{m}^2/\text{s}$$

Ans.

Also,

$$H_O = r_{OA} \times mV_A$$

$$= \left\{ \begin{array}{ccc} i & j & k \\ 2 & -1.5 & 2 \\ \frac{6}{\sqrt{25.25}} & \frac{-27}{\sqrt{25.25}} & \frac{-12}{\sqrt{25.25}} \end{array} \right\}$$

$$= \{ -21.4928i + 21.4928j + 37.6124k \} \text{ kg} \cdot \text{m}^2/\text{s}$$

$$= \{ -21.5i + 21.5j + 37.6k \} \text{ kg} \cdot \text{m}^2/\text{s}$$

Ans.

Ans:

$$\{ -21.5i + 21.5j + 37.6 \} \text{ kg} \cdot \text{m}^2/\text{s}$$
15–99. Determine the angular momentum \( \mathbf{H}_p \) of the 3-kg particle about point \( P \).

**SOLUTION**

**Position and Velocity Vectors.** The coordinates of points \( A, B \) and \( P \) are \( A(2, -1.5, 2) \) m, \( B(3, 3, 0) \) m and \( P(-1, 1.5, 2) \) m.

\[
\mathbf{r}_{PA} = (2 - (-1))\mathbf{i} + (-1.5 - 1.5)\mathbf{j} + (2 - 2)\mathbf{k} = 3\mathbf{i} - 3\mathbf{j} \text{ m}
\]

\[
\mathbf{r}_{PB} = (3 - (-1))\mathbf{i} + (3 - 1.5)\mathbf{j} + (0 - 2)\mathbf{k} = 4\mathbf{i} + 1.5\mathbf{j} - 2\mathbf{k} \text{ m}
\]

\[
\mathbf{V}_A = v_A \mathbf{r}_{AB} / \mathbf{r}_{AB} = 6 \left[ \frac{(3 - 2)\mathbf{i} + [3 - (-1.5)]\mathbf{j} + (0 - 2)\mathbf{k}}{\sqrt{(3 - 2)^2 + [3 - (-1.5)]^2 + (0 - 2)^2}} \right]
\]

\[
= \left\{ \frac{6}{\sqrt{25.25}} \mathbf{i} + \frac{27}{\sqrt{25.25}} \mathbf{j} - \frac{12}{\sqrt{25.25}} \mathbf{k} \right\} \text{ m/s}
\]

**Angular Momentum about Point \( P \).** Applying Eq. 15

\[
\mathbf{H}_P = \mathbf{r}_{PA} \times m\mathbf{V}_A
\]

\[
= \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
3 & -3 & 0 \\
\frac{6}{\sqrt{25.25}} & \frac{27}{\sqrt{25.25}} & \frac{-12}{\sqrt{25.25}}
\end{vmatrix}
\]

\[
= \{ 21.4928\mathbf{i} + 21.4928\mathbf{j} + 59.1052\mathbf{k} \} \text{ kg} \cdot \text{m}^2/\text{s}
\]

\[
= \{ 21.5\mathbf{i} + 21.5\mathbf{j} + 59.1\mathbf{k} \} \text{ kg} \cdot \text{m}^2/\text{s} \quad \text{Ans.}
\]

Also,

\[
\mathbf{H}_P = \mathbf{r}_{PB} \times m\mathbf{V}_A
\]

\[
= \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
4 & 1.5 & -2 \\
\frac{6}{\sqrt{25.25}} & \frac{27}{\sqrt{25.25}} & \frac{-12}{\sqrt{25.25}}
\end{vmatrix}
\]

\[
= \{ 21.4928\mathbf{i} + 21.4928\mathbf{j} + 59.1052\mathbf{k} \} \text{ kg} \cdot \text{m}^2/\text{s}
\]

\[
= \{ 21.5\mathbf{i} + 21.5\mathbf{j} + 59.1\mathbf{k} \} \text{ kg} \cdot \text{m}^2/\text{s} \quad \text{Ans.}
\]

**Ans:**

\[
\{ 21.5\mathbf{i} + 21.5\mathbf{j} + 59.1\mathbf{k} \} \text{ kg} \cdot \text{m}^2/\text{s}
\]
Each ball has a negligible size and a mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the rod is subjected to a torque $M = (t^2 + 2) \text{ N} \cdot \text{m}$, where $t$ is in seconds, determine the speed of each ball when $t = 3$ s. Each ball has a speed $v = 2 \text{ m/s}$ when $t = 0$.

**SOLUTION**

*Principle of Angular Impulse and Momentum.* Referring to the FBD of the assembly, Fig. a

$$(H_Z)_1 + \sum \int_{H_1}^{H_2} M_Z \, dt = (H_Z)_2$$

$$2[0.5(10)(2)] + \int_0^{3s} (t^2 + 2) \, dt = 2[0.5(10v)]$$

$$v = 3.50 \text{ m/s}$$

**Ans.**
15–101.

The 800-lb roller-coaster car starts from rest on the track having the shape of a cylindrical helix. If the helix descends 8 ft for every one revolution, determine the speed of the car when \( t = 4 \) s. Also, how far has the car descended in this time? Neglect friction and the size of the car.

**SOLUTION**

\[ \theta = \tan^{-1}\left(\frac{\theta}{2\pi(8)}\right) = 9.043^\circ \]

\[ \sum F_y = 0; \quad N - 800 \cos 9.043^\circ = 0 \]

\[ N = 790.1 \text{ lb} \]

\[ H_1 + \int M \, dt = H_2 \]

\[ 0 + \int_0^4 8(790.1 \sin 9.043^\circ)dt = \frac{800}{32.2}(8)v_t \]

\[ v_t = 20.0 \text{ ft/s} \]

\[ v = \frac{20.0}{\cos 9.043^\circ} = 20.2 \text{ ft/s} \]

\[ T_1 + \sum U_{1-2} = T_2 \]

\[ 0 + 800h = \frac{1}{2} \left(\frac{800}{32.2}\right)(20.2)^2 \]

\[ h = 6.36 \text{ ft} \]
15–102.

The 800-lb roller-coaster car starts from rest on the track having the shape of a cylindrical helix. If the helix descends 8 ft for every one revolution, determine the time required for the car to attain a speed of 60 ft/s. Neglect friction and the size of the car.

**SOLUTION**

\[
\theta = \tan^{-1}\left(\frac{8}{2\pi(8)}\right) = 9.043^\circ
\]

\[\Sigma F_y = 0; \quad N - 800 \cos 9.043^\circ = 0\]

\[N = 790.1 \text{ lb}\]

\[v = \frac{v_t}{\cos 9.043^\circ}\]

\[60 = \frac{v_t}{\cos 9.043^\circ}\]

\[v_t = 59.254 \text{ ft/s}\]

\[H_1 + \int M \, dt = H_z\]

\[0 + \int_0^t 8(790.1 \sin 9.043^\circ) \, dt = \frac{800}{32.2}(8)(59.254)\]

\[t = 11.9 \text{ s}\]

Ans.
15–103.

A 4-lb ball \( B \) is traveling around in a circle of radius \( r_1 = 3 \) ft with a speed \( (v_B)_1 = 6 \) ft/s. If the attached cord is pulled down through the hole with a constant speed \( v_r = 2 \) ft/s, determine the ball’s speed at the instant \( r_2 = 2 \) ft. How much work has to be done to pull down the cord? Neglect friction and the size of the ball.

**SOLUTION**

\[
H_1 = H_2
\]

\[
\frac{4}{32.2}(6)(3) = \frac{4}{32.2} \cdot v_B(2)
\]

\( v_B = 9 \) ft/s

\( v_2 = \sqrt{9^2 + 2^2} = 9.22 \) ft/s

\( T_1 + \Sigma U_{1-2} = T_2 \)

\[
\frac{1}{2} \cdot \frac{4}{32.2}(6)^2 + \Sigma U_{1-2} = \frac{1}{2} \cdot \frac{4}{32.2}(9.22)^2
\]

\( \Sigma U_{1-2} = 3.04 \text{ ft}\cdot\text{lb} \)

Ans.

Ans:

\[ v_2 = 9.22 \text{ ft/s} \]

\[ \Sigma U_{1-2} = 3.04 \text{ ft}\cdot\text{lb} \]
A 4-lb ball $B$ is traveling around in a circle of radius $r_1 = 3$ ft with a speed $(v_B)_1 = 6$ ft/s. If the attached cord is pulled down through the hole with a constant speed $v_r = 2$ ft/s, determine how much time is required for the ball to reach a speed of 12 ft/s. How far $r_2$ is the ball from the hole when this occurs? Neglect friction and the size of the ball.

**SOLUTION**

\[ v = \sqrt{(v_0)^2 + (2)^2} \]
\[ v_B = 11.832 \text{ ft/s} \]
\[ H_1 = H_2 \]
\[ \frac{4}{32.2}(6)(3) = \frac{4}{32.2}(11.832)(r_2) \]
\[ r_2 = 1.5213 = 1.52 \text{ ft} \]
\[ \Delta r = v_r t \]
\[ (3 - 1.5213) = 2t \]
\[ t = 0.739 \text{ s} \]

**Ans:**
\[ r_2 = 1.52 \text{ ft} \]
\[ t = 0.739 \text{ s} \]
The two blocks $A$ and $B$ each have a mass of 400 g. The blocks are fixed to the horizontal rods, and their initial velocity along the circular path is 2 m/s. If a couple moment of $M = (0.6) \text{ N} \cdot \text{m}$ is applied about $CD$ of the frame, determine the speed of the blocks when $t = 3$ s. The mass of the frame is negligible, and it is free to rotate about $CD$. Neglect the size of the blocks.

**SOLUTION**

\[
(H_o)_1 + \sum \int_{t_1}^{t_2} M_\theta dt = (H_o)_2
\]

\[
2[0.3(0.4)(2)] + 0.6(3) = 2[0.3(0.4)v]
\]

\[
v = 9.50 \text{ m/s}
\]

*Ans.*
A small particle having a mass $m$ is placed inside the semicircular tube. The particle is placed at the position shown and released. Apply the principle of angular momentum about point $O (\Sigma M_O = H_O)$, and show that the motion of the particle is governed by the differential equation $\theta + \left(\frac{g}{R}\right) \sin \theta = 0$.

**SOLUTION**

\[ \zeta + \Sigma M_O = \frac{dH_O}{dt} = -Rmg \sin \theta = \frac{d}{dt}(mvR) \]

\[ g \sin \theta = \frac{dv}{dt} = -\frac{d^2s}{dt^2} \]

But, $s = R\theta$

Thus, $g \sin \theta = -R\ddot{\theta}$

or, $\dot{\theta} + \left(\frac{g}{R}\right) \sin \theta = 0$

Q.E.D.
If the rod of negligible mass is subjected to a couple moment of $M = (30t^2) \text{ N} \cdot \text{m}$ and the engine of the car supplies a traction force of $F = (15t) \text{ N}$ to the wheels, where $t$ is in seconds, determine the speed of the car at the instant $t = 5 \text{ s}$. The car starts from rest. The total mass of the car and rider is 150 kg. Neglect the size of the car.

**SOLUTION**

**Free-Body Diagram:** The free-body diagram of the system is shown in Fig. a. Since the moment reaction $M_z$ has no component about the $z$ axis, the force reaction $F_z$ acts through the $z$ axis, and the line of action of $W$ and $N$ are parallel to the $z$ axis, they produce no angular impulse about the $z$ axis.

**Principle of Angular Impulse and Momentum:**

$$\left( H_1 \right)_z + \sum F_z \int_{t_1}^{t_2} M_z \, dt = \left( H_2 \right)_z$$

$$0 + \int_{0}^{5} 30t^2 \, dt + \int_{0}^{5} 15t(4) \, dt = 150v(4)$$

$$v = 3.33 \text{ m/s}$$

**Ans:**

$v = 3.33 \text{ m/s}$
*15–108.

When the 2-kg bob is given a horizontal speed of 1.5 m/s, it begins to rotate around the horizontal circular path A. If the force \( F \) on the cord is increased, the bob rises and then rotates around the horizontal circular path B. Determine the speed of the bob around path B. Also, find the work done by force \( F \).

**SOLUTION**

**Equations of Motion:** By referring to the free-body diagram of the bob shown in Fig. a,

\[ + F_r = 0; \quad F \cos \theta - 2(9.81) = 0 \quad (1) \]

\[ - \sum F_n = ma_n; \quad F \sin \theta = 2 \left( \frac{v^2}{l \sin \theta} \right) \quad (2) \]

Eliminating \( F \) from Eqs. (1) and (2) yields

\[ \frac{\sin^2 \theta}{\cos \theta} = \frac{v^2}{9.81l} \]

\[ 1 - \cos^2 \theta = \frac{v^2}{9.81l} \quad (3) \]

When \( l = 0.6 \text{ m} \), \( v = v_1 = 5 \text{ m/s} \). Using Eq. (3), we obtain

\[ \frac{1 - \cos^2 \theta_1}{\cos \theta_1} = \frac{1.5^2}{9.81(0.6)} \]

\[ \cos^2 \theta_1 + 0.3823 \cos \theta_1 - 1 = 0 \]

Solving for the root \( \theta_1 \), we obtain

\[ \theta_1 = 34.21^\circ \]

**Conservation of Angular Momentum:** By observing the free-body diagram of the system shown in Fig. b, notice that \( W \) and \( F \) are parallel to the \( z \) axis. \( M_s \) has no \( z \) component, and \( F_s \) acts through the \( z \) axis. Thus, they produce no angular impulse about the \( z \) axis. As a result, the angular momentum of the system is conserved about the \( z \) axis. When \( \theta = \theta_1 = 34.21^\circ \) and \( \theta = \theta_2 \), \( r = r_1 = 0.6 \sin 34.21^\circ = 0.3373 \text{ m} \) and \( r = r_2 = 0.3 \sin \theta_2 \). Thus,

\[ (H_z)_1 = (H_z)_2 \]

\[ r_1mv_1 = r_2mv_2 \]

\[ 0.3373(2)(1.5) = 0.3 \sin \theta_2 (2)v_2 \]

\[ v_2 \sin \theta_2 = 1.6867 \]
Substituting \( l = 0.3 \) and \( \theta = \theta_2 \) into Eq. (3) yields

\[
\frac{1 - \cos^2 \theta_2}{\cos \theta_2} = \frac{v_2}{9.81(0.3)}
\]

\[
\frac{1 - \cos^2 \theta_2}{\cos \theta_2} = \frac{v_2^2}{2.943}
\]

(Eq. 5)

Eliminating \( v_2 \) from Eqs. (4) and (5),

\[
\sin^3 \theta_2 \tan \theta_2 - 0.9667 = 0
\]

Solving the above equation by trial and error, we obtain

\( \theta_2 = 57.866^\circ \)

Substituting the result of \( \theta_2 \) into Eq. (4), we obtain

\( v_2 = 1.992 \text{ m/s} = 1.99 \text{ m/s} \)

Ans.

**Principle of Work and Energy:** When \( \theta \) changes from \( \theta_1 \) to \( \theta_2 \), \( W \) displaces vertically upward \( h = 0.6 \cos 34.21^\circ - 0.3 \cos 57.866^\circ = 0.3366 \text{ m} \). Thus, \( W \) does negatives work.

\[
T_1 + \sum U_{1-2} = T_2
\]

\[
\frac{1}{2} mv_1^2 + U_F + (-Wh) = \frac{1}{2} mv_2^2
\]

\[
\frac{1}{2} (2)(1.5^2) + U_F - 2(9.81)(0.3366) = \frac{1}{2} (2)(1.992)^2
\]

\( U_F = 8.32 \text{ N} \cdot \text{m} \)

Ans.

Ans:

\( v_2 = 1.99 \text{ m/s} \)

\( U_F = 8.32 \text{ N} \cdot \text{m} \)
15–109.

The elastic cord has an unstretched length \( l_0 = 1.5 \) ft and a stiffness \( k = 12 \text{ lb/ft} \). It is attached to a fixed point at \( A \) and a block at \( B \), which has a weight of 2 lb. If the block is released from rest from the position shown, determine its speed when it reaches point \( C \) after it slides along the smooth guide. After leaving the guide, it is launched onto the smooth horizontal plane. Determine if the cord becomes unstretched. Also, calculate the angular momentum of the block about point \( A \), at any instant after it passes point \( C \).

**SOLUTION**

\[
T_B + V_B = T_C + V_C
\]

\[
0 + \frac{1}{2} (12)(5 - 1.5)^2 = \frac{1}{2} (32.2) v_C^2 + \frac{1}{2} (12)(3 - 1.5)^2
\]

\[v_C = 43.95 = 44.0 \text{ ft/s}\] \(\text{Ans.}\)

There is a central force about \( A \), and angular momentum about \( A \) is conserved.

\[H_A = \frac{2}{32.2} (43.95)(3) = 8.19 \text{ slug \cdot ft}^2/\text{s}\] \(\text{Ans.}\)

If cord is slack \( AD = 1.5 \) ft

\[(H_A)_1 = (H_A)_2\]

\[8.19 = \frac{2}{32.2} (v_D)(1.5)\]

\[(v_D) = 88 \text{ ft/s}\]

But

\[
T_C + V_C = T_D + V_D
\]

\[
\frac{1}{2} (32.2)(43.95)^2 + \frac{1}{2} (12)(3 - 1.5)^2 = \frac{1}{2} (32.2)(v_D)^2 + 0
\]

\[v_D = 48.6 \text{ ft/s}\]

Since \( v_D < (v_D)_D \) cord will not unstretch. \(\text{Ans.}\)

\[v_C = 44.0 \text{ ft/s}\]

\[H_A = 8.19 \text{ slug \cdot ft}^2/\text{s}\]

The cord will not unstretch.

\[\text{Ans:}\]

\[\text{The cord will not unstretch.}\]
15–110.

The amusement park ride consists of a 200-kg car and passenger that are traveling at 3 m/s along a circular path having a radius of 8 m. If at $t = 0$, the cable $OA$ is pulled in toward $O$ at 0.5 m/s, determine the speed of the car when $t = 4$ s. Also, determine the work done to pull in the cable.

**SOLUTION**

*Conservation of Angular Momentum.* At $t = 4$ s,

$$r_2 = 8 - 0.5(4) = 6 \text{ m}.$$  

$$(H_0)_1 = (H_0)_2$$  

$$r_1mv_1 = r_2m(v_2),$$  

$$8[200(3)] = 6[200(v_2)],$$  

$$(v_2)_r = 4.00 \text{ m/s}$$  

Here, $(v_2)_r = 0.5 \text{ m/s}$. Thus

$$v_2 = \sqrt{(v_2)_r^2 + (v_2)_t^2} = \sqrt{4.00^2 + 0.5^2} = 4.031 \text{ m/s} = 4.03 \text{ m/s} \quad \text{Ans.}$$

*Principle of Work and Energy.*

$$T_1 + \sum U_{1-2} = T_2$$  

$$\frac{1}{2}(200)(3^2) + \sum U_{1-2} = \frac{1}{2}(200)(4.031)^2$$  

$$\sum U_{1-2} = 725 \text{ J} \quad \text{Ans.}$$

**Ans:**

$$v_2 = 4.03 \text{ m/s}$$  

$$\sum U_{1-2} = 725 \text{ J}$$
15–111.

A box having a weight of 8 lb is moving around in a circle of radius \( r_A = 2 \text{ ft} \) with a speed of \( (v_A)_1 = 5 \text{ ft/s} \) while connected to the end of a rope. If the rope is pulled inward with a constant speed of \( v_r = 4 \text{ ft/s} \), determine the speed of the box at the instant \( r_B = 1 \text{ ft} \). How much work is done after pulling in the rope from \( A \) to \( B \)? Neglect friction and the size of the box.

**SOLUTION**

\[
(H_c)_A = (H_c)_B; \quad \left( \frac{8}{32.2} \right) (2)(5) = \left( \frac{8}{32.2} \right) (1)(v_B)_{\text{tangent}}
\]

\( (v_B)_{\text{tangent}} = 10 \text{ ft/s} \)

\[ v_B = \sqrt{(10)^2 + (4)^2} = 10.77 = 10.8 \text{ ft/s} \]

\[
\sum U_{AB} = T_B - T_A \quad U_{AB} = \frac{1}{2} \left( \frac{8}{32.2} \right) (10.77)^2 - \frac{1}{2} \left( \frac{8}{32.2} \right) (5)^2
\]

\[ U_{AB} = 11.3 \text{ ft} \cdot \text{lb} \]

**Ans:**

\[ v_B = 10.8 \text{ ft/s} \]

\[ U_{AB} = 11.3 \text{ ft} \cdot \text{lb} \]
A toboggan and rider, having a total mass of 150 kg, enter horizontally tangent to a 90° circular curve with a velocity of \( v_A = 70\ \text{km/h} \). If the track is flat and banked at an angle of 60°, determine the speed \( v_B \) and the angle \( \theta \) of “descent,” measured from the horizontal in a vertical \( x-z \) plane, at which the toboggan exists at \( B \). Neglect friction in the calculation.

**SOLUTION**

\[
\begin{align*}
v_A &= 70\ \text{km/h} = 19.44\ \text{m/s} \\
(H_A)_z &= (H_B)_z \\
150(19.44)(60) &= 150(v_B)\cos\theta(57)
\end{align*}
\]

Datum at \( B \):

\[
T_A + V_A = T_B + V_B
\]

\[
\frac{1}{2}(150)(19.44)^2 + 150(9.81)h = \frac{1}{2}(150)(v_B)^2 + 0
\]

Since \( h = (r_A - r_B)\tan 60° = (60 - 57)\tan 60° = 5.196 \)

Solving Eq. (1) and Eq (2):

\[
\begin{align*}
v_B &= 21.9\ \text{m/s} & \text{Ans.} \\
\theta &= 20.9 & \text{Ans.}
\end{align*}
\]

**Ans:**

\[
\begin{align*}
v_B &= 21.9\ \text{m/s} \\
\theta &= 20.9
\end{align*}
\]
15–113.

An earth satellite of mass 700 kg is launched into a free-flight trajectory about the earth with an initial speed of \( v_A = 10 \text{ km/s} \) when the distance from the center of the earth is \( r_A = 15 \text{ Mm} \). If the launch angle at this position is \( \phi_A = 70^\circ \), determine the speed \( v_B \) of the satellite and its closest distance \( r_B \) from the center of the earth. The earth has a mass \( M_e = 5.976(10^{24}) \) kg. \( \text{Hint:} \) Under these conditions, the satellite is subjected only to the earth’s gravitational force, \( F = \frac{G M_e m_s}{r^2} \), Eq. 13–1. For part of the solution, use the conservation of energy.

**SOLUTION**

\[
(H_O)_1 = (H_O)_2
\]

\[
m_s (v_A \sin \phi_A) r_A = m_s (v_B) r_B
\]

\[
700[10(10^3) \sin 70^\circ](15)(10^6) = 700(v_B)(r_B)
\]

\[
T_A + V_A = T_B + V_B
\]

\[
\frac{1}{2} m_s (v_A)^2 - \frac{G M_e m_s}{r_A} = \frac{1}{2} m_s (v_B)^2 - \frac{G M_e m_s}{r_B}
\]

\[
\frac{1}{2} (700)[10(10^3)]^2 = \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{[15(10^6)]} - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{r_B}
\]

Solving,

\( v_B = 10.2 \text{ km/s} \) \hspace{1cm} \text{Ans.}

\( r_B = 13.8 \text{ Mm} \) \hspace{1cm} \text{Ans.}

\text{Ans:}

\( v_B = 10.2 \text{ km/s} \)

\( r_B = 13.8 \text{ Mm} \)
15–114.

The fire boat discharges two streams of seawater, each at a flow of 0.25 m³/s and with a nozzle velocity of 50 m/s. Determine the tension developed in the anchor chain needed to secure the boat. The density of seawater is ρ_{sw} = 1020 kg/m³.

**SOLUTION**

**Steady Flow Equation:** Here, the mass flow rate of the sea water at nozzles A and B are \( \frac{dm_A}{dt} = \frac{dm_B}{dt} = \rho_{sw} Q = 1020(0.25) = 225 \text{ kg/s} \). Since the sea water is collected from the larger reservoir (the sea), the velocity of the sea water entering the control volume can be considered zero. By referring to the free-body diagram of the control volume (the boat),

\[
F_x = \sum \frac{dm_A}{dt} (v_{A})_x + \frac{dm_B}{dt} (v_{B})_x;
\]

\[
T \cos 60° = 225(50 \cos 30°) + 225(50 \cos 45°)
\]

\[
T = 40.114.87 \text{ N} = 40.1 \text{ kN}
\]

Ans: \( T = 40.1 \text{ kN} \)
The chute is used to divert the flow of water, $Q = 0.6 \text{ m}^3/\text{s}$. If the water has a cross-sectional area of 0.05 m$^2$, determine the force components at the pin $D$ and roller $C$ necessary for equilibrium. Neglect the weight of the chute and weight of the water on the chute. $\rho_w = 1 \text{ Mg/m}^3$.

**SOLUTION**

**Equations of Steady Flow:** Here, the flow rate $Q = 0.6 \text{ m}^3/\text{s}$. Then, $v = \frac{Q}{A} = \frac{0.6}{0.05} = 12.0 \text{ m/s}$. Also, $\frac{dm}{dt} = \rho_w Q = 1000 \times (0.6) = 600 \text{ kg/s}$. Applying Eqs. 15–26 and 15–28, we have

$$\zeta + \Sigma M_A = \frac{dm}{dt} (d_{DB} v_B - d_{DA} v_A);$$

$$-C_x (2) = 600 [0 - 1.38(12.0)] \quad C_x = 4968 \text{ N} = 4.97 \text{ kN} \quad \text{Ans.}$$

$$\sum F_x = \frac{dm}{dt} (v_{Bx} - v_{Ax});$$

$$D_x + 4968 = 600 (12.0 - 0) \quad D_x = 2232 \text{ N} = 2.23 \text{ kN} \quad \text{Ans.}$$

$$\sum F_y = \frac{dm}{dt} (v_{out} - v_{in});$$

$$D_y = 600 [0 - (-12.0)] \quad D_y = 7200 \text{ N} = 7.20 \text{ kN} \quad \text{Ans.}$$

**Ans:**

$C_x = 4.97 \text{ kN}$

$D_x = 2.23 \text{ kN}$

$D_y = 7.20 \text{ kN}$
The 200-kg boat is powered by the fan which develops a slipstream having a diameter of 0.75 m. If the fan ejects air with a speed of 14 m/s, measured relative to the boat, determine the initial acceleration of the boat if it is initially at rest. Assume that air has a constant density of \( \rho_a = 1.22 \text{ kg/m}^3 \) and that the entering air is essentially at rest. Neglect the drag resistance of the water.

**SOLUTION**

*Equations of Steady Flow:* Initially, the boat is at rest hence \( v_B = v_{a/b} \)

\[
= 14 \text{ m/s. Then, } Q = v_B A = 14 \left[ \frac{\pi}{4} (0.75^2) \right] = 6.185 \text{ m}^3/\text{s} \text{ and } \frac{dm}{dt} = \rho_a Q
\]

\[= 1.22(6.185) = 7.546 \text{ kg/s. Applying Eq. 15–26, we have}
\]

\[\Sigma F_x = \frac{dm}{dt}(v_B - v_A); \quad -F = 7.546(-14 - 0) \quad F = 105.64 \text{ N}
\]

*Equation of Motion:*

\[\rightarrow \Sigma F_x = ma; \quad 105.64 = 200a \quad a = 0.528 \text{ m/s}^2 \quad \text{Ans.}
\]
15–117.

The nozzle discharges water at a constant rate of 2 ft³/s. The cross-sectional area of the nozzle at A is 4 in², and at B the cross-sectional area is 12 in². If the static gauge pressure due to the water at B is 2 lb/in², determine the magnitude of force which must be applied by the coupling at B to hold the nozzle in place. Neglect the weight of the nozzle and the water within it. \( \gamma_w = 62.4 \text{ lb/ft}^3 \).

**SOLUTION**

\[
\frac{dm}{dt} = pQ = \left( \frac{62.4}{32.2} \right)(2) = 3.876 \text{ slug/s}
\]

\[
(v_{Bi}) = \frac{Q}{A_B} = \frac{2}{12/144} = 24 \text{ ft/s} \quad (v_{Bi}) = 0
\]

\[
(v_{Ai}) = \frac{Q}{A_A} = \frac{2}{4/144} = 72 \text{ ft/s} \quad (v_{Ai}) = 0
\]

\[F_B = p_B A_B = 2(12) = 24 \text{ lb}\]

Equations of steady flow:

\[\downarrow \sum F_x = \frac{dm}{dt} (v_{Ax} - v_{Bx}) ; \quad F_x = 3.876(0 - 24) = 117.01 \text{ lb}\]

\[\uparrow \sum F_y = \frac{dm}{dt} (v_{Ay} - v_{By}) ; \quad F_y = 3.876(72 - 0) = 279.06 \text{ lb}\]

\[F = \sqrt{F_x^2 + F_y^2} = \sqrt{117.01^2 + 279.06^2} = 303 \text{ lb} \quad \text{Ans.}\]
The blade divides the jet of water having a diameter of 4 in. If one-half of the water flows to the right while the other half flows to the left, and the total flow is $Q = 1.5 \text{ ft}^3/\text{s}$, determine the vertical force exerted on the blade by the jet, $\gamma_w = 62.4 \text{ lb/ft}^3$.

**SOLUTION**

*Equation of Steady Flow.* Here $\frac{dm}{dt} = \rho_wQ = \left(\frac{62.4}{32.2}\right)(1.5) = 2.9068 \text{ slug/s}$. The velocity of the water jet is $v_J = \frac{Q}{A} = \frac{1.5}{\frac{1}{2} \pi (\frac{1}{12})^2} = \frac{54}{\pi} \text{ ft/s}$. Referring to the FBD of the control volume shown in Fig. a,

$$+\Sigma F_y = \frac{dm}{dt}[(v_B)_y - (v_A)_y];$$

$$F = 2.9068 \left[ 0 - \left( \frac{54}{\pi} \right) \right] = 49.96 \text{ lb} = 50.0 \text{ lb} \quad \text{Ans.}$$

**Ans:**

$F = 50.0 \text{ lb}$
15–119.

The blade divides the jet of water having a diameter of 3 in. If one-fourth of the water flows downward while the other three-fourths flows upwards, and the total flow is \( Q = 0.5 \text{ ft}^3/\text{s} \), determine the horizontal and vertical components of force exerted on the blade by the jet, \( \gamma_w = 62.4 \text{ lb/ft}^3 \).

**SOLUTION**

_Equations of Steady Flow:_ Here, the flow rate \( Q = 0.5 \text{ ft}^3/\text{s} \). Then,

\[
\frac{\nu}{A} = \frac{Q}{\frac{n}{4} \left(\frac{3}{12}\right)^2} = 10.19 \text{ ft/s}. \text{ Also, } \frac{dm}{dt} = \rho_w Q = \frac{62.4}{32.2} \times (0.5) = 0.9689 \text{ slug/s.}
\]

Applying Eq. 15–25 we have

\[
\sum F_x = \sum \frac{dm}{dt} (v_{out_x} - v_{in_x}) ; - F_x = 0 - 0.9689 \times 10.19 \quad F_x = 9.87 \text{ lb} \quad \text{**Ans.**}
\]

\[
\sum F_y = \sum \frac{dm}{dt} (v_{out_y} - v_{in_y}) ; F_y = \frac{3}{4} \times (0.9689) \times 10.19 + \frac{1}{4} \times (0.9689) \times (-10.19)
\]

\[
F_y = 4.93 \text{ lb} \quad \text{**Ans.**}
\]

**Ans:**

\[
F_x = 9.87 \text{ lb} \\
F_y = 4.93 \text{ lb}
\]
The gauge pressure of water at A is 150.5 kPa. Water flows through the pipe at A with a velocity of 18 m/s, and out the pipe at B and C with the same velocity \( v \). Determine the horizontal and vertical components of force exerted on the elbow necessary to hold the pipe assembly in equilibrium. Neglect the weight of water within the pipe and the weight of the pipe. The pipe has a diameter of 50 mm at A, and at B and C the diameter is 30 mm, \( \rho_w = 1000 \text{ kg/m}^3 \).

**SOLUTION**

**Continuity.** The flow rate at B and C are the same since the pipe have the same diameter there. The flow rate at A is

\[
Q_A = v_A A_A = (18)[\pi (0.025^2)] = 0.01125 \pi \text{ m}^3/\text{s}
\]

Continuity negatives that

\[
Q_A = Q_B + Q_C; \quad 0.01125 \pi = 2Q
\]

Thus,

\[
v_c = v_B = \frac{Q}{A} = \frac{0.005625 \pi}{\pi (0.015^2)} = 25 \text{ m/s}
\]

**Equation of Steady Flow.** The force due to the pressure at A is

\[
P = \rho_A A_A = (150.5)(10^3)[\pi (0.025^2)] = 94,062.5 \pi \text{ N.}
\]

Here, \( \frac{dm_A}{dt} = \rho_w Q_A \)

\[
= 1000(0.01125 \pi) = 11.25 \pi \text{ kg/s} \quad \text{and} \quad \frac{dm_A}{dt} = \frac{dM_c}{dt} = \rho_w Q = 1000(0.005625 \pi)
\]

\[
= 5.625 \pi \text{ kg/s}.
\]

\[
\Sigma F_x = \frac{dm_B}{dt}(v_B)_x + \frac{dm_c}{dt}(v_c)_x - \frac{dm_A}{dt}(v_A)_x;
\]

\[
F_x = (5.625 \pi)(25) + (5.625 \pi) \left[ 25 \left( \frac{4}{5} \right) \right] - (11.25 \pi)(0)
\]

\[
= 795.22 \text{ N} = 795 \text{ N}
\]

\[
\Sigma F_y = \frac{dm_B}{dt}(v_B)_y + \frac{dm_c}{dt}(v_c)_y - \frac{dm_A}{dt}(v_A)_y;
\]

\[
94,062.5 \pi - F_y = (5.625 \pi)(0) + (5.625 \pi) \left[ -25 \left( \frac{3}{5} \right) \right] - (11.25 \pi)(18)
\]

\[
F_y = 1196.75 \text{ N} = 1.20 \text{ kN}
\]

**Ans:**

\[
F_x = 795 \text{ N}
\]

\[
F_y = 1.20 \text{ kN}
\]
The gauge pressure of water at \( C \) is 40 lb/in\(^2\). If water flows out of the pipe at \( A \) and \( B \) with velocities \( v_A = 12 \text{ ft/s} \) and \( v_B = 25 \text{ ft/s} \), determine the horizontal and vertical components of force exerted on the elbow necessary to hold the pipe assembly in equilibrium. Neglect the weight of water within the pipe and the weight of the pipe. The pipe has a diameter of 0.75 in. at \( C \), and at \( A \) and \( B \) the diameter is 0.5 in. \( \gamma_w = 62.4 \text{ lb/ft}^3 \).

**SOLUTION**

\[
\frac{dm_A}{dt} = \frac{62.4}{32.2} (12)(\pi) \left(\frac{0.25}{12}\right)^2 = 0.03171 \text{ slug/s}
\]

\[
\frac{dm_B}{dt} = \frac{62.4}{32.2} (25)(\pi) \left(\frac{0.25}{12}\right)^2 = 0.06606 \text{ slug/s}
\]

\[
\frac{dm_C}{dt} = 0.03171 + 0.06606 = 0.09777 \text{ slug/s}
\]

\[
v_C A_C = v_A A_A + v_B A_B
\]

\[
v_C (\pi) \left(\frac{0.375}{12}\right)^2 = 12(\pi) \left(\frac{0.25}{12}\right)^2 + 25(\pi) \left(\frac{0.25}{12}\right)^2
\]

\[
v_C = 16.44 \text{ ft/s}
\]

\[
\sum F_x = \frac{dm_B}{dt} v_B + \frac{dm_A}{dt} v_A - \frac{dm_C}{dt} v_C,
\]

\[
40(\pi)(0.375)^2 - F_x = 0 - 0.03171(12) \left(\frac{3}{5}\right) - 0.09777(16.44)
\]

\[F_x = 19.5 \text{ lb}\]

\[
\sum F_y = \frac{dm_B}{dt} v_B + \frac{dm_A}{dt} v_A - \frac{dm_C}{dt} v_C,
\]

\[
F_y = 0.06606(25) + 0.03171 \left(\frac{4}{5}\right)(12) - 0
\]

\[F_y = 1.9559 = 1.96 \text{ lb}\]
15–122.

The fountain shoots water in the direction shown. If the water is discharged at 30° from the horizontal, and the cross-sectional area of the water stream is approximately 2 in², determine the force it exerts on the concrete wall at B. \( \gamma_w = 62.4 \text{ lb/ft}^3 \).

**SOLUTION**

\[
\begin{align*}
(\downarrow) & \quad s = s_0 + v_0 t \\
& \quad 20 = 0 + v_A \cos 30° t \\
(\uparrow) & \quad v = v_0 + a_t \\
& \quad -(v_A \sin 30°) = (v_A \sin 30°) - 32.2 t
\end{align*}
\]

Solving,

\[ t = 0.8469 \text{ s} \]

\[ v_A = v_B = 27.27 \text{ ft/s} \]

At B:

\[
\begin{align*}
\frac{dm}{dt} &= \rho v A \\
&= \left(62.4 \text{ lb/ft}^3\right)\left(27.27 \text{ ft/s}\right)\left(\frac{2}{144}\right) \\
&= 0.7340 \text{ slug/s}
\end{align*}
\]

\[
\begin{align*}
\gamma \Sigma F &= \frac{dm}{dt}(v_A - v_B) \\
&= 0.7340(0 - 27.27) \\
F &= 20.0 \text{ lb}
\end{align*}
\]

Ans: 

\[ F = 20.0 \text{ lb} \]
A plow located on the front of a locomotive scoops up snow at the rate of 10 ft³/s and stores it in the train. If the locomotive is traveling at a constant speed of 12 ft/s, determine the resistance to motion caused by the shoveling. The specific weight of snow is \( \gamma_s = 6 \text{ lb/ft}^3 \).

**SOLUTION**

\[
\Sigma F_s = m \frac{dv}{dt} + v \frac{dm}{dt}
\]

\[
F = 0 + (12 - 0) \left( \frac{10(6)}{32.2} \right)
\]

\( F = 22.4 \text{ lb} \)  

**Ans:**

\( F = 22.4 \text{ lb} \)
The boat has a mass of 180 kg and is traveling forward on a river with a constant velocity of 70 km/h, measured relative to the river. The river is flowing in the opposite direction at 5 km/h. If a tube is placed in the water, as shown, and it collects 40 kg of water in the boat in 80 s, determine the horizontal thrust $T$ on the tube that is required to overcome the resistance due to the water collection and yet maintain the constant speed of the boat. $\rho_w = 1 \text{ Mg/m}^3$.

**SOLUTION**

\[
\frac{dm}{dt} = \frac{40}{80} = 0.5 \text{ kg/s}
\]

\[
v_{D/t} = (70) \left( \frac{1000}{3600} \right) = 19.444 \text{ m/s}
\]

\[
\sum F_s = m\frac{dv}{dt} + v_{D/t}\frac{dm}{dt}
\]

\[
T = 0 + 19.444(0.5) = 9.72 \text{ N}
\]

Ans: $T = 9.72 \text{ N}$
Water is discharged from a nozzle with a velocity of 12 m/s and strikes the blade mounted on the 20-kg cart. Determine the tension developed in the cord, needed to hold the cart stationary, and the normal reaction of the wheels on the cart. The nozzle has a diameter of 50 mm and the density of water is $\rho_w = 1000 \text{ kg/m}^3$.

**SOLUTION**

**Steady Flow Equation:** Here, the mass flow rate at sections $A$ and $B$ of the control volume is $\frac{dm}{dt} = \rho_w Q = \rho_w Av = \frac{\pi}{4} (0.05^2) (12) = 7.5\pi \text{ kg/s}$

Referring to the free-body diagram of the control volume shown in Fig. a,

$\sum F_x = \frac{dm}{dt} [(v_B)_x - (v_A)_x]; \quad -F_x = 7.5\pi(12 \cos 45^\circ - 12)$

$F_x = 82.81 \text{ N}$

$\sum F_y = \frac{dm}{dt} [(v_B)_y - (v_A)_y]; \quad F_y = 7.5\pi(12 \sin 45^\circ - 0)$

$F_y = 199.93 \text{ N}$

**Equilibrium:** Using the results of $F_x$ and $F_y$, and referring to the free-body diagram of the cart shown in Fig. b,

$\sum F_x = 0; \quad 82.81 - T = 0 \quad \Rightarrow T = 82.8 \text{ N} \quad \text{Ans.}$

$\sum F_y = 0; \quad N - 20(9.81) - 199.93 = 0 \quad \Rightarrow N = 396 \text{ N} \quad \text{Ans.}$
A snowblower having a scoop $S$ with a cross-sectional area of $A_s = 0.12 \text{ m}^2$ is pushed into snow with a speed of $v_s = 0.5 \text{ m/s}$. The machine discharges the snow through a tube $T$ that has a cross-sectional area of $A_T = 0.03 \text{ m}^2$ and is directed 60° from the horizontal. If the density of snow is $\rho_s = 104 \text{ kg/m}^3$, determine the horizontal force $P$ required to push the blower forward, and the resultant frictional force $F$ of the wheels on the ground, necessary to prevent the blower from moving sideways. The wheels roll freely.

**SOLUTION**

\[
\frac{dm}{dt} = \rho v_s A_s = (104)(0.5)(0.12) = 6.24 \text{ kg/s}
\]

\[
v_s = \frac{dm}{dt} \left( \frac{1}{\rho A_s} \right) = \left( \frac{6.24}{104(0.03)} \right) = 2.0 \text{ m/s}
\]

\[
\Sigma F_x = \frac{dm}{dt} (v_{T_x} - v_{S_x})
\]

\[-F = 6.24(-2\cos 60° - 0)
\]

\[F = 6.24 \text{ N} \quad \text{Ans.}
\]

\[
\Sigma F_y = \frac{dm}{dt} (v_{T_y} - v_{S_y})
\]

\[-P = 6.24(0 - 0.5)
\]

\[P = 3.12 \text{ N} \quad \text{Ans.}
\]
15–127.

The fan blows air at 6000 ft³/min. If the fan has a weight of 30 lb and a center of gravity at $G$, determine the smallest diameter $d$ of its base so that it will not tip over. The specific weight of air is $\gamma = 0.076$ lb/ft³.

**SOLUTION**

*Equations of Steady Flow:* Here $Q = \left( \frac{6000 \text{ ft}^3}{\text{min}} \right) \times \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 100 \text{ ft}^3/\text{s}$. Then,

\[ v = \frac{Q}{A} = \frac{100}{\pi (1.5^2)} = 56.59 \text{ ft/s}. \]

Also,

\[ \frac{dm}{dt} = \rho _a Q = \frac{0.076}{32.2} (100) = 0.2360 \text{ slug/s}. \]

Applying Eq. 15–26 we have

\[ a + \Sigma M_O = \frac{dm}{dt} \left( d_{OB} v_B - d_{OA} v_A \right); \quad 30 \left( 0.5 + \frac{d}{2} \right) = 0.2360 \left[ 4(56.59) - 0 \right] \]

\[ d = 2.56 \text{ ft} \quad \text{Ans.} \]
The nozzle has a diameter of 40 mm. If it discharges water uniformly with a downward velocity of 20 m/s against the fixed blade, determine the vertical force exerted by the water on the blade. $\rho_w = 1$ Mg/m$^3$.

**SOLUTION**

\[
\frac{dm}{dt} = \rho v A = (1000)(20)(\pi)(0.02)^2 = 25.13 \text{ kg/s}
\]

\[
\sum F_y = \frac{dm}{dt} (v_y - v_y) + \sum F_y
\]

\[
F = (25.13)(20 \sin 45^\circ - (-20))
\]

\[
F = 858 \text{ N}
\]

*Ans:* $F = 858$ N
The water flow enters below the hydrant at C at the rate of 0.75 m³/s. It is then divided equally between the two outlets at A and B. If the gauge pressure at C is 300 kPa, determine the horizontal and vertical force reactions and the moment reaction on the fixed support at C. The diameter of the two outlets at A and B is 75 mm, and the diameter of the inlet pipe at C is 150 mm. The density of water is \( \rho_w = 1000 \text{ kg/m}^3 \). Neglect the mass of the contained water and the hydrant.

**SOLUTION**

*Free-Body Diagram:* The free-body diagram of the control volume is shown in Fig. a. The force exerted on section A due to the water pressure is \( F_A = p_A A_C = 300(10^5) \left[ \frac{\pi}{4}(0.15^2) \right] = 5301.44 \text{ N} \). The mass flow rate at sections A, B, and C, are

\[
\begin{align*}
\frac{dm_A}{dt} &= \frac{dm_B}{dt} = \rho_w \left( \frac{Q}{2} \right) = 1000 \left( \frac{0.75}{2} \right) = 375 \text{ kg/s} \\
\frac{dm_C}{dt} &= \rho_w Q = 1000(0.75) = 750 \text{ kg/s}.
\end{align*}
\]

The speed of the water at sections A, B, and C are

\[
\begin{align*}
v_A &= v_B = \frac{Q/2}{A_A} = \frac{0.75/2}{\pi \left[ \frac{4}{4}(0.075^2) \right]} = 84.88 \text{ m/s} \\
v_C &= \frac{Q}{A_C} = \frac{0.75}{\pi \left[ \frac{4}{4}(0.15^2) \right]} = 42.44 \text{ m/s}.
\end{align*}
\]

*Steady Flow Equation:* Writing the force steady flow equations along the x and y axes,

\[
\begin{align*}
\downarrow \Sigma F_x &= \frac{dm_A}{dt} (v_A)_x + \frac{dm_B}{dt} (v_B)_x - \frac{dm_C}{dt} (v_C)_x; \\
C_x &= -375(84.88 \cos 30^\circ) + 375(84.88) - 0 \\
&= 4264.54 \text{ N} = 4.26 \text{ kN} \quad \text{Ans.}
\end{align*}
\]

\[
\begin{align*}
\downarrow \Sigma F_y &= \frac{dm_A}{dt} (v_A)_y + \frac{dm_B}{dt} (v_B)_y - \frac{dm_C}{dt} (v_C)_y; \\
-C_y + 5301.44 &= 375(84.88 \sin 30^\circ) + 0 - 750(42.44) \\
C_y &= 21216.93 \text{ N} = 2.12 \text{ kN} \quad \text{Ans.}
\end{align*}
\]

Writing the steady flow equation about point C,

\[
\begin{align*}
\downarrow \Sigma M_C &= \frac{dm_A}{dt} d v_A + \frac{dm_B}{dt} d v_B - \frac{dm_C}{dt} d v_C; \\
-375(0.65)(84.88 \cos 30^\circ) - 375(0.25)(84.88 \sin 30^\circ) \\
&\quad + \left[ -375(0.6)(84.88) \right] - 0 \\
M_C &= 5159.28 \text{ N} \cdot \text{m} = 5.16 \text{ kN} \cdot \text{m} \quad \text{Ans.}
\end{align*}
\]

\[\text{Ans:}\]

\[
\begin{align*}
C_x &= 4.26 \text{ kN} \\
C_y &= 2.12 \text{ kN} \\
M_C &= 5.16 \text{ kN} \cdot \text{m}
\end{align*}
\]
Sand drops onto the 2-Mg empty rail car at 50 kg/s from a conveyor belt. If the car is initially coasting at 4 m/s, determine the speed of the car as a function of time.

**SOLUTION**

**Gains Mass System.** Here the sand drops vertically onto the rail car. Thus \((v_i)_x = 0\).

Then

\[
V_D = V_i + V_{D/\delta}
\]

\[
(\downarrow) \quad v = (v_i)_x + (v_{D/\delta})_x
\]

\[
v = 0 + (v_{D/\delta})_x
\]

\[
(v_{D/\delta})_x = v
\]

Also, \(\frac{dm_i}{dt} = 50 \text{ kg/s} \) and \( m = 2000 + 50t \)

\[
\Sigma F_i = m \frac{dv}{dt} + (v_{D/\delta})_x \frac{dm_i}{dt},
\]

\[
0 = (2000 + 50t) \frac{dv}{dt} + v(50)
\]

\[
\frac{dv}{v} = -\frac{50}{2000 + 50t} dt
\]

Integrate this equation with initial condition \( v = 4 \text{ m/s} \) at \( t = 0 \).

\[
\int_{4 \text{ m/s}}^{v} \frac{dv}{v} = -50 \int_{0}^{t} \frac{dt}{2000 + 50t}
\]

\[
\ln v \bigg|_{4 \text{ m/s}}^{v} = -\ln \left( \frac{2000 + 50t}{2000} \right) \bigg|_{0}
\]

\[
\ln \frac{v}{4} = -\ln \left( \frac{2000 + 50t}{2000} \right)
\]

\[
\frac{v}{4} = \frac{2000}{2000 + 50t}
\]

\[
v = \left\{ \frac{8000}{2000 + 50t} \right\} \text{ m/s}
\]

**Ans:**

\[
v = \left\{ \frac{8000}{2000 + 50t} \right\} \text{ m/s}
\]
Sand is discharged from the silo at A at a rate of 50 kg/s with a vertical velocity of 10 m/s onto the conveyor belt, which is moving with a constant velocity of 1.5 m/s. If the conveyor system and the sand on it have a total mass of 750 kg and center of mass at point G, determine the horizontal and vertical components of reaction at the pin support B roller support A. Neglect the thickness of the conveyor.

**SOLUTION**

**Steady Flow Equation:** The moment steady flow equation will be written about point B to eliminate $B_x$ and $B_y$: Referring to the free-body diagram of the control volume shown in Fig. a,

$$+ \sum M_B = \frac{dm}{dt}(v_B - v_A);$$

$$750(9.81)(4) - A_y(8) = 50[0 - 8(5)]$$

$$A_y = 4178.5 \text{ N} = 4.18 \text{ kN} \quad \text{Ans.}$$

Writing the force steady flow equation along the x and y axes,

$$\sum F_x = \frac{dm}{dt}[(v_B)_x - (v_A)_x];$$

$$-B_x = 50(1.5 \cos 30° - 0)$$

$$B_x = |-64.95 \text{ N}| = 65.0 \text{ N} \quad \text{Ans.}$$

$$\sum F_y = \frac{dm}{dt}[(v_B)_y - (v_A)_y];$$

$$B_y + 4178.5 - 750(9.81)$$

$$= 50[1.5 \sin 30° - (-10)]$$

$$B_y = 3716.25 \text{ N} = 3.72 \text{ kN} \uparrow \quad \text{Ans.}$$
Sand is deposited from a chute onto a conveyor belt which is moving at 0.5 m/s. If the sand is assumed to fall vertically onto the belt at \( A \) at the rate of 4 kg/s, determine the belt tension \( F_B \) to the right of \( A \). The belt is free to move over the conveyor rollers and its tension to the left of \( A \) is \( F_C = 400 \) N.

**SOLUTION**

\[
(\pm a) \sum F_x = \frac{dm}{dt} (v_{Bx} - v_{Ax})
\]

\[
F_B - 400 = 4(0.5 - 0)
\]

\[
F_B = 2 + 400 = 402 \text{ N}
\]

**Ans:**

\[
F_B = 402 \text{ N}
\]
The tractor together with the empty tank has a total mass of 4 Mg. The tank is filled with 2 Mg of water. The water is discharged at a constant rate of 50 kg/s with a constant velocity of 5 m/s, measured relative to the tractor. If the tractor starts from rest, and the rear wheels provide a resultant traction force of 250 N, determine the velocity and acceleration of the tractor at the instant the tank becomes empty.

**SOLUTION**

The free-body diagram of the tractor and water jet is shown in Fig. a. The pair of thrusts T cancel each other since they are internal to the system. The mass of the tractor and the tank at any instant $t$ is given by $m = (4000 + 2000) - 50t = (6000 - 50t)$ kg.

$$\sum F = m \frac{dv}{dt} - v \frac{dm}{dt}; \quad 250 = (6000 - 50t) \frac{dv}{dt} - 5(50)$$

$$a = \frac{dv}{dt} = \frac{10}{120 - t} \quad (1)$$

The time taken to empty the tank is $t = \frac{2000}{50} = 40$ s. Substituting the result of $t$ into Eq. (1),

$$a = \frac{10}{120 - 40} = 0.125 \text{m/s}^2 \quad \text{Ans.}$$

Integrating Eq. (1),

$$\int_0^v dv = \int_0^{40} \frac{10}{120 - t} dt$$

$$v = -10 \ln(120 - t) \bigg|_0^{40}$$

$$v = 4.05 \text{ m/s} \quad \text{Ans.}$$
A rocket has an empty weight of 500 lb and carries 300 lb of fuel. If the fuel is burned at the rate of 15 lb/s and ejected with a relative velocity of 4400 ft/s, determine the maximum speed attained by the rocket starting from rest. Neglect the effect of gravitation on the rocket.

**SOLUTION**

\[ + \sum F = m \frac{dv}{dt} - v \frac{dm_e}{dt} \]

At time \( t \), \( m = m_0 - ct \), where \( c = \frac{dm_e}{dt} \). In space the weight of the rocket is zero.

\[ 0 = (m_0 - ct) \frac{dv}{dt} - v \frac{dm_e}{dt} \]

\[ \int_0^v dv = \int_0^t \left( \frac{cv}{m_0 - ct} \right) dt \]

\[ v = v \frac{m_0 - ct}{m_0} \ln \left( \frac{m_0}{m_0 - ct} \right) \] (1)

The maximum speed occurs when all the fuel is consumed, that is, when \( t = \frac{300}{15} = 20 \) s. Here, \( m_0 = \frac{500 + 300}{32.2} = 24.8447 \text{ slug}, c = \frac{15}{32.2} = 0.4658 \text{ slug/s}, \)

\( v = 4400 \text{ ft/s}. \) Substitute the numerical values into Eq. (1):

\[ v_{\text{max}} = 4400 \ln \left( \frac{24.8447}{24.8447 - 0.4658(20)} \right) \]

\[ v_{\text{max}} = 2068 \text{ ft/s} = 2.07 \left( 10^3 \right) \text{ ft/s} \]

Ans.
A power lawn mower hovers very close over the ground. This is done by drawing air in at a speed of 6 m/s through an intake unit $A$, which has a cross-sectional area of $A_A = 0.25 \text{ m}^2$, and then discharging it at the ground, $B$, where the cross-sectional area is $A_B = 0.35 \text{ m}^2$. If air at $A$ is subjected only to atmospheric pressure, determine the air pressure which the lawn mower exerts on the ground when the weight of the mower is freely supported and no load is placed on the handle. The mower has a mass of 15 kg with center of mass at $G$. Assume that air has a constant density of $\rho_a = 1.22 \text{ kg/m}^3$.

**SOLUTION**

\[
\frac{dm}{dt} = \rho A_A v_A = 1.22(0.25)(6) = 1.83 \text{ kg/s}
\]

\[
\nabla \Sigma F_y = \frac{dm}{dt} ((v_B)_y - (v_A)_y)
\]

pressure = (0.35) – 15(9.81) = 1.83(0 – (−6))

pressure = 452 Pa

**Ans:**

452 Pa
*15–136.

The rocket car has a mass of 2 Mg (empty) and carries 120 kg of fuel. If the fuel is consumed at a constant rate of 6 kg/s and ejected from the car with a relative velocity of 800 m/s, determine the maximum speed attained by the car starting from rest. The drag resistance due to the atmosphere is $F_D = (6.8v^2)$ N, where $v$ is the speed in m/s.

**SOLUTION**

$$\Sigma F = m \frac{dv}{dt} - v^2$$

At time $t_1$ the mass of the car is $m_0 - ct$ where $c = \frac{dm}{dt} = 6$ kg/s

Set $F = kv^2$, then

$$-kv^2 = (m_0 - ct) \frac{dv}{dt} - vD/c$$

$$\int_0^v dv = \int_0^{m_0 - ct} \frac{dt}{ct + k}$$

$$\left( \frac{1}{2\sqrt{cvD/k}} \right) \ln \left[ \sqrt{cvD/k} + \frac{v}{k} \right] = -\frac{1}{c} \ln(m_0 - ct)$$

Maximum speed occurs at the instant the fuel runs out

$$t = \frac{120}{6} = 20 \text{ s}$$

Thus,

$$\left( \frac{1}{2\sqrt{(6)(800)(6.8)}} \right) \ln \left( \frac{(6)(800)}{6.8} + \frac{v}{6.8} \right) = -\frac{1}{6} \ln \left( \frac{2120 - 6(20)}{2120} \right)$$

Solving,

$$v = 25.0 \text{ m/s}$$

**Ans:**

$$v = 25.0 \text{ m/s}$$
15–137.

If the chain is lowered at a constant speed \( v = 4 \text{ ft/s} \), determine the normal reaction exerted on the floor as a function of time. The chain has a weight of 5 lb/ft and a total length of 20 ft.

**SOLUTION**

At time \( t \), the weight of the chain on the floor is \( W = mg( vt) \)

\[
dv = 0, \quad m_i = m( vt)
\]

\[
\frac{dm_i}{dt} = mv
\]

\[
\Sigma F_i = m \frac{dv}{dt} + v \frac{dm_i}{dt}
\]

\( R - mg( vt) = 0 + v( mv) \)

\( R = m(gvt + v^2) \)

\[
R = \frac{5}{32.2} \left( 32.2(4)(t) + (4)^2 \right)
\]

\( R = (20t + 2.48) \text{ lb} \)

 Ans. 

\[
R = [20t + 2.48] \text{ lb}
\]
The second stage of a two-stage rocket weighs 2000 lb (empty) and is launched from the first stage with a velocity of 3000 mi/h. The fuel in the second stage weighs 1000 lb. If it is consumed at the rate of 50 lb/s and ejected with a relative velocity of 8000 ft/s, determine the acceleration of the second stage just after the engine is fired. What is the rocket's acceleration just before all the fuel is consumed? Neglect the effect of gravitation.

SOLUTION

Initially,

\[ \sum F_i = m \frac{dv}{dt} - v_d \left( \frac{dm}{dt} \right) \]

\[ 0 = \frac{3000}{32.2} a - 8000 \left( \frac{50}{32.2} \right) \]

\[ a = 133 \text{ ft/s}^2 \quad \text{Ans.} \]

Finally,

\[ 0 = \frac{2000}{32.2} a - 8000 \left( \frac{50}{32.2} \right) \]

\[ a = 200 \text{ ft/s}^2 \quad \text{Ans.} \]

Ans:

\[ a_i = 133 \text{ ft/s}^2 \]

\[ a_f = 200 \text{ ft/s}^2 \]
The missile weighs 40,000 lb. The constant thrust provided by the turbojet engine is $T = 15,000$ lb. Additional thrust is provided by two rocket boosters $B$. The propellant in each booster is burned at a constant rate of 150 lb/s, with a relative exhaust velocity of 3000 ft/s. If the mass of the propellant lost by the turbojet engine can be neglected, determine the velocity of the missile after the 4-s burn time of the boosters. The initial velocity of the missile is 300 mi/h.

**SOLUTION**

\[ \sum F_s = m\frac{dv}{dt} - v_{D/e} \frac{dm_e}{dt} \]

At a time $t$, $m = m_0 - ct$, where $c = \frac{dm_e}{dt}$.

\[ T = (m_0 - ct)\frac{dv}{dt} - v_{D/e}c \]

\[ \int_{v_0}^{v} dv = \int_{0}^{t} \left( \frac{T + cv_{D/e}}{m_0 - ct} \right) dt \]

\[ v = \left( \frac{T + cv_{D/e}}{c} \right) \ln \left( \frac{m_0}{m_0 - ct} \right) + v_0 \]  
\hspace{1cm} (1)

Here, $m_0 = \frac{40,000}{32.2} = 1242.24$ slug, $c = 2\left( \frac{150}{32.2} \right) = 9.3168$ slug/s, $v_{D/e} = 3000$ ft/s, $t = 4$ s, $v_0 = \frac{300(5280)}{3600} = 440$ ft/s.

Substitute the numerical values into Eq. (1):

\[ v_{max} = \left( \frac{15,000 + 9.3168(3000)}{9.3168} \right) \ln \left( \frac{1242.24}{1242.24 - 9.3168(4)} \right) + 440 \]

\[ v_{max} = 580 \text{ ft/s} \]

**Ans:**

\[ v_{max} = 580 \text{ ft/s} \]
The jet is traveling at a speed of 720 km/h. If the fuel is being spent at 0.8 kg/s, and the engine takes in air at 200 kg/s, whereas the exhaust gas (air and fuel) has a relative speed of 12 000 m/s, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = (55 \, v^2)$, where the speed is measured in m/s. The jet has a mass of 7 Mg.

**SOLUTION**

Since the mass enters and exits the plane at the same time, we can combine Eqs. 15-29 and 15-30 which resulted in

$$\sum F_i = m \frac{dv}{dt} - v_{D/e} \frac{dm_e}{dt} + v_{D/i} \frac{dm_i}{dt}$$

Here $m = 7000$ kg, $\frac{dv}{dt} = a$, $v_{D/e} = 12000$ m/s, $\frac{dm_e}{dt} = 0.8 + 200 = 200.8$ kg/s

$$v = \left( \frac{720 \, \text{km}}{\text{h}} \right) \left( \frac{1000 \, \text{m}}{1 \, \text{km}} \right) \left( \frac{1 \, \text{h}}{3600 \, \text{s}} \right) = 200 \, \text{m/s}, \quad v_{D/i} = v = 200 \, \text{m/s},$$

$$\frac{dm_i}{dt} = 200 \, \text{kg/s}$$

and $F_D = 55 (200^2) = 2.2 \times 10^6$ N. Referring to the FBD of the jet, Fig. a

$$\sum F_i = m \frac{dv}{dt} - v_{D/e} \frac{dm_e}{dt} + v_{D/i} \frac{dm_i}{dt};$$

$$-2.2 \times 10^6 = 7000a - 12000(200.8) + 200(200)$$

$$a = 24.23 \, \text{m/s}^2 = 24.2 \, \text{m/s}^2$$

**Ans:**

$$a = 24.2 \, \text{m/s}^2$$
15–141.  

The rope has a mass $m'$ per unit length. If the end length $y = h$ is draped off the edge of the table, and released, determine the velocity of its end $A$ for any position $y$, as the rope uncoils and begins to fall.

**SOLUTION**

\[ \sum F_i = m \frac{dv}{dt} + v \frac{dm_i}{dt} \]

At a time $t, m = m' y$ and $\frac{dm_i}{dt} = m' \frac{dy}{dt} = m' v$. Here, $v_{D/y} = v, \frac{dv}{dt} = g$.

\[ m'gy = m' y \frac{dv}{dt} + v(m'v) \]

\[ gy = \frac{v}{dt} + v^2 \quad \text{since} \quad v = \frac{dy}{dt} \quad \text{then} \quad dt = \frac{dy}{v} \]

\[ gy = v \frac{dy}{dy} + v^2 \]

Multiply both sides by $2y dy$

\[ 2gy^2 dy = 2vy^2 dv + 2vy^2 dy \]

\[ \int 2gy^2 dy = \int dv(v^2y^2) \]

\[ \frac{2}{3} gy^3 + C = v^3 y^2 \]

\[ v = 0 \quad \text{at} \quad y = h \quad \frac{2}{3} gh^3 + C = 0 \quad C = -\frac{2}{3} gh^3 \]

\[ \frac{2}{3} gy^3 - \frac{2}{3} gh^3 = v^3 y^2 \]

\[ v = \sqrt{\frac{2}{3} g \left( \frac{y^3 - h^3}{y^3} \right)} \quad \text{Ans.} \]
The 12-Mg jet airplane has a constant speed of 950 km/h when it is flying along a horizontal straight line. Air enters the intake scoops \( S \) at the rate of \( 50 \text{ m}^3/\text{s} \). If the engine burns fuel at the rate of 0.4 kg/s and the gas (air and fuel) is exhausted relative to the plane with a speed of 450 km/h, determine the resultant drag force exerted on the plane by air resistance. Assume that air has a constant density of 1.22 kg/m\(^3\). Hint: Since mass both enters and exits the plane, Eqs. 15–28 and 15–29 must be combined to yield

\[
\Sigma F_s = m \frac{dv}{dt} - v_{D/E} \frac{dm_e}{dt} + v_{D/i} \frac{dm_i}{dt}.
\]

**SOLUTION**

\[
\Sigma F_s = m \frac{dv}{dt} - v_{D/E} \frac{dm_e}{dt} + v_{D/i} \frac{dm_i}{dt}.
\]

\( v = 950 \text{ km/h} = 0.2639 \text{ km/s} \), \( \frac{dv}{dt} = 0 \)

\( v_{D/E} = 0.45 \text{ km/s} \)

\( v_{D/i} = 0.2639 \text{ km/s} \)

\[
\frac{dm_e}{dt} = 50(1.22) = 61.0 \text{ kg/s}
\]

\[
\frac{dm_i}{dt} = 0.4 + 61.0 = 61.4 \text{ kg/s}
\]

Forces \( T \) and \( R \) are incorporated into Eq. (1) as the last two terms in the equation.

\[
( \frac{dv}{dt} ) - F_D = 0 - (0.45)(61.4) + (0.2639)(61)
\]

\[
F_D = 11.5 \text{ kN}
\]

Ans.

\( F_D = 11.5 \text{ kN} \)
15–143.

The jet is traveling at a speed of 500 mi/h, 30° with the horizontal. If the fuel is being spent at 3 lb/s, and the engine takes in air at 400 lb/s, whereas the exhaust gas (air and fuel) has a relative speed of 32 800 ft/s, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = (0.7v^2)$ lb, where the speed is measured in ft/s. The jet has a weight of 15 000 lb. Hint: See Prob. 15–142.

**SOLUTION**

\[
\frac{dm_i}{dt} = \frac{400}{32.2} = 12.42 \text{ slug/s}
\]

\[
\frac{dm_e}{dt} = \frac{403}{32.2} = 12.52 \text{ slug/s}
\]

\[v = v_{D/e} = 733.3 \text{ ft/s}
\]

\[v = v_{D/i} = 500 \text{ mi/h} = 733.3 \text{ ft/s}
\]

\[\sum F = m \frac{dv}{dt} - v_{D/e} \frac{dm_e}{dt} + v_{D/i} \frac{dm_i}{dt}
\]

\[-(15 000) \sin 30° - 0.7(733.3)^2 = \frac{15 000 \frac{dv}{dt}}{32.2} - 32 800(12.52) + 733.3(12.42)
\]

\[a = \frac{dv}{dt} = 37.5 \text{ ft/s}^2\]  \text{ Ans.}
A four-engine commercial jumbo jet is cruising at a constant speed of 800 km/h in level flight when all four engines are in operation. Each of the engines is capable of discharging combustion gases with a velocity of 775 m/s relative to the plane. If during a test two of the engines, one on each side of the plane, are shut off, determine the new cruising speed of the jet. Assume that air resistance (drag) is proportional to the square of the speed, that is, \( F_D = cv^2 \), where \( c \) is a constant to be determined. Neglect the loss of mass due to fuel consumption.

**SOLUTION**

**Steady Flow Equation:** Since the air is collected from a large source (the atmosphere), its entrance speed into the engine is negligible. The exit speed of the air from the engine is

\[
\left( \downarrow \right) \quad v_e = v_p + v_{e/p}
\]

When the four engines are in operation, the airplane has a constant speed of

\[
v_p = \left[ 800 \times 10^3 \frac{m}{h} \right] \left( \frac{1 \frac{h}{3600 \ s}}{} \right) = 222.22 \frac{m}{s}.
\]

Thus,

\[
\left( \downarrow \right) \quad v_e = -222.22 + 775 = 552.78 \frac{m}{s} \quad \rightarrow
\]

Referring to the free-body diagram of the airplane shown in Fig. a,

\[
\begin{align*}
\downarrow \Sigma F_x &= \frac{dm}{dt} \left[ (v_B)_{x} - (v_A)_{x} \right]; \quad C(222.22^2) = 4 \frac{dm}{dt} (552.78 - 0) \\
C &= 0.044775 \frac{dm}{dt}
\end{align*}
\]

When only two engines are in operation, the exit speed of the air is

\[
\left( \downarrow \right) \quad v_e = -v_p + 775
\]

Using the result for \( C \),

\[
\begin{align*}
\downarrow \Sigma F_x &= \frac{dm}{dt} \left[ (v_B)_{x} - (v_A)_{x} \right]; \quad \left( 0.044775 \frac{dm}{dt} \left( v_p^2 \right) = 2 \frac{dm}{dt} [-v_p + 775] - 0 \right) \\
0.044775v_p^2 + 2v_p - 1550 &= 0
\end{align*}
\]

Solving for the positive root,

\[ v_p = 165.06 \frac{m}{s} = 594 \frac{km}{h} \quad \text{Ans.} \]
15–145.

The 10-Mg helicopter carries a bucket containing 500 kg of water, which is used to fight fires. If it hovers over the land in a fixed position and then releases 50 kg/s of water at 10 m/s, measured relative to the helicopter, determine the initial upward acceleration the helicopter experiences as the water is being released.

**SOLUTION**

\[ + \sum \vec{F}_t = m \frac{dv}{dt} - \rho_v \frac{dm}{dt} \]

Initially, the bucket is full of water, hence \( m = 10(10^3) + 0.5(10^3) = 10.5(10^3) \) kg

\[ 0 = 10.5(10^3) a - (10)(50) \]

\[ a = 0.0476 \text{ m/s}^2 \quad \text{Ans.} \]
A rocket has an empty weight of 500 lb and carries 300 lb of fuel. If the fuel is burned at the rate of 1.5 lb/s and ejected with a velocity of 4400 ft/s relative to the rocket, determine the maximum speed attained by the rocket starting from rest. Neglect the effect of gravitation on the rocket.

SOLUTION

\[ + \sum F_t = \frac{dv}{dt} - v_{D/e} \frac{dm_e}{dt} \]

At a time \( t \), \( m = m_0 - ct \), where \( c = \frac{dm_e}{dt} \). In space the weight of the rocket is zero.

\[ 0 = (m_0 - ct) \frac{dv}{dt} - v_{D/e} c \]

\[ \int_0^v dv = \int_0^t \left( \frac{cv_{D/e}}{m_0 - ct} \right) dt \]

\[ v = v_{D/e} \ln \left( \frac{m_0}{m_0 - ct} \right) \]  \hspace{1cm} (1)

The maximum speed occurs when all the fuel is consumed, that is, when \( t = \frac{300}{1.5} = 200 \) s.

Here, \( m_0 = \frac{500 + 300}{32.2} = 24.8447 \) slug, \( c = \frac{15}{32.2} = 0.4658 \) slug/s, \( v_{D/e} = 4400 \) ft/s.

Substitute the numerical into Eq. (1):

\[ v_{max} = 4400 \ln \left( \frac{24.8447}{24.8447 - (0.04658(200))} \right) \]

\[ v_{max} = 2068 \text{ ft/s} \]  \hspace{1cm} \text{Ans.}
Determine the magnitude of force $F$ as a function of time, which must be applied to the end of the cord at $A$ to raise the hook $H$ with a constant speed $v = 0.4 \text{ m/s}$. Initially the chain is at rest on the ground. Neglect the mass of the cord and the hook. The chain has a mass of 2 kg/m.

**SOLUTION**

\[
\frac{dv}{dt} = 0, \quad y = vt
\]

\[
m_i = my = mvt
\]

\[
\frac{dm_i}{dt} = mv
\]

\[
+ \sum F = m \frac{dv}{dt} + v_D \left( \frac{dm_i}{dt} \right)
\]

\[
F - mgvt = 0 + v(mv)
\]

\[
F = m(gvt + v^2)
\]

\[
= 2[9.81(0.4)t + (0.4)^2]
\]

\[
F = (7.85t + 0.320) \text{ N}
\]

**Ans.**

\[
F = (7.85t + 0.320) \text{ N}
\]
The truck has a mass of 50 Mg when empty. When it is unloading 5 m³ of sand at a constant rate of 0.8 m³/s, the sand flows out the back at a speed of 7 m/s, measured relative to the truck, in the direction shown. If the truck is free to roll, determine its initial acceleration just as the load begins to empty. Neglect the mass of the wheels and any frictional resistance to motion. The density of sand is \( \rho_s = 1520 \text{ kg/m}^3 \).

**SOLUTION**

**A System That Loses Mass:** Initially, the total mass of the truck is

\[
m = 50(10^3) + 5(1520) = 57.6(10^3) \text{ kg} \quad \text{and} \quad \frac{dm}{dt} = 0.8(1520) = 1216 \text{ kg/s}.
\]

Applying Eq. 15–29, we have

\[
\Sigma F_x = m \frac{dv}{dt} - v \frac{dm}{dt} = 0 = 57.6(10^3)a - (0.8 \cos 45^\circ)(1216)
\]

\[
a = 0.104 \text{ m/s}^2 \quad \text{Ans.}
\]
The car has a mass $m_0$ and is used to tow the smooth chain having a total length $l$ and a mass per unit of length $m'$. If the chain is originally piled up, determine the tractive force $F$ that must be supplied by the rear wheels of the car, necessary to maintain a constant speed $v$ while the chain is being drawn out.

**SOLUTION**

$$\sum F = m\frac{dv}{dt} + v_{D/t}\frac{dm_i}{dt}$$

At a time $t$, $m = m_0 + ct$, where $c = \frac{dm_i}{dt} = \frac{m'dx}{dt} = m'v$.

Here, $v_{D/t} = v$, $\frac{dv}{dt} = 0$.

$$F = (m_0 - m'v)(0) + v(m'v) = m'v^2$$

**Ans:**

$$F = m'v^2$$