17–1.

Determine the moment of inertia $I_y$ for the slender rod. The rod’s density $\rho$ and cross-sectional area $A$ are constant. Express the result in terms of the rod’s total mass $m$.

SOLUTION

$$I_y = \int x^2 \, dm$$

$$= \int_0^l x^2 (\rho A \, dx)$$

$$= \frac{1}{3} \rho A l^3$$

$$m = \rho A l$$

Thus,

$$I_y = \frac{1}{3} m l^2$$

Ans: $I_y = \frac{1}{3} m l^2$
17–2.

The solid cylinder has an outer radius $R$, height $h$, and is made from a material having a density that varies from its center as $\rho = k + ar^2$, where $k$ and $a$ are constants. Determine the mass of the cylinder and its moment of inertia about the $z$ axis.

**SOLUTION**

Consider a shell element of radius $r$ and mass

$$dm = \rho dV = \rho(2\pi r dr)h$$

$$m = \int_0^R (k + ar^2)(2\pi r dr)h$$

$$m = 2\pi h\left(kR^2 + \frac{aR^4}{4}\right)$$

$$m = \pi h R^2\left(k + \frac{aR^2}{2}\right)$$

$$dI = r^2 dm = r^2(\rho)(2\pi r dr)h$$

$$I_z = \int_0^R r^2(k + ar^2)(2\pi r dr)h$$

$$I_z = 2\pi h\int_0^R (k r^3 + a r^5) dr$$

$$I_z = 2\pi h\left[k \frac{R^4}{4} + \frac{aR^6}{6}\right]$$

$$I_z = \frac{\pi h R^4}{2}\left[k + \frac{2aR^2}{3}\right]$$

**Ans:**

$$m = \pi h R^2\left(k + \frac{aR^2}{2}\right)$$

$$I_z = \frac{\pi h R^4}{2}\left[k + \frac{2aR^2}{3}\right]$$
17–3.

Determine the moment of inertia of the thin ring about the z axis. The ring has a mass \( m \).

**SOLUTION**

\[
I_z = \int_{0}^{2\pi} \rho A (R \, d\theta) R^2 = 2\pi \, \rho \, A \, R^3
\]

\[
m = \int_{0}^{2\pi} \rho A \, R \, d\theta = 2\pi \, \rho \, A \, R
\]

Thus,

\[
I_z = mR^2 \quad \text{Ans.}
\]
The paraboloid is formed by revolving the shaded area around the x axis. Determine the radius of gyration $k_x$. The density of the material is $\rho = 5 \text{ Mg/m}^3$.

**SOLUTION**

$$dm = \rho \pi y^2 \, dx = \rho \pi (50x) \, dx$$

$$I_x = \int \frac{1}{2} y^2 \, dm = \frac{1}{2} \int_0^{200} 50x \{\pi \rho (50x)\} \, dx$$

$$= \rho \pi \left(\frac{50^2}{2}\right) \left[\frac{1}{3}x^3\right]_0^{200}$$

$$= \rho \pi \left(\frac{50^2}{6}\right)(200)^3$$

$$m = \int dm = \int_0^{200} \pi \rho (50x) \, dx$$

$$= \rho \pi (50) \left[\frac{1}{2} x^2\right]_0^{200}$$

$$= \rho \pi \left(\frac{50}{2}\right)(200)^2$$

$$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{50}{3}} (200) = 57.7 \text{ mm} \quad \text{Ans.}$$
17–5.

Determine the radius of gyration $k_x$ of the body. The specific weight of the material is $\gamma = 380$ lb/ft$^3$.

**SOLUTION**

$$dm = \rho dV = \rho \pi y^2 dx$$

$$dI_x = \frac{1}{2}(dm)y^2 = \frac{1}{2} \rho \pi y^4 dx$$

$$I_x = \int_0^8 \frac{1}{2} \rho \pi x^{4/3} dx = 86.17 \rho$$

$$m = \int_0^8 \rho \pi x^{2/3} dx = 60.32 \rho$$

$$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{86.17 \rho}{60.32 \rho}} = 1.20 \text{ in.}$$

**Ans:**

$$k_x = 1.20 \text{ in.}$$
17–6.

The sphere is formed by revolving the shaded area around the x axis. Determine the moment of inertia \( I_x \) and express the result in terms of the total mass \( m \) of the sphere. The material has a constant density \( \rho \).

**SOLUTION**

\[
dI_x = \frac{y^2}{2} dm
\]

\[
dm = \rho \, dV = \rho (\pi y^2 \, dx) = \rho \pi (r^2 - x^2) \, dx
\]

\[
dI_x = \frac{1}{2} \rho \pi (r^2 - x^2)^2 \, dx
\]

\[
I_x = \int_{-r}^{r} \frac{1}{2} \rho \pi (r^2 - x^2)^2 \, dx
\]

\[
= \frac{8}{15} \pi \rho r^5
\]

\[
m = \int_{-r}^{r} \rho \pi (r^2 - x^2) \, dx
\]

\[
= \frac{4}{3} \pi \rho r^3
\]

Thus,

\[
I_x = \frac{2}{5} m r^2 \quad \text{Ans.}
\]
17–7.

The frustum is formed by rotating the shaded area around the x axis. Determine the moment of inertia $I_x$ and express the result in terms of the total mass $m$ of the frustum. The frustum has a constant density $\rho$.

**SOLUTION**

$$dm = \rho \, dV = \rho \pi y^2 \, dx = \rho \pi \left( \frac{b^2}{a^2} x^2 + \frac{2b^2}{a} x + b^2 \right) dx$$

$$dI_x = \frac{1}{2} dmy^2 = \frac{1}{2} \rho \pi y^4 \, dx$$

$$dI_x = \frac{1}{2} \rho \pi \left( \frac{b^4}{a^4} x^4 + \frac{4b^4}{a^3} x^3 + \frac{6b^4}{a^2} x^2 + \frac{4b^4}{a} x + b^4 \right) dx$$

$$I_x = \int dI_x = \frac{1}{2} \rho \pi \int_0^a \left( \frac{b^4}{a^4} x^4 + \frac{4b^4}{a^3} x^3 + \frac{6b^4}{a^2} x^2 + \frac{4b^4}{a} x + b^4 \right) dx$$

$$= \frac{31}{10} \rho \pi ab^4$$

$$m = \int dm = \rho \pi \int_0^a \left( \frac{b^2}{a^2} x^2 + \frac{2b^2}{a} x + b^2 \right) dx = \frac{7}{3} \rho \pi ab^2$$

$$I_x = \frac{93}{70} mb^2$$

**Ans:**

$$I_x = \frac{93}{70} mb^2$$
*17–8.

The hemisphere is formed by rotating the shaded area around the $y$ axis. Determine the moment of inertia $I_y$ and express the result in terms of the total mass $m$ of the hemisphere. The material has a constant density $\rho$.

**SOLUTION**

$$m = \int \rho \, dV = \rho \int_0^r \pi x^2 \, dy = \rho \pi \int_0^r (r^2 - y^2) \, dy$$

$$= \rho \pi \left[ r^2 y - \frac{1}{3} y^3 \right]_0^r = \frac{2}{3} \rho \pi \, r^3$$

$$I_y = \int m \left( \frac{1}{2} (dm) x^2 \right) = \frac{\rho}{2} \int_0^r \pi x^4 \, dy = \frac{\rho \pi}{2} \int_0^r (r^2 - y^2)^2 \, dy$$

$$= \frac{\rho \pi}{2} \left[ r^4 y - \frac{2}{3} r^2 y^3 + \frac{y^5}{5} \right]_0^r = \frac{4 \rho \pi}{15} \, r^5$$

Thus,

$$I_y = \frac{2}{5} m \, r^2$$

**Ans:**

$$I_y = \frac{2}{5} m r^2$$
17–9.

Determine the moment of inertia of the homogeneous triangular prism with respect to the \( y \) axis. Express the result in terms of the mass \( m \) of the prism. \textit{Hint:} For integration, use thin plate elements parallel to the \( x-y \) plane and having a thickness \( dz \).

**SOLUTION**

\[ dV = bx \, dz = b(a)(1 - \frac{z}{h}) \, dz \]

\[ dI_y = dI_y + (dm)[(\frac{y}{2})^2 + z^2] \]

\[ = \frac{1}{12} \, dm(x^2) + dm(\frac{x^2}{4}) + dmz^2 \]

\[ = dm(\frac{x^2}{3} + z^2) \]

\[ = [b(a)(1 - \frac{z}{h})dz][\rho][\frac{d^2}{3}(1 - \frac{z}{h})^2 + z^2] \]

\[ I_y = abp \int_0^h \left[ \frac{a^3}{3} \left( 1 - \frac{z}{h} \right) \right]^3 + z^2(1 - \frac{z}{h})dz \]

\[ = abp[a^3 \left( h^4 - \frac{3}{2} h^4 + h^4 - \frac{1}{4} h^4 \right) + \frac{1}{h} \left( \frac{1}{3} h^4 - \frac{1}{4} h^4 \right)] \]

\[ = \frac{1}{12} \, abhp(a^2 + h^2) \]

\[ m = \rho V = \frac{1}{2} \, abhp \]

Thus,

\[ I_y = \frac{m}{6} (a^2 + h^2) \quad \text{Ans.} \]
17–10.

The pendulum consists of a 4-kg circular plate and a 2-kg slender rod. Determine the radius of gyration of the pendulum about an axis perpendicular to the page and passing through point $O$.

**SOLUTION**

Using the parallel axis theorem by referring to Fig. $a$,

$$I_O = \sum (I_G + md^2)$$

$$= \left[ \frac{1}{12}(2^2) + 2(1^2) \right] + \left[ \frac{1}{2}(4)(0.5^2) + 4(2.5^2) \right]$$

$$= 28.17 \text{ kg \cdot m}^2$$

Thus, the radius of gyration is

$$k_O = \sqrt{\frac{I_O}{m}} = \sqrt{\frac{28.17}{4 + 2}} = 2.167 \text{ m} = 2.17 \text{ m}$$

Ans: $k_O = 2.17 \text{ m}$
17–11.

The assembly is made of the slender rods that have a mass per unit length of 3 kg/m. Determine the mass moment of inertia of the assembly about an axis perpendicular to the page and passing through point $O$.

SOLUTION

Using the parallel axis theorem by referring to Fig. $a$,

$$I_O = \Sigma (I_G + md^2)$$

$$= \left\{ \frac{1}{12} [3(1.2)](1.2^2) + [3(1.2)](0.2^2) \right\}$$

$$+ \left\{ \frac{1}{12} [3(0.4)](0.4^2) + [3(0.4)](0.8^2) \right\}$$

$$= 1.36 \text{ kg} \cdot \text{m}^2$$

Ans:

$$I_O = 1.36 \text{ kg} \cdot \text{m}^2$$
*17–12.

Determine the moment of inertia of the solid steel assembly about the $x$ axis. Steel has a specific weight of $\gamma_s = 490 \text{ lb/ft}^3$.

SOLUTION

\[ I_x = \frac{1}{2} m_1 (0.5)^2 + \frac{3}{10} m_2 (0.5)^2 - \frac{3}{10} m_3 (0.25)^2 \]

\[ = \left[ \frac{1}{2} \pi (0.5)^2 (3)(0.5)^2 + \frac{3}{10} \left( \frac{1}{2} \right) \pi (0.5)^2 (4)(0.5)^2 - \frac{3}{10} \left( \frac{1}{2} \right) \pi (0.25)^2 (2)(0.25)^2 \right] \left( \frac{490}{32.2} \right) \]

\[ = 5.64 \text{ slug} \cdot \text{ft}^2 \]

Ans.

Ans:

\[ I_x = 5.64 \text{ slug} \cdot \text{ft}^2 \]
17–13.

The wheel consists of a thin ring having a mass of 10 kg and four spokes made from slender rods, each having a mass of 2 kg. Determine the wheel’s moment of inertia about an axis perpendicular to the page and passing through point $A$.

**SOLUTION**

\[
I_A = I_o + md^3
\]

\[
= \left[ 2 \left( \frac{1}{12} (4)(1)^2 \right) + 10(0.5)^2 \right] + 18(0.5)^2
\]

\[= 7.67 \text{ kg} \cdot \text{m}^2\]  

*Ans.*
17–14.

If the large ring, small ring and each of the spokes weigh 100 lb, 15 lb, and 20 lb, respectively, determine the mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point $A$.

**SOLUTION**

*Composite Parts:* The wheel can be subdivided into the segments shown in Fig. $a$. The spokes which have a length of $(4 - 1) = 3$ ft and a center of mass located at a distance of $\left(1 + \frac{3}{2}\right)$ ft = 2.5 ft from point $O$ can be grouped as segment (2).

*Mass Moment of Inertia:* First, we will compute the mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point $O$.

$$I_O = \left(\frac{100}{32.2}\right)(4^2) + 8\left[\frac{20}{32.2}\right](3^2) + \left(\frac{20}{32.2}\right)(2.5^2) + \left(\frac{15}{32.2}\right)(1^2)$$

$$= 84.94 \text{ slug} \cdot \text{ft}^2$$

The mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point $A$ can be found using the parallel-axis theorem $I_A = I_O + md^2$, where $m = \frac{100}{32.2} + 8\left(\frac{20}{32.2}\right) + \frac{15}{32.2} = 8.5404 \text{ slug}$ and $d = 4 \text{ ft}$. Thus,

$$I_A = 84.94 + 8.5404(4^2) = 221.58 \text{ slug} \cdot \text{ft}^2 = 222 \text{ slug} \cdot \text{ft}^2$$

**Ans:**

$I_A = 222 \text{ slug} \cdot \text{ft}^2$
17–15.

Determine the moment of inertia about an axis perpendicular to the page and passing through the pin at O. The thin plate has a hole in its center. Its thickness is 50 mm, and the material has a density \( \rho = 50 \text{ kg/m}^3 \).

**SOLUTION**

\[
I_O = \frac{1}{12} \left[ 50(1.4)(0.05) \right]\left[ (1.4)^2 + (1.4)^2 \right] - \frac{1}{2} \left[ 50(\pi)(0.15)^2(0.05) \right](0.15)^2
\]

\[
= 1.5987 \text{ kg} \cdot \text{m}^2
\]

\[
I_O = I_G + md^2
\]

\[
m = 50(1.4)(1.4)(0.05) - 50(\pi)(0.15)^2(0.05) = 4.7233 \text{ kg}
\]

\[
I_O = 1.5987 + 4.7233(1.4 \sin 45^\circ)^2 = 6.23 \text{ kg} \cdot \text{m}^2 \quad \text{Ans.}
\]
17–16.

Determine the mass moment of inertia of the thin plate about an axis perpendicular to the page and passing through point $O$. The material has a mass per unit area of $20 \text{ kg/m}^2$.

**SOLUTION**

*Composite Parts:* The plate can be subdivided into two segments as shown in Fig. $a$. Since segment (2) is a hole, it should be considered as a negative part. The perpendicular distances measured from the center of mass of each segment to the point $O$ are also indicated.

*Mass Moment of Inertia:* The moment of inertia of segments (1) and (2) are computed as $m_1 = \pi(0.2^2)(20) = 0.8\pi \text{ kg}$ and $m_2 = (0.2)(0.2)(20) = 0.8 \text{ kg}$. The moment of inertia of the plate about an axis perpendicular to the page and passing through point $O$ for each segment can be determined using the parallel-axis theorem.

$$I_O = \Sigma I_O + md^2$$

$$= \left[ \frac{1}{2} (0.8\pi)(0.2^2) + 0.8\pi(0.2^2) \right] - \left[ \frac{1}{12} (0.8)(0.2^2 + 0.2^2) + 0.8(0.2^2) \right]$$

$$= 0.113 \text{ kg} \cdot \text{m}^2$$

Ans: $I_O = 0.113 \text{ kg} \cdot \text{m}^2$
17–17.
Determine the location \( y \) of the center of mass \( G \) of the assembly and then calculate the moment of inertia about an axis perpendicular to the page and passing through \( G \). The block has a mass of 3 kg and the semicylinder has a mass of 5 kg.

**SOLUTION**

Moment inertia of the semicylinder about its center of mass:

\[
(I_G)_{cy} = \frac{1}{2} mR^2 - m \left( \frac{4R}{3\pi} \right)^2 = 0.3199mR^2
\]

\[
\bar{y} = \frac{\sum y \cdot m}{\sum m} = \frac{0.2 \cdot \frac{4(0.2)}{3\pi} (5) + 0.35(3)}{5 + 3} = 0.2032 \text{ m} = 0.203 \text{ m}
\]

\[
I_G = (I_G)_{cy} + 5 \left[ 0.2032 - \left( \frac{4(0.2)}{3\pi} \right) \right]^2 + \frac{1}{12} (3) (0.3^2 + 0.4^2) + 3(0.35 - 0.2032)^2
\]

\[
= 0.230 \text{ kg} \cdot \text{m}^2
\]

\[\text{Ans.:} \quad I_G = 0.230 \text{ kg} \cdot \text{m}^2\]
17–18.
Determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through point $O$. The block has a mass of 3 kg, and the semicylinder has a mass of 5 kg.

**SOLUTION**

$$(I_G)_{cy} = \frac{1}{2}mR^2 - m\left(\frac{4R}{3\pi}\right)^2 = 0.3199mR^2$$

$$I_O = 0.3199(5)(0.2)^2 + \frac{5}{2} \left(0.2 - \frac{4(0.2)}{3\pi}\right)^2 + \frac{1}{12}(3)((0.3)^2 + (0.4)^2) + 3(0.350)^2$$

$$= 0.560 \text{ kg} \cdot \text{m}^2 \quad \text{Ans.}$$

Also from the solution to Prob. 17–22,

$$I_O = I_G + md^2$$

$$= 0.230 + 8(0.2032)^2$$

$$= 0.560 \text{ kg} \cdot \text{m}^2 \quad \text{Ans.}$$
17–19.

Determine the moment of inertia of the wheel about an axis which is perpendicular to the page and passes through the center of mass \( G \). The material has a specific weight \( \gamma = 90 \text{ lb/ft}^3 \).

**SOLUTION**

\[
I_G = \frac{1}{2} \left[ \frac{90}{32.2} (\pi)(2)^2 (0.25)^2 \right] (2)^2 + \frac{1}{2} \left[ \frac{90}{32.2} (\pi)(2.5)^2 (1)^2 \right] (2.5)^2 \\
- \frac{1}{2} \left[ \frac{90}{32.2} (\pi)(2)^2 (1)^2 \right] (2)^2 - 4 \left[ \frac{1}{2} \left( \frac{90}{32.2} \right)(\pi)(0.25)^2 (0.25)^2 \right] (0.25)^2 \\
- 4 \left[ \left( \frac{90}{32.2} \right)(\pi)(0.25)^2 (0.25)^2 \right] (1)^2 \\
= 118.25 \text{ slug} \cdot \text{ft}^2
\]

**Ans:**

\[ I_G = 118 \text{ slug} \cdot \text{ft}^2 \]
Determine the moment of inertia of the wheel about an axis which is perpendicular to the page and passes through point $O$. The material has a specific weight $\gamma = 90$ lb/ft$^3$.

**SOLUTION**

\[
m = \frac{90}{32.2} \left[ \pi (2)^2 (0.25) + \pi \left\{ (2.5)^2 (1) - (2)^2 (1) \right\} - 4\pi (0.25)^2 (0.25) \right] = 27.99 \text{ slug}
\]

From the solution to Prob. 17–18,

$\int = 118.25 \text{ slug} \cdot \text{ft}^2$

\[
\int = 118.25 + 27.99 (2.5)^2 = 293 \text{ slug} \cdot \text{ft}^2
\]

**Ans.**

$\int = 293 \text{ slug} \cdot \text{ft}^2$
17–21.

The pendulum consists of the 3-kg slender rod and the 5-kg thin plate. Determine the location $\bar{y}$ of the center of mass $G$ of the pendulum; then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through $G$.

**SOLUTION**

$$\bar{y} = \frac{\sum y m}{\sum m} = \frac{1(3) + 2.25(5)}{3 + 5} = 1.781 \text{ m} = 1.78 \text{ m}$$  \hspace{1cm} \text{Ans.}

$$I_G = \Sigma I_G + md^2$$

$$= \frac{1}{12} (3)(2)^2 + 3(1.781 - 1)^2 + \frac{1}{12} (5)(0.5^2 + 1^2) + 5(2.25 - 1.781)^2$$

$$= 4.45 \text{ kg} \cdot \text{m}^2$$  \hspace{1cm} \text{Ans.}

\[
\begin{align*}
\text{Ans:} \\
\bar{y} &= 1.78 \text{ m} \\
I_G &= 4.45 \text{ kg} \cdot \text{m}^2
\end{align*}
\]
17–22.

Determine the moment of inertia of the overhung crank about the $x$ axis. The material is steel having a density of $\rho = 7.85 \text{ Mg/m}^3$.

**SOLUTION**

$m_c = 7.85 \times 10^3 \times (0.05) \pi (0.01)^2 = 0.1233 \text{ kg}$

$m_p = 7.85 \times 10^3 \times (0.03)(0.180)(0.02) = 0.8478 \text{ kg}$

$I_x = 2 \left[ \frac{1}{2} (0.1233)(0.01)^2 + (0.1233)(0.06)^2 \right] + \left[ \frac{1}{12} (0.8478) (0.03)^2 + (0.180)^2 \right]$

$= 0.00325 \text{ kg} \cdot \text{m}^2 = 3.25 \text{ g} \cdot \text{m}^2$

**Ans:**

$I_x = 3.25 \text{ g} \cdot \text{m}^2$
17–23.

Determine the moment of inertia of the overhung crank about the \( x' \) axis. The material is steel having a density of \( \rho = 7.85 \text{ Mg/m}^3 \).

\[ \text{Ans.:} \quad I_{x'} = 7.19 \text{ g \cdot m}^2 \]
The door has a weight of 200 lb and a center of gravity at $G$. Determine how far the door moves in 2 s, starting from rest, if a man pushes on it at $C$ with a horizontal force $F = 30$ lb. Also, find the vertical reactions at the rollers $A$ and $B$.

**SOLUTION**

\[ \sum F_x = m(a_G)_x; \quad 30 = \left( \frac{200}{32.2} \right) a_G \]

\[ a_G = 4.83 \text{ ft/s}^2 \]

\[ \sum M_A = \sum (M_k)_A; \quad N_B (12) - 200(6) + 30(9) = \left( \frac{200}{32.2} \right) (4.83)(7) \]

\[ N_B = 95.0 \text{ lb} \]

\[ + \sum F_y = m(a_G)_y; \quad N_A + 95.0 - 200 = 0 \]

\[ N_A = 105 \text{ lb} \]

\[ (\Rightarrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_G t^2 \]

\[ s = 0 + 0 + \frac{1}{2} (4.83)(2)^2 = 9.66 \text{ ft} \]
17–25.

The door has a weight of 200 lb and a center of gravity at \( G \). Determine the constant force \( F \) that must be applied to the door to push it open 12 ft to the right in 5 s, starting from rest. Also, find the vertical reactions at the rollers \( A \) and \( B \).

SOLUTION

\( (\vec{a})s = x_0 + v_0t + \frac{1}{2}a_Gt^2 \)

\[ 12 = 0 + 0 + \frac{1}{2}a_G(5)^2 \]

\( a_c = 0.960 \text{ ft/s}^2 \)

\( \vec{a} = \sum F_x = m(a_G)x ; \quad F = \frac{200}{32.2}(0.960) \)

\( F = 5.9627 \text{ lb} = 5.96 \text{ lb} \)

\( \sum M_A = \sum (M_k)_A ; \quad N_B(12) - 200(6) + 5.9627(9) = \frac{200}{32.2}(0.960)(7) \)

\( N_B = 99.0 \text{ lb} \)

\( \sum F_y = m(a_G)y ; \quad N_A + 99.0 - 200 = 0 \)

\( N_A = 101 \text{ lb} \)

\( F = 5.96 \text{ lb} \)
\( N_B = 99.0 \text{ lb} \)
\( N_A = 101 \text{ lb} \)

The jet aircraft has a total mass of 22 Mg and a center of mass at \( G \). Initially at take-off the engines provide a thrust \( 2T = 4 \text{ kN} \) and \( T' = 1.5 \text{ kN} \). Determine the acceleration of the plane and the normal reactions on the nose wheel and each of the two wing wheels located at \( B \). Neglect the mass of the wheels and, due to low velocity, neglect any lift caused by the wings.

\[ T = 1.5 \text{ kN} \]
\[ T' = 4 \text{ kN} \]

**SOLUTION**

\[ \sum F_x = ma_x ; \quad 1.5 + 4 = 22a_G \]
\[ \sum F_y = 0 ; \quad 2B_y + A_y - 22(9.81) = 0 \]
\[ \sum M_B = \Sigma(M_K)_B ; \quad 4(2.3) - 1.5(2.5) - 22(9.81)(3) + A_y(9) = -22a_G(1.2) \]

\[ A_y = 72.6 \text{ kN} \]
\[ B_y = 71.6 \text{ kN} \]
\[ a_G = 0.250 \text{ m/s}^2 \]

**Ans:**

\[ A_y = 72.6 \text{ kN} \]
\[ B_y = 71.6 \text{ kN} \]
\[ a_G = 0.250 \text{ m/s}^2 \]
The sports car has a weight of 4500 lb and center of gravity at \( G \). If it starts from rest it causes the rear wheels to slip as it accelerates. Determine how long it takes for it to reach a speed of 10 ft/s. Also, what are the normal reactions at each of the four wheels on the road? The coefficients of static and kinetic friction at the road are \( \mu_s = 0.5 \) and \( \mu_k = 0.3 \), respectively. Neglect the mass of the wheels.

**SOLUTION**

\[ \sum M_A = \sum (Mc)_A: \quad -2N_B(6) + 4500(2) = \frac{4500}{32.2} a_G(2.5) \]

\[ \sum F_x = m(a_G)_x: \quad 0.3(2N_B) = \frac{4500}{32.2} a_G \]

\[ \sum F_y = m(a_G)_y: \quad 2N_B + 2N_A - 4500 = 0 \]

Solving,

\[ N_A = 1393 \text{ lb} \]  \hspace{1cm} \text{Ans.}

\[ N_B = 857 \text{ lb} \]  \hspace{1cm} \text{Ans.}

\[ a_G = 3.68 \text{ ft/s}^2 \]

\[ v = v_0 + a_G t \]

\[ 10 = 0 + 3.68 t \]

\[ t = 2.72 \text{ s} \]  \hspace{1cm} \text{Ans.}
**17–28.**

The assembly has a mass of 8 Mg and is hoisted using the boom and pulley system. If the winch at $B$ draws in the cable with an acceleration of $2 \text{ m/s}^2$, determine the compressive force in the hydraulic cylinder needed to support the boom. The boom has a mass of 2 Mg and mass center at $G$.

**SOLUTION**

$\sum s_B + 2s_L = l$

$a_B = -2a_L$

$2 = -2a_L$

$a_L = -1 \text{ m/s}^2$

Assembly:

$\sum F_y = ma_y; \quad 2T - 8(10^3)(9.81) = 8(10^3)(1)$

$T = 43.24 \text{ kN}$

Boom:

$\sum M_A = 0; \quad F_{CD}(2) - 2(10^3)(9.81)(6 \cos 60^\circ) - 2(43.24)(10^3)(12 \cos 60^\circ) = 0$

$F_{CD} = 289 \text{ kN}$

**Ans:**

$F_{CD} = 289 \text{ kN}$
The assembly has a mass of 4 Mg and is hoisted using the winch at B. Determine the greatest acceleration of the assembly so that the compressive force in the hydraulic cylinder supporting the boom does not exceed 180 kN. What is the tension in the supporting cable? The boom has a mass of 2 Mg and mass center at G.

**SOLUTION**

**Boom:**

\[ \sum M_A = 0; \quad 180(10^3)(2) - 2(10^3)(9.81)(6 \cos 60^\circ) - 2T(12 \cos 60^\circ) = 0 \]

\[ T = 25095 \text{ N} = 25.1 \text{ kN} \quad \text{Ans.} \]

**Assembly:**

\[ \sum F_y = ma; \quad 2(25095) - 4(10^3)(9.81) = 4(10^3) a \]

\[ a = 2.74 \text{ m/s}^2 \quad \text{Ans.} \]

\[ a \quad \text{Ans.} \]

\[ T = 25.1 \text{ kN} \]

\[ a = 2.74 \text{ m/s}^2 \]
17–30.

The uniform girder $AB$ has a mass of 8 Mg. Determine the internal axial, shear, and bending-moment loadings at the center of the girder if a crane gives it an upward acceleration of 3 m/s².

**SOLUTION**

Girder:

$+\Sigma F_y = ma_y$; \hspace{1cm} $2T \sin 60^\circ - 8000(9.81) = 8000(3)$

$T = 59\ 166.86\ N$

Segment:

$\pm\Sigma F_x = ma_x$; \hspace{1cm} $59\ 166.86\ \cos 60^\circ - N = 0$

$N = 29.6\ kN$

$+\Sigma F_y = ma_y$; \hspace{1cm} $59\ 166.86\ \sin 60^\circ - 4000(9.81) + V = 4000(3)$

$V = 0$

$\zeta + \Sigma M = \Sigma (M_k)_C$; \hspace{1cm} $M + 4000(9.81)(1) - 59\ 166.86\ \sin 60^\circ(2) = -4000(3)(1)$

$M = 51.2\ kN \cdot m$

Ans.

$N = 29.6\ kN$

$V = 0$

$M = 51.2\ kN \cdot m$
A car having a weight of 4000 lb begins to skid and turn with the brakes applied to all four wheels. If the coefficient of kinetic friction between the wheels and the road is \( \mu_k = 0.8 \), determine the maximum critical height \( h \) of the center of gravity \( G \) such that the car does not overturn. Tipping will begin to occur after the car rotates 90° from its original direction of motion and, as shown in the figure, undergoes translation while skidding. \textit{Hint:} Draw a free-body diagram of the car viewed from the front. When tipping occurs, the normal reactions of the wheels on the right side (or passenger side) are zero.

\textbf{SOLUTION}

\( N_A \) represents the reaction for both the front and rear wheels on the left side.

\[
\sum F_x = m(a_G)_x; \quad 0.8 N_A = \frac{4000}{32.2} a_G
\]

\[
+ \uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 4000 = 0
\]

\[
\zeta + \Sigma M_A = \Sigma (M_k)_A; \quad 4000(2.5) = \frac{4000}{32.2} (a_G) (h)
\]

Solving,

\( N_A = 4000 \text{ lb} \)

\( a_G = 25.76 \text{ ft/s}^2 \)

\( h = 3.12 \text{ ft} \)

\textbf{Ans:} \\
\( h = 3.12 \text{ ft} \)
A force of $P = 300$ N is applied to the 60-kg cart. Determine the reactions at both the wheels at $A$ and both the wheels at $B$. Also, what is the acceleration of the cart? The mass center of the cart is at $G$.

**SOLUTION**

**Equations of Motions.** Referring to the FBD of the cart, Fig. a,

\[ \sum F_x = m(a_G)_x; \quad 300 \cos 30^\circ = 60a \]
\[ a = 4.3301 \, \text{m/s}^2 = 4.33 \, \text{m/s}^2 \quad \text{Ans.} \]

\[ \sum F_y = m(a_G)_y; \quad N_A + N_B + 300 \sin 30^\circ - 60(9.81) = 60(0) \]
\[ (1) \]

\[ \sum M_G = 0; \quad N_B(0.2) - N_A(0.3) + 300 \cos 30^\circ(0.1) - 300 \sin 30^\circ(0.38) = 0 \]
\[ (2) \]

Solving Eqs. (1) and (2),

\[ N_A = 113.40 \, \text{N} = 113 \, \text{N} \quad \text{Ans.} \]
\[ N_B = 325.20 \, \text{N} = 325 \, \text{N} \quad \text{Ans.} \]

\[ a = 4.33 \, \text{m/s}^2 \quad \text{Ans.} \]

\[ N_A = 113 \, \text{N} \]
\[ N_B = 325 \, \text{N} \]
17–33.

Determine the largest force $P$ that can be applied to the 60-kg cart, without causing one of the wheel reactions, either at $A$ or at $B$, to be zero. Also, what is the acceleration of the cart? The mass center of the cart is at $G$.

**SOLUTION**

**Equations of Motions.** Since $(0.38 \text{ m}) \tan 30^\circ = 0.22 \text{ m} > 0.1 \text{ m}$, the line of action of $P$ passes below $G$. Therefore, $P$ tends to rotate the cart clockwise. The wheels at $A$ will leave the ground before those at $B$. Then, it is required that $N_A = 0$. Referring, to the FBD of the cart, Fig. $a$

$$ P \sin 30^\circ - 60(9.81) = 60(0) \tag{1} $$

$$ P \cos 30^\circ - N_B(0.38) + N_B(0.2) = 0 \tag{2} $$

Solving Eqs. (1) and (2)

$$ P = 578.77 \text{ N} = 579 \text{ N} \quad \text{Ans.} $$

$$ N_B = 299.22 \text{ N} \quad \text{Ans.} $$
17–34.

The trailer with its load has a mass of 150 kg and a center of mass at \( G \). If it is subjected to a horizontal force of \( P = 600 \) N, determine the trailer’s acceleration and the normal force on the pair of wheels at \( A \) and at \( B \). The wheels are free to roll and have negligible mass.

**SOLUTION**

*Equations of Motion:* Writing the force equation of motion along the \( x \) axis,

\[
\pm \sum F_x = m(a_G)_x; \quad 600 = 150a \quad \Rightarrow \quad a = 4 \text{ m/s}^2 \quad \text{Ans.}
\]

Using this result to write the moment equation about point \( A \),

\[
\zeta + \sum M_A = (M_k)_A; \quad 150(9.81)(1.25) - 600(0.5) - N_B(2) = -150(4)(1.25)
\]

\[
N_B = 1144.69 \text{ N} = 1.14 \text{ kN} \quad \text{Ans.}
\]

Using this result to write the force equation of motion along the \( y \) axis,

\[
+ \sum F_y = m(a_G)_y; \quad N_A + 1144.69 - 150(9.81) = 150(0)
\]

\[
N_A = 326.81 \text{ N} = 327 \text{ N} \quad \text{Ans.}
\]
17–35.
The desk has a weight of 75 lb and a center of gravity at \( G \).
Determine its initial acceleration if a man pushes on it with a force \( F = 60 \) lb. The coefficient of kinetic friction at \( A \) and \( B \) is \( \mu_k = 0.2 \).

**SOLUTION**

\[
\sum F_x = ma_x; \quad 60 \cos 30^\circ - 0.2N_A - 0.2N_B = \frac{75}{32.2} a_G
\]

\[
\sum F_y = ma_y; \quad N_A + N_B - 75 - 60 \sin 30^\circ = 0
\]

\[
\sum M_G = 0; \quad 60 \sin 30^\circ(2) - 60 \cos 30^\circ(1) - N_A(2) + N_B(2) - 0.2N_A(2) - 0.2N_B(2) = 0
\]

Solving,

\[ a_G = 13.3 \text{ ft/s}^2 \]

\[ N_A = 44.0 \text{ lb} \]

\[ N_B = 61.0 \text{ lb} \]
The desk has a weight of 75 lb and a center of gravity at $G$. Determine the initial acceleration of a desk when the man applies enough force $F$ to overcome the static friction at $A$ and $B$. Also, find the vertical reactions on each of the two legs at $A$ and $B$. The coefficients of static and kinetic friction at $A$ and $B$ are $\mu_s = 0.5$ and $\mu_k = 0.2$, respectively.

**SOLUTION**

Force required to start desk moving:

\[ \sum F_x = 0; \quad F \cos 30^\circ - 0.5N_A - 0.5N_B = 0 \]
\[ \sum F_y = 0; \quad N_A + N_B - F \sin 30^\circ - 75 = 0 \]

Solving for $F$ by eliminating $N_A + N_B$,

\[ F = 60.874 \text{ lb} \]

Desk starts to slide.

\[ \sum F_x = m(a_G); \quad 60.874 \cos 30^\circ - 0.2N_A - 0.2N_B = \frac{75}{32.2} a_G \]
\[ \sum F_y = m(a_G); \quad N_A + N_B - 60.874 \sin 30^\circ - 75 = 0 \]

Solving for $a_G$ by eliminating $N_A + N_B$,

\[ a_G = 13.58 \approx 13.6 \text{ ft/s}^2 \]

\[ \sum M_A = \sum (M_k)_A; \quad N_B(4) - 75(2) - 60.874 \cos 30^\circ(3) = \frac{-75}{32.2}(13.58)(2) \]

\[ N_B = 61.2 \text{ lb} \]

So that

\[ N_A = 44.2 \text{ lb} \]

For each leg,

\[ N_A' = \frac{44.2}{2} = 22.1 \text{ lb} \]

\[ N_B' = \frac{61.2}{2} = 30.6 \text{ lb} \]
17–37.

The 150-kg uniform crate rests on the 10-kg cart. Determine the maximum force $P$ that can be applied to the handle without causing the crate to tip on the cart. Slipping does not occur.

**SOLUTION**

**Equation of Motion.** Tipping will occur about edge $A$. Referring to the FBD and kinetic diagram of the crate, Fig. $a$,

$$\sum \tau_A + \sum M_A = \sum (M_k)_A: \quad 150(9.81)(0.25) = (150a)(0.5)$$

$$a = 4.905 \text{ m/s}^2$$

Using the result of $a$ and refer to the FBD of the crate and cart, Fig. $b$,

$$\sum F_x = m(a_G)_x \quad P = (150 + 10)(4.905) = 784.8 \text{ N} = 785 \text{ N} \quad \text{Ans.}$$
17–38.

The 150-kg uniform crate rests on the 10-kg cart. Determine the maximum force $P$ that can be applied to the handle without causing the crate to slip or tip on the cart. The coefficient of static friction between the crate and cart is $\mu_s = 0.2$.

**SOLUTION**

**Equation of Motion.** Assuming that the crate slips before it tips, then $F_f = \mu_s N = 0.2$ N. Referring to the FBD and kinetic diagram of the crate, Fig. a

\[ + \sum F_y = ma_y; \quad N - 150(9.81) = 150(0) \quad N = 1471.5 \text{ N} \]
\[ - \sum F_x = m(a_G)_y; \quad 0.2(1471.5) = 150a \quad a = 1.962 \text{ m/s}^2 \]
\[ \zeta + \sum M_A = (M_k)_A; \quad 150(9.81)(x) = 150(1.962)(0.5) \quad x = 0.1 \text{ m} \]

Since $x = 0.1 \text{ m} < 0.25 \text{ m}$, the crate indeed slips before it tips. Using the result of $a$ and refer to the FBD of the crate and cart, Fig. b,

\[ - \sum F_x = m(a_G)_y; \quad P = (150 + 10)(1.962) = 313.92 \text{ N} = 314 \text{ N} \quad \text{Ans.} \]
17–39.

The bar has a weight per length \( w \) and is supported by the smooth collar. If it is released from rest, determine the internal normal force, shear force, and bending moment in the bar as a function of \( x \).

**SOLUTION**

**Entire bar:**

\[
\sum F_x = m(a_G) \cos 30^\circ; \quad \text{wl} \cos 30^\circ = \frac{wl}{g}(a_G)
\]

\( a_G = g \cos 30^\circ \)

**Segment:**

\( \sum F_x = m(a_G) \cos 30^\circ; \quad N = (wx \cos 30^\circ) \sin 30^\circ = 0.433wx \)

\( \sum F_y = m(a_G) \); \quad \text{wx} - V = \text{wx} \cos 30^\circ \cos 30^\circ \)

\( V = 0.25wx \)

\( \sum M_S = \sum (M_k)_S; \quad \text{wx} \frac{x}{2} - M = \text{wx} \cos 30^\circ \cos 30^\circ \frac{x}{2} \)

\( M = 0.125wx^2 \)

\( N = 0.433wx \)

\( V = 0.25wx \)

\( M = 0.125wx^2 \)
The smooth 180-lb pipe has a length of 20 ft and a negligible diameter. It is carried on a truck as shown. Determine the maximum acceleration which the truck can have without causing the normal reaction at A to be zero. Also determine the horizontal and vertical components of force which the truck exerts on the pipe at B.

**SOLUTION**

\[ \sum F_i = ma_i; \quad B_x = \frac{180}{32.2}a_T \]

\[ + \sum F_y = 0; \quad B_y - 180 = 0 \]

\[ \sum M_B = \sum (M_B)_B; \quad 180(10)\left(\frac{12}{13}\right) = \frac{180}{32.2}a_T(10)\left(\frac{5}{13}\right) \]

Solving,

\[ B_x = 432 \text{ lb} \]
\[ B_y = 180 \text{ lb} \]
\[ a_T = 77.3 \text{ ft/s}^2 \]

*Ans.*

\[ B_x = 432 \text{ lb} \]
\[ B_y = 180 \text{ lb} \]
\[ a_T = 77.3 \text{ ft/s}^2 \]
17–41.

The smooth 180-lb pipe has a length of 20 ft and a negligible diameter. It is carried on a truck as shown. If the truck accelerates at $a = 5 \text{ ft/s}^2$, determine the normal reaction at $A$ and the horizontal and vertical components of force which the truck exerts on the pipe at $B$.

**SOLUTION**

\[ \Sigma F_x = ma_x; \quad B_x - N_A \left( \frac{5}{13} \right) = \frac{180}{32.2} (5) \]

\[ \Sigma F_y = 0; \quad B_y - 180 + N_A \left( \frac{12}{13} \right) = 0 \]

\[ \Sigma M_B = \Sigma (M_k)_B; \quad -180 \left( \frac{12}{13} \right) + N_A (13) = -\frac{180}{32.2} (5)(10) \left( \frac{5}{13} \right) \]

Solving,

\[ B_x = 73.9 \text{ lb} \quad \text{Ans.} \]
\[ B_y = 69.7 \text{ lb} \quad \text{Ans.} \]
\[ N_A = 120 \text{ lb} \quad \text{Ans.} \]
17–42.

The uniform crate has a mass of 50 kg and rests on the cart having an inclined surface. Determine the smallest acceleration that will cause the crate either to tip or slip relative to the cart. What is the magnitude of this acceleration? The coefficient of static friction between the crate and the cart is $\mu_s = 0.5$.

**SOLUTION**

*Equations of Motion:* Assume that the crate slips, then $F_f = \mu_s N = 0.5N$.

\[ \sum F_y = m(a_G)_y; \quad N - 50(9.81) \cos 15^\circ - 50(9.81) \sin 15^\circ(0.5) = 50 \cos 15^\circ(0.5) + 50 \sin 15^\circ(x) \]

\[ \sum F_x = m(a_G)_x; \quad 50(9.81) \sin 15^\circ - 0.5N = -50 \sin 15^\circ \]

Solving Eqs. (1), (2), and (3) yields

\[ N = 447.81 \text{ N} \quad x = 0.250 \text{ m} \]

\[ a = 2.01 \text{ m/s}^2 \]

Since $x < 0.3 \text{ m}$, then crate will not tip. Thus, the crate slips.

**Ans:**

\[ a = 2.01 \text{ m/s}^2 \]

The crate slips.
17–43.

Determine the acceleration of the 150-lb cabinet and the normal reaction under the legs \( A \) and \( B \) if \( P = 35 \text{ lb} \). The coefficients of static and kinetic friction between the cabinet and the plane are \( \mu_s = 0.2 \) and \( \mu_k = 0.15 \), respectively. The cabinet’s center of gravity is located at \( G \).

**SOLUTION**

**Equations of Equilibrium:** The free-body diagram of the cabinet under the static condition is shown in Fig. \( a \), where \( P \) is the unknown minimum force needed to move the cabinet. We will assume that the cabinet slides before it tips. Then, \( F_A = \mu_s N_A = 0.2 N_A \) and \( F_B = \mu_s N_B = 0.2 N_B \).

\[
\begin{align*}
\sum F_x & = 0; \quad P - 0.2 N_A - 0.2 N_B = 0 \quad (1) \\
\sum F_y & = 0; \quad N_A + N_B - 150 = 0 \quad (2) \\
\sum M_A & = 0; \quad N_B(2) - 150(1) - P(4) = 0 \quad (3)
\end{align*}
\]

Solving Eqs. (1), (2), and (3) yields

\[ P = 30 \text{ lb} \quad N_A = 15 \text{ lb} \quad N_B = 135 \text{ lb} \]

Since \( P < 35 \text{ lb} \) and \( N_A \) is positive, the cabinet will slide.

**Equations of Motion:** Since the cabinet is in motion, \( F_A = \mu_k N_A = 0.15 N_A \) and \( F_B = \mu_k N_B = 0.15 N_B \). Referring to the free-body diagram of the cabinet shown in Fig. \( b \),

\[
\begin{align*}
\sum F_x & = m(a_G)x; \quad 35 - 0.15 N_A - 0.15 N_B = \left( \frac{150}{32.2} \right) a \quad (4) \\
\sum F_y & = m(a_G)y; \quad N_A + N_B - 150 = 0 \quad (5) \\
\sum M_G & = 0; \quad N_B(1) - 0.15 N_B(3.5) - 0.15 N_A(3.5) - N_A(1) - 35(0.5) = 0 \quad (6)
\end{align*}
\]

Solving Eqs. (4), (5), and (6) yields

\[ a = 2.68 \text{ ft/s}^2 \quad \text{Ans.} \]
\[ N_A = 26.9 \text{ lb} \quad N_B = 123 \text{ lb} \quad \text{Ans.} \]
The uniform bar of mass $m$ is pin connected to the collar, which slides along the smooth horizontal rod. If the collar is given a constant acceleration of $a$, determine the bar’s inclination angle $\theta$. Neglect the collar’s mass.

**SOLUTION**

*Equations of Motion:* Writing the moment equation of motion about point $A$,

$$+ \Sigma M_A = (M_A)_A; \quad mg \sin \theta \left( \frac{L}{2} \right) = ma \cos \theta \left( \frac{L}{2} \right)$$

$$\theta = \tan^{-1} \left( \frac{a}{g} \right)$$

**Ans:**

$$\theta = \tan^{-1} \left( \frac{a}{g} \right)$$
17–45.

The drop gate at the end of the trailer has a mass of 1.25 Mg and mass center at $G$. If it is supported by the cable $AB$ and hinge at $C$, determine the tension in the cable when the truck begins to accelerate at 5 m/s$^2$. Also, what are the horizontal and vertical components of reaction at the hinge $C$?

**SOLUTION**

\[ \zeta + \Sigma M_C = \Sigma (M_k)_{C}; \quad T \sin 30^\circ(2.5) - 12 \ 262.5(1.5 \cos 45^\circ) = 1250(5)(1.5 \sin 45^\circ) \]

\[ T = 15 \ 708.4 \text{ N} = 15.7 \text{ kN} \quad \text{Ans.} \]

\[ \Leftrightarrow \Sigma F_x = m(a_G)_{x}; \quad -C_x + 15 \ 708.4 \cos 15^\circ = 1250(5) \]

\[ C_x = 8.92 \text{ kN} \quad \text{Ans.} \]

\[ \rightarrow \Sigma F_y = m(a_G)_{y}; \quad C_y - 12 \ 262.5 - 15 \ 708.4 \sin 15^\circ = 0 \]

\[ C_y = 16.3 \text{ kN} \quad \text{Ans.} \]
17–46.

The drop gate at the end of the trailer has a mass of 1.25 Mg and mass center at \( G \). If it is supported by the cable \( AB \) and hinge at \( C \), determine the maximum deceleration of the truck so that the gate does not begin to rotate forward. What are the horizontal and vertical components of reaction at the hinge \( C \)?

**SOLUTION**

\[ \zeta + \Sigma M_C = \Sigma (M_k)_C; \quad -12262.5(1.5 \cos 45^\circ) = -1250(a)(1.5 \sin 45^\circ) \]

\[ a = 9.81 \text{ m/s}^2 \]

\[ \pm \Sigma F_x = m(a_G)_x; \quad C_x = 1250(9.81) \]

\[ C_x = 12.3 \text{ kN} \]

\[ + \Sigma F_y = m(a_G)_y; \quad C_y - 12262.5 = 0 \]

\[ C_y = 12.3 \text{ kN} \]
17–47.

The snowmobile has a weight of 250 lb, centered at $G_1$, while the rider has a weight of 150 lb, centered at $G_2$. If the acceleration is $a = 20 \text{ ft/s}^2$, determine the maximum height $h$ of $G_2$ of the rider so that the snowmobile’s front skid does not lift off the ground. Also, what are the traction (horizontal) force and normal reaction under the rear tracks at $A$?

**SOLUTION**

*Equations of Motion:* Since the front skid is required to be on the verge of lift off, $N_B = 0$. Writing the moment equation about point $A$ and referring to Fig. $a$,

$$\sum M_A = (M_k)_A; \quad 250(1.5) + 150(0.5) = \frac{150}{32.2}(20)(h_{\text{max}}) + \frac{250}{32.2}(20)(1)$$

$$h_{\text{max}} = 3.163 \text{ ft} = 3.16 \text{ ft} \quad \text{Ans.}$$

Writing the force equations of motion along the $x$ and $y$ axes,

$$\sum F_x = m(a_G)_x; \quad F_A = \frac{150}{32.2}(20) + \frac{250}{32.2}(20)$$

$$F_A = 248.45 \text{ lb} = 248 \text{ lb} \quad \text{Ans.}$$

$$\sum F_y = m(a_G)_y; \quad N_A - 250 - 150 = 0$$

$$N_A = 400 \text{ lb} \quad \text{Ans.}$$

$$h_{\text{max}} = 3.16 \text{ ft}$$
$$F_A = 248 \text{ lb}$$
$$N_A = 400 \text{ lb}$$
The snowmobile has a weight of 250 lb, centered at \( G_1 \), while the rider has a weight of 150 lb, centered at \( G_2 \). If \( h = 3 \text{ ft} \), determine the snowmobile’s maximum permissible acceleration \( a \) so that its front skid does not lift off the ground. Also, find the traction (horizontal) force and the normal reaction under the rear tracks at \( A \).

**SOLUTION**

*Equations of Motion:* Since the front skid is required to be on the verge of lift off, \( N_B = 0 \). Writing the moment equation about point \( A \) and referring to Fig. \( a \),

\[
\sum M_A = (M_k)_A ; \quad 250(1.5) + 150(0.5) = \left( \frac{150}{32.2} a_{\text{max}} \right) (3) + \left( \frac{250}{32.2} a_{\text{max}} \right) (1)
\]

\[a_{\text{max}} = 20.7 \text{ ft/s}^2 \quad \text{Ans.}\]

Writing the force equations of motion along the \( x \) and \( y \) axes and using this result, we have

\[
\sum F_x = m(a_G)_x ; \quad F_A = \frac{150}{32.2} (20.7) + \frac{250}{32.2} (20.7)
\]

\[F_A = 257.14 \text{ lb} = 257 \text{ lb} \quad \text{Ans.}\]

\[
\sum F_y = m(a_G)_y ; \quad N_A - 150 - 250 = 0
\]

\[N_A = 400 \text{ lb} \quad \text{Ans.}\]

*Ans:*
\[
a_{\text{max}} = 20.7 \text{ ft/s}^2
\]
\[
F_A = 257 \text{ lb}
\]
\[
N_A = 400 \text{ lb}
\]
17–49.

If the cart’s mass is 30 kg and it is subjected to a horizontal force of \( P = 90 \text{ N} \), determine the tension in cord \( AB \) and the horizontal and vertical components of reaction on end \( C \) of the uniform 15-kg rod \( BC \).

**SOLUTION**

*Equations of Motion:* The acceleration \( a \) of the cart and the rod can be determined by considering the free-body diagram of the cart and rod system shown in Fig. \( a \).

\[
\Rightarrow \sum F_x = m(a_G)_x; \quad 90 = (15 + 30)a \quad a = 2 \text{ m/s}^2
\]

The force in the cord can be obtained directly by writing the moment equation of motion about point \( C \) by referring to Fig. \( b \).

\[
+ \sum M_C = (M_k)_C; \quad F_{AB} \sin 30^\circ(1) - 15(9.81) \cos 30^\circ(0.5) = -15(2) \sin 30^\circ(0.5)
\]

\[
F_{AB} = 112.44 \text{ N} = 112 \text{ N} \quad \text{Ans.}
\]

Using this result and applying the force equations of motion along the \( x \) and \( y \) axes,

\[
\Rightarrow \sum F_x = m(a_G)_x; \quad -C_x + 112.44 \sin 30^\circ = 15(2)
\]

\[
C_x = 26.22 \text{ N} = 26.2 \text{ N} \quad \text{Ans.}
\]

\[
+ \sum F_y = m(a_G)_y; \quad C_y + 112.44 \cos 30^\circ - 15(9.81) = 0
\]

\[
C_y = 49.78 \text{ N} = 49.8 \text{ N} \quad \text{Ans.}
\]

\[
\text{Ans:} \quad F_{AB} = 112 \text{ N}
\]

\[
C_x = 26.2 \text{ N}
\]

\[
C_y = 49.8 \text{ N}
\]
17–50.

If the cart’s mass is 30 kg, determine the horizontal force $P$ that should be applied to the cart so that the cord $AB$ just becomes slack. The uniform rod $BC$ has a mass of 15 kg.

**SOLUTION**

*Equations of Motion:* Since cord $AB$ is required to be on the verge of becoming slack, $F_{AB} = 0$. The corresponding acceleration $a$ of the rod can be obtained directly by writing the moment equation of motion about point $C$. By referring to Fig. $a$.

$$+ \Sigma M_C = \Sigma (M_C)_A; \quad -15(9.81) \cos 30°(0.5) = -15a \sin 30°(0.5)$$

$$a = 16.99 \text{ m/s}^2$$

Using this result and writing the force equation of motion along the $x$ axis and referring to the free-body diagram of the cart and rod system shown in Fig. $b$,

$$\left( \pm \right) \Sigma F_x = m(a_G); \quad P = (30 + 15)(16.99)$$

$$= 764.61 \text{ N} = 765 \text{ N}$$

Ans: $P = 765 \text{ N}$
17–51.

The pipe has a mass of 800 kg and is being towed behind the truck. If the acceleration of the truck is $a_t = 0.5 \text{ m/s}^2$, determine the angle $\theta$ and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is $\mu_k = 0.1$.

**SOLUTION**

\[ \sum F_x = ma_c; \quad -0.1N_c + T \cos 45^\circ = 800(0.5) \]

\[ \sum F_y = ma_v; \quad N_c - 800(9.81) + T \sin 45^\circ = 0 \]

\[ \sum M_G = 0; \quad -0.1N_c(0.4) + T \sin \phi(0.4) = 0 \]

$N_c = 6770.9 \text{ N}$

$T = 1523.24 \text{ N} = 1.52 \text{ kN}$

\[ \sin \phi = \frac{0.1(6770.9)}{1523.24} \quad \phi = 26.39^\circ \]

$\theta = 45^\circ - \phi = 18.6^\circ$

Ans: $T = 1.52 \text{ kN}$

$\theta = 18.6^\circ$
*17–52.

The pipe has a mass of 800 kg and is being towed behind a truck. If the angle $\theta = 30^\circ$, determine the acceleration of the truck and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is $\mu_k = 0.1$.

**SOLUTION**

\[ \Sigma F_x = ma_x; \quad T \cos 45° - 0.1N_C = 800a \]
\[ + \Sigma F_y = ma_y; \quad N_C - 800(9.81) + T \sin 45° = 0 \]
\[ \zeta + \Sigma M_G = 0; \quad T \sin 15°(0.4) - 0.1N_C(0.4) = 0 \]

- $N_C = 6161$ N
- $T = 2382$ N = 2.38 kN
- $a = 1.33$ m/s$^2$

Ans: $T = 2.38$ kN  
$a = 1.33$ m/s$^2$
17–53.

The crate \( C \) has a weight of 150 lb and rests on the truck elevator for which the coefficient of static friction is \( \mu_s = 0.4 \). Determine the largest initial angular acceleration \( \alpha \), starting from rest, which the parallel links \( AB \) and \( DE \) can have without causing the crate to slip. No tipping occurs.

**SOLUTION**

\[ \sum F_x = ma_x; \quad 0.4N_C = \frac{150}{32.2} (a) \cos 30^\circ \]

\[ \sum F_y = ma_y; \quad N_C - 150 = \frac{150}{32.2} (a) \sin 30^\circ \]

\( N_C = 195.0 \text{ lb} \)

\( a = 19.34 \text{ ft/s}^2 \)

\( 19.34 = 2a \)

\( a = 9.67 \text{ rad/s}^2 \)

\[ \text{Ans:} \quad \alpha = 9.67 \text{ rad/s}^2 \]
The crate \( C \) has a weight of 150 lb and rests on the truck elevator. Determine the initial friction and normal force of the elevator on the crate if the parallel links are given an angular acceleration \( \alpha = 2 \text{ rad/s}^2 \) starting from rest.

**SOLUTION**

\[ \alpha = 2 \text{ rad/s}^2 \]
\[ a = 2\alpha = 4 \text{ rad/s}^2 \]

\[ + \sum F_x = ma; \quad F_C = \frac{150}{32.2} (a) \cos 30^\circ \]

\[ + \sum F_y = ma; \quad N_C - 150 = \frac{150}{32.2} (a) \sin 30^\circ \]

\[ F_C = 16.1 \text{ lb} \]
\[ N_C = 159 \text{ lb} \]

Ans.

Ans.
17–55.

The 100-kg uniform crate \( C \) rests on the elevator floor where the coefficient of static friction is \( \mu_s = 0.4 \). Determine the largest initial angular acceleration \( \alpha \), starting from rest at \( \theta = 90^\circ \), without causing the crate to slip. No tipping occurs.

**SOLUTION**

**Equations of Motion.** The crate undergoes curvilinear translation. At \( \theta = 90^\circ \), \( \omega = 0 \). Thus, \( (a_G)_n = \omega^2 r = 0 \). However; \( (a_G)_t = ar = \alpha (1.5) \). Assuming that the crate slides before it tips, then, \( F_f = \mu_s N = 0.4 \text{ N} \).

\[
\sum F_n = m(a_G)_n; \quad 100(9.81) - N = 100(0) \quad N = 981 \text{ N}
\]

\[
\sum F_t = m(a_G)_t; \quad 0.4(981) = 100[\alpha (1.5)] \quad \alpha = 2.616 \text{ rad/s}^2 = 2.62 \text{ rad/s}^2 \textbf{ Ans.}
\]

\[
\zeta + \sum M_G = 0; \quad 0.4(981)(0.6) - 981(x) = 0
\]

\[
x = 0.24 \text{ m}
\]

Since \( x < 0.3 \text{ m} \), the crate indeed slides before it tips, as assumed.

Ans: \( \alpha = 2.62 \text{ rad/s}^2 \)
The two uniform 4-kg bars $DC$ and $EF$ are fixed (welded) together at $E$. Determine the normal force $N_E$, shear force $V_E$, and moment $M_E$, which $DC$ exerts on $EF$ at $E$ if at the instant $\theta = 60^\circ$ BC has an angular velocity $\omega = 2 \text{ rad/s}$ and an angular acceleration $\alpha = 4 \text{ rad/s}^2$ as shown.

**SOLUTION**

**Equations of Motion.** The rod assembly undergoes curvilinear motion. Thus, $(a_G)_x = \alpha r = 4(2) = 8 \text{ m/s}^2$ and $(a_G)_y = \omega^2 r = (2^2)(2) = 8 \text{ m/s}^2$. Referring to the FBD and kinetic diagram of rod $EF$, Fig. a

\[
\begin{align*}
\sum F_x &= m(a_G)_x; \quad V_E = 4(8) \cos 30^\circ + 4(8) \cos 60^\circ \\
&= 43.71 \text{ N} = 43.7 \text{ N} \quad \text{Ans.} \\
+ \sum F_y &= m(a_G)_y; \quad N_E - 4(9.81) = 4(8) \sin 30^\circ - 4(8) \sin 60^\circ \\
&= 27.53 \text{ N} = 27.5 \text{ N} \quad \text{Ans.} \\
\sum M_E &= \sum (M_E)_x; \quad M_E = 4(8) \cos 30^\circ(0.75) + 4(8) \cos 60^\circ(0.75) \\
&= 32.78 \text{ N} \cdot \text{m} = 32.8 \text{ N} \cdot \text{m} \quad \text{Ans.}
\end{align*}
\]

\[\begin{array}{l}
\text{Ans:} \\
V_E = 43.7 \text{ N} \\
N_E = 27.5 \text{ N} \\
M_E = 32.8 \text{ N} \cdot \text{m}
\end{array}\]
17–57.

The 10-kg wheel has a radius of gyration \( k_A = 200 \text{ mm} \). If the wheel is subjected to a moment \( M = (5t) \text{ N} \cdot \text{m} \), where \( t \) is in seconds, determine its angular velocity when \( t = 3 \text{ s} \) starting from rest. Also, compute the reactions which the fixed pin \( A \) exerts on the wheel during the motion.

**SOLUTION**

\[ \Rightarrow \Sigma F_x = m(a_x)_x; \quad A_x = 0 \]
\[ + \uparrow \Sigma F_y = m(a_y)_y; \quad A_y - 10(9.81) = 0 \]
\[ \zeta + \Sigma M_A = I_A \alpha; \quad 5t = 10(0.2)^2 \alpha \]

\[ \alpha = \frac{d\omega}{dt} = 12.5t \]

\[ \omega = \int_0^3 12.5t \, dt = \frac{12.5}{2}(3)^2 \]

\[ \omega = 56.2 \text{ rad/s} \quad \text{Ans.} \]
\[ A_x = 0 \quad \text{Ans.} \]
\[ A_y = 98.1 \text{ N} \quad \text{Ans.} \]

**Ans:**

\( \omega = 56.2 \text{ rad/s} \)
\( A_x = 0 \)
\( A_y = 98.1 \text{ N} \)
17–58.

The uniform 24-kg plate is released from rest at the position shown. Determine its initial angular acceleration and the horizontal and vertical reactions at the pin A.

**SOLUTION**

**Equations of Motion.** The mass moment of inertia of the plate about its center of gravity \( G \) is \( I_G = \frac{1}{12}(24)(0.5^2 + 0.5^2) = 1.00 \text{ kg} \cdot \text{m}^2 \). Since the plate is at rest initially \( \omega = 0 \). Thus, \( (a_G)_N = \omega^2 r_G = 0 \). Here \( r_G = \sqrt{0.25^2 + 0.25^2} = 0.25\sqrt{2} \text{ m} \).

Thus, \( (a_G)_t = ar_G = \alpha(0.25\sqrt{2}) \). Referring to the FBD and kinetic diagram of the plate,

\[
\zeta + \sum M_A = (M_G)_A; \quad -24(9.81)(0.25) = -24\big[\alpha\big(0.25\sqrt{2}\big)\big]\big(0.25\sqrt{2}\big) - 1.00 \alpha \nn \alpha = 14.715 \text{ rad/s}^2 = 14.7 \text{ rad/s}^2
\]

Ans.

Also, the same result can be obtained by applying \( \Sigma M_A = I_A \alpha \) where

\[
I_A = \frac{1}{12}(24)(0.5^2 + 0.5^2) + 24(0.25\sqrt{2})^2 = 4.00 \text{ kg} \cdot \text{m}^2
\]

\[
\zeta + \sum M_A = I_A \alpha; \quad -24(9.81)(0.25) = -4.00 \alpha \nn \alpha = 14.715 \text{ rad/s}^2
\]

\[
\sum F_x = m(a_G)_x; \quad A_x = 24\big[14.715(0.25\sqrt{2})\big] \cos 45^\circ = 88.29 \text{ N} = 88.3 \text{ N} \quad \text{Ans.}
\]

\[
\sum F_y = m(a_G)_y; \quad A_y = -24(9.81) = -24\big[14.715(0.25\sqrt{2})\big] \sin 45^\circ
\]

\[
A_y = 147.15 \text{ N} = 147 \text{ N} \quad \text{Ans.}
\]
17–59.

The uniform slender rod has a mass \( m \). If it is released from rest when \( \theta = 0^\circ \), determine the magnitude of the reactive force exerted on it by pin \( B \) when \( \theta = 90^\circ \).

**SOLUTION**

*Equations of Motion:* Since the rod rotates about a fixed axis passing through point \( B \), (\( a_G \))\( = \alpha r_G = \alpha \left( \frac{L}{6} \right) \) and (\( a_G \))\( = \omega^2 r_G = \omega^2 \left( \frac{L}{6} \right) \). The mass moment of inertia of the rod about its \( G \) is \( I_G = \frac{1}{12} mL^2 \). Writing the moment equation of motion about point \( B \),

\[
+ \Sigma M_B = \Sigma (M_a)_B; \quad -mg \cos \theta \left( \frac{L}{6} \right) = -m \left[ \alpha \left( \frac{L}{6} \right) \left( \frac{L}{6} \right) - \left( \frac{1}{12} mL^2 \right) \alpha \right]
\]

\[
\alpha = \frac{3g}{2L} \cos \theta
\]

This equation can also be obtained by applying \( \Sigma M_B = I_B \alpha \), where \( I_B = \frac{1}{12} mL^2 + m \left( \frac{L}{6} \right)^2 = \frac{1}{9} mL^2 \). Thus,

\[
+ \Sigma M_B = I_B \alpha; \quad -mg \cos \theta \left( \frac{L}{6} \right) = - \left( \frac{1}{9} mL^2 \right) \alpha
\]

\[
\alpha = \frac{3g}{2L} \cos \theta
\]

Using this result and writing the force equation of motion along the \( n \) and \( t \) axes,

\[
\Sigma F_t = m(a_G)_t; \quad mg \cos \theta - B_t = m \left[ \omega^2 \left( \frac{L}{6} \right) \right]
\]

\[
B_t = \frac{3}{4} mg \cos \theta
\]

\[
(1)
\]

\[
\Sigma F_n = m(a_G)_n; \quad B_n - mg \sin \theta = m \left[ \omega^2 \left( \frac{L}{6} \right) \right]
\]

\[
B_n = \frac{1}{6} m \omega^2 L + mg \sin \theta
\]

\[
(2)
\]

*Kinematics:* The angular velocity of the rod can be determined by integrating

\[
\int_0^\omega \omega d\omega = \int_0^\alpha d\theta
\]

\[
\int_0^\omega \omega d\omega = \int_0^\alpha \frac{3g}{2L} \cos \theta d\theta
\]

\[
\omega = \sqrt{\frac{3g}{L} \sin \theta}
\]

When \( \theta = 90^\circ \), \( \omega = \sqrt{\frac{3g}{L}} \). Substituting this result and \( \theta = 90^\circ \) into Eqs. (1) and (2),

\[
B_t = \frac{3}{4} mg \cos 90^\circ = 0
\]

\[
B_n = \frac{1}{6} m \left[ \frac{3g}{L} \right] L + mg \sin 90^\circ = \frac{3}{2} mg
\]

\[
F_A = \sqrt{A_t^2 + A_n^2} = \sqrt{0^2 + \left( \frac{3}{2} mg \right)^2} = \frac{3}{2} mg
\]

Ans: \( F_A = \frac{3}{2} mg \)
The bent rod has a mass of 2 kg/m. If it is released from rest in the position shown, determine its initial angular acceleration and the horizontal and vertical components of reaction at A.

**SOLUTION**

**Equations of Motion.** Referring to Fig. a, the location of center of gravity $G$ of the bent rod is at

$$
\bar{x} = \frac{\sum x m}{\sum m} = \frac{2(0.75)(1.5)(2) + 1.5(2)(1.5)}{3(1.5)(2)} = 1.00 \text{ m}
$$

$$
\bar{y} = \frac{1.5}{2} = 0.75 \text{ m}
$$

The mass moment of inertia of the bent rod about its center of gravity is

$$
I_G = 2 \left( \frac{1}{12} (3)(1.5^2) + 3(0.25^2 + 0.75^2) \right) + \frac{1}{12} (3)(1.5^2) + 3(0.5^2) = 6.1875 \text{ kg} \cdot \text{m}^2.
$$

Here, $r_G = \sqrt{1.00^2 + 0.75^2} = 1.25 \text{ m}$. Since the bent rod is at rest initially, $\omega = 0$. Thus, $(a_G)_n = \omega^2 r_G = 0$. Also, $(a_G)_t = a r_G = a(1.25)$. Referring to the FBD and kinetic diagram of the plate,

$$
\zeta + \Sigma M_A = (M_k)_A; \quad 9(9.81)(1) = 9[a(1.25)](1.25) + 6.1875 a
$$

$$
\alpha = 4.36 \text{ rad/s}^2
$$

Ans.

Also, the same result can be obtained by applying $\Sigma M_A = I_A \alpha$ where

$$
I_A = \frac{1}{12} (3)(1.5^2) + 3(0.75^2) + \frac{1}{12} (3)(1.5^2) + 3(1.5^2 + 0.75^2)
$$

$$
+ \frac{1}{12} (3)(1.5^2) + 3(1.5^2 + 0.75^2) = 20.25 \text{ kg} \cdot \text{m}^2;
$$

$$
\zeta + \Sigma M_A = I_A \alpha, \quad 9(9.81)(1) = 20.25 \alpha \quad \alpha = 4.36 \text{ rad/s}^2
$$

Ans.

$$
\downarrow \Sigma F_x = m(a_G)_t; \quad A_x = 9[4.36(1.25)]\left(\frac{3}{5}\right) = 29.43 \text{ N} = 29.4 \text{ N}
$$

Ans.

$$
+ \Sigma F_y = m(a_G)_t; \quad A_y - 9(9.81) = -9[4.36(1.25)]\left(\frac{4}{5}\right)
$$

$$
A_y = 49.05 \text{ N} = 49.1 \text{ N}
$$

Ans.
17–61.

If a horizontal force of $P = 100$ N is applied to the 300-kg reel of cable, determine its initial angular acceleration. The reel rests on rollers at $A$ and $B$ and has a radius of gyration of $k_O = 0.6$ m.

**SOLUTION**

**Equations of Motions.** The mass moment of inertia of the reel about $O$ is $I_O = M k_O^2 = 300 \times (0.6^2) = 108$ kg · m². Referring to the FBD of the reel, Fig. a,

$$
\sum M_O = I_O \alpha; \quad -100(0.75) = 108(-\alpha)
$$

$$
\alpha = 0.6944 \text{ rad/s}^2
$$

$$
= 0.694 \text{ rad/s}^2
$$

Ans: $\alpha = 0.694 \text{ rad/s}^2$
The 10-lb bar is pinned at its center $O$ and connected to a torsional spring. The spring has a stiffness $k = 5 \text{ lb} \cdot \text{ft}/\text{rad}$, so that the torque developed is $M = (5\theta) \text{ lb} \cdot \text{ft}$, where $\theta$ is in radians. If the bar is released from rest when it is vertical at $\theta = 90^\circ$, determine its angular velocity at the instant $\theta = 0^\circ$.

**SOLUTION**

\[
\zeta + \sum M_D = I_O \alpha; \quad 5\theta = \left[ \frac{1}{12} \left( \frac{10}{32.2} \right) (2)^2 \right] \alpha \\
- 48.3 \theta = \alpha \\
\alpha \, d\theta = \omega \, d\omega \\
- \int_0^\omega 48.3 \theta \, d\theta = \int_0^\omega \omega \, d\omega \\
\frac{48.3}{2} \left( \frac{\pi}{2} \right)^2 = \frac{1}{2} \omega^2 \\
\omega = 10.9 \text{ rad/s} \quad \text{Ans.}
\]
17–63.

The 10-lb bar is pinned at its center \( O \) and connected to a torsional spring. The spring has a stiffness \( k = 5 \) lb \( \cdot \) ft/rad, so that the torque developed is \( M = (5\theta) \) lb \( \cdot \) ft, where \( \theta \) is in radians. If the bar is released from rest when it is vertical at \( \theta = 90^\circ \), determine its angular velocity at the instant \( \theta = 45^\circ \).

**SOLUTION**

\[ \zeta + 2M_O = I_0\alpha; \quad 5\theta = \left[ \frac{1}{12}(\frac{10}{3.22})(2)^2 \right] \alpha \]

\[ \alpha = -48.3\theta \]

\[ \alpha \, d\theta = \omega \, d\omega \]

\[ -\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} 48.3\theta \, d\theta = \int_0^\omega \omega \, d\omega \]

\[ -24.15 \left( \left( \frac{\pi}{4} \right)^2 - \left( \frac{\pi}{2} \right)^2 \right) = \frac{1}{2} \omega^2 \]

\[ \omega = 9.45 \text{ rad/s} \quad \text{Ans.} \]
A cord is wrapped around the outer surface of the 8-kg disk. If a force of \( F = \left( \frac{1}{4} \theta^2 \right) \) N, where \( \theta \) is in radians, is applied to the cord, determine the disk’s angular acceleration when it has turned 5 revolutions. The disk has an initial angular velocity of \( \omega_0 = 1 \text{ rad/s} \).

**SOLUTION**

**Equations of Motion.** The mass moment inertia of the disk about \( O \) is

\[
I_O = \frac{1}{2} mr^2 = \frac{1}{2} (8)(0.3^2) = 0.36 \text{ kg} \cdot \text{m}^2.
\]

Referring to the FBD of the disk, Fig. \( a \),

\[
\zeta + \sum M_O = I_O \alpha; \quad \left( \frac{1}{4} \theta^2 \right)(0.3) = 0.36 \alpha
\]

\[
\alpha = 0.2083 \theta^2 \text{ rad/s}^2
\]

**Kinematics.** Using the result of \( \alpha \), integrate \( \omega d\omega = \alpha d\theta \) with the initial condition \( \omega = 0 \) when \( \theta = 0 \),

\[
\int_0^\omega \omega d\omega = \int_0^{5(2\pi)} 0.2083 \theta^2 d\theta
\]

\[
\left( \frac{1}{2} \right)(\omega^2 - 1) = 0.06944 \theta^3 \bigg|_0^{5(2\pi)}
\]

\[
\omega = 65.63 \text{ rad/s} = 65.6 \text{ rad/s}
\]

**Ans:**

\[
\omega = 65.6 \text{ rad/s}
\]
17–65.

Disk A has a weight of 5 lb and disk B has a weight of 10 lb. If no slipping occurs between them, determine the couple moment $M$ which must be applied to disk A to give it an angular acceleration of 4 rad/s².

**SOLUTION**

Disk A:

$$
\zeta + \sum M_A = I_A \alpha_A; \quad M - F_D(0.5) = \left[ \frac{1}{2} \left( \frac{5}{32.2} \right) (0.5)^2 \right] (4)
$$

Disk B:

$$
+ \sum M_B = I_B \alpha_B; \quad F_D(0.75) = \left[ \frac{1}{2} \left( \frac{10}{32.2} \right) (0.75)^2 \right] \alpha_B
$$

\[
\begin{align*}
\frac{r_A \alpha_A}{r_B \alpha_B} &= 0.5(4) = 0.75 \alpha_B \\
\alpha_B &= 2.67 \text{ rad/s}^2; \quad F_D = 0.311 \text{ lb}
\end{align*}
\]

Solving:

\[
\alpha_B = 2.67 \text{ rad/s}^2; \quad F_D = 0.311 \text{ lb}
\]

$$
M = 0.233 \text{ lb} \cdot \text{ft}
$$

Ans:

$$
M = 0.233 \text{ lb} \cdot \text{ft}
$$
The kinetic diagram representing the general rotational motion of a rigid body about a fixed axis passing through $O$ is shown in the figure. Show that $I_G \alpha$ may be eliminated by moving the vectors $m(a_G)_h$ and $m(a_G)_n$ to point $P$, located a distance $r_{GP} = k_G/r_{OG}$ from the center of mass $G$ of the body. Here $k_G$ represents the radius of gyration of the body about an axis passing through $G$. The point $P$ is called the center of percussion of the body.

**SOLUTION**

$$m(a_G)_h r_{OG} + I_G \alpha = m(a_G)_h r_{OG} + (mk_G^2) \alpha$$

However,

$$k_G^2 = r_{OG} r_{GP} \text{ and } \alpha = \frac{(a_G)_h}{r_{OG}}$$

$$m(a_G)_h r_{OG} + I_G \alpha = m(a_G)_h r_{OG} + (mr_{OG} r_{GP}) \frac{(a_G)_h}{r_{OG}}$$

$$= m(a_G)_h (r_{OG} + r_{GP}) \quad \text{Q.E.D.}$$

**Ans:**

$$m(a_G)_h r_{OG} + I_G \alpha = m(a_G)_h (r_{OG} + r_{GP})$$
17–67.

If the cord at $B$ suddenly fails, determine the horizontal and vertical components of the initial reaction at the pin $A$, and the angular acceleration of the 120-kg beam. Treat the beam as a uniform slender rod.

**SOLUTION**

**Equations of Motion.** The mass moment of inertia of the beam about $A$ is $I_A = \frac{1}{12}(120)(4^2) + 120(2^2) = 640 \text{ kg} \cdot \text{m}^2$. Initially, the beam is at rest, $\omega = 0$. Thus, $(a_G)_h = \omega^2 r = 0$. Also, $(a_G)_v = a\tau_G = a(2) = 2a$. Referring to the FBD of the beam, Fig. a

\[ \sum M_A = I_A \alpha; \quad 800(4) + 120(9.81)(2) = 640 \alpha \]

\[ \alpha = 8.67875 \text{ rad}/\text{s}^2 = 8.68 \text{ rad}/\text{s}^2 \quad \text{Ans.} \]

\[ \sum F_n = m(a_G)_n; \quad A_n = 0 \quad \text{Ans.} \]

\[ \sum F_i = m(a_G)_v; \quad 800 + 120(9.81) + A_i = 120[2(8.67875)] \]

\[ A_i = 105.7 \text{ N} = 106 \text{ N} \quad \text{Ans.} \]
The device acts as a pop-up barrier to prevent the passage of a vehicle. It consists of a 100-kg steel plate $AC$ and a 200-kg counterweight solid concrete block located as shown. Determine the moment of inertia of the plate and block about the hinged axis through $A$. Neglect the mass of the supporting arms $AB$. Also, determine the initial angular acceleration of the assembly when it is released from rest at $\theta = 45^\circ$.

**SOLUTION**

*Mass Moment of Inertia:*

$$I_A = \frac{1}{12} (100)(1.25^2) + 100(0.625^2)$$

$$+ \frac{1}{12}(200)(0.5^2 + 0.3^2) + 200(\sqrt{0.75^2 + 0.15^2})^2$$

$$= 174.75 \text{ kg} \cdot \text{m}^2 = 175 \text{ kg} \cdot \text{m}^2$$

*Equation of Motion:* Applying Eq. 17–16, we have

$$\zeta + \sum M_A = I_A \alpha; \quad 100(9.81)(0.625) + 200(9.81) \sin 45^\circ(0.15)$$

$$-200(9.81) \cos 45^\circ(0.75) = -174.75 \alpha$$

$$\alpha = 1.25 \text{ rad/s}^2$$

*Ans:*

$$I_A = 175 \text{ kg} \cdot \text{m}^2$$

$$\alpha = 1.25 \text{ rad/s}^2$$
17–69.

The 20-kg roll of paper has a radius of gyration \( k_A = 90 \text{ mm} \) about an axis passing through point \( A \). It is pin supported at both ends by two brackets \( AB \). If the roll rests against a wall for which the coefficient of kinetic friction is \( \mu_k = 0.2 \) and a vertical force \( F = 30 \text{ N} \) is applied to the end of the paper, determine the angular acceleration of the roll as the paper unrolls.

**SOLUTION**

\[
\begin{align*}
\Sigma F_x &= m(a_G)_x ; \\
N_C - T_{AB} \cos 67.38^\circ &= 0 \\
\Sigma F_y &= m(a_G)_y ; \\
T_{AB} \sin 67.38^\circ - 0.2N_C - 20(9.81) - 30 &= 0 \\
\Sigma M_A &= I_A \alpha ; \\
-0.2N_c(0.125) + 30(0.125) &= 20(0.09^2)\alpha \\
\end{align*}
\]

Solving:

\[
\begin{align*}
N_C &= 103 \text{ N} \\
T_{AB} &= 267 \text{ N} \\
\alpha &= 7.28 \text{ rad/s}^2 \\
\end{align*}
\]

Ans:

\[\alpha = 7.28 \text{ rad/s}^2\]
17–70.

The 20-kg roll of paper has a radius of gyration \( k_A = 90 \text{ mm} \) about an axis passing through point \( A \). It is pin supported at both ends by two brackets \( AB \). If the roll rests against a wall for which the coefficient of kinetic friction is \( \mu_k = 0.2 \), determine the constant vertical force \( F \) that must be applied to the roll to pull off 1 m of paper in \( t = 3 \text{ s} \) starting from rest. Neglect the mass of paper that is removed.

**SOLUTION**

\[
( + \downarrow ) \ s = s_0 + v_0 t + \frac{1}{2} a_C t^2
\]

\[
1 = 0 + 0 + \frac{1}{2} a_C (3)^2
\]

\[
a_C = \frac{2.22 \text{ m/s}^2}{0.125} = 1.778 \text{ rad/s}^2
\]

\[
\alpha = \frac{a_C}{0.125} = 14.22\text{ rad/s}^2
\]

\[
\sum F_x = m(a_G)_x: \quad N_C - T_{AB} \cos 67.38^\circ = 0
\]

\[
\sum F_y = m(a_G)_y: \quad T_{AB} \sin 67.38^\circ - 0.2N_C - 20(9.81) - F = 0
\]

\[
\sum M_A = I_A \alpha: \quad -0.2N_C(0.125) + F(0.125) = 20(0.09)(1.778)
\]

Solving:

\[
N_C = 99.3 \text{ N}
\]

\[
T_{AB} = 258 \text{ N}
\]

\[
F = 22.1 \text{ N}
\]

Ans: \( F = 22.1 \text{ N} \)
17–71. The reel of cable has a mass of 400 kg and a radius of gyration of $k_A = 0.75$ m. Determine its angular velocity when $t = 2$ s, starting from rest, if the force $P = (20t^2 + 80)$ N, when $t$ is in seconds. Neglect the mass of the unwound cable, and assume it is always at a radius of 0.5 m.

**SOLUTION**

**Equations of Motion.** The mass moment of inertia of the reel about $A$ is $I_A = Mk_A^2 = 400(0.75^2) = 225$ kg·m$^2$. Referring to the FBD of the reel, Fig. a

$$\zeta + \Sigma M_A = I_A \alpha; \quad -(20t^2 + 80)(0.5) = 225(-\alpha)$$

$$\alpha = \frac{2}{45}(r^2 + 4) \text{ rad/s}^2$$

**Kinematics.** Using the result of $\alpha$, integrate $d\omega = a dt$, with the initial condition $\omega = 0$ at $t = 0$,

$$\int_0^\omega d\omega = \int_0^{2t} \frac{2}{45} (r^2 + 4) \, dt$$

$$\omega = 0.4741 \text{ rad/s} = 0.474 \text{ rad/s} \quad \text{Ans.}$$

$\omega = 0.474 \text{ rad/s}$
The 30-kg disk is originally spinning at \( \omega = 125 \text{ rad/s} \). If it is placed on the ground, for which the coefficient of kinetic friction is \( \mu_c = 0.5 \), determine the time required for the motion to stop. What are the horizontal and vertical components of force which the member \( AB \) exerts on the pin at \( A \) during this time? Neglect the mass of \( AB \).

**Solution**

**Equations of Motion.** The mass moment of inertia of the disk about \( B \) is \( I_B = \frac{1}{2}mr^2 = \frac{1}{2}(30)(0.3^2) = 1.35 \text{ kg} \cdot \text{m}^2 \). Since it is required to slip at \( C \), \( F_f = \mu_cN_C = 0.5 N_C \). Referring to the FBD of the disk, Fig. \( a \),

\[
\begin{align*}
\sum F_x &= m(a_C)_x; \quad 0.5N_C - F_{AB} \cos 45^\circ = 30(0) \\
\sum F_y &= m(a_C)_y; \quad N_C - F_{AB} \sin 45^\circ - 30(9.81) = 30(0)
\end{align*}
\]

(1) (2)

Solving Eqs. (1) and (2),

\( N_C = 588.6 \text{ N} \quad F_{AB} = 416.20 \text{ N} \)

Subsequently,

\[ \zeta + \sum M_B = I_B\alpha; \quad 0.5(588.6)(0.3) = 1.35\alpha \]

\[ \alpha = 65.4 \text{ rad/s}^2 \]

Referring to the FBD of pin \( A \), Fig. \( b \),

\[ \sum F_x = 0; \quad 416.20 \cos 45^\circ - A_x = 0 \quad A_x = 294.3 \text{ N} = 294 \text{ N} \]

(Ans.)

\[ \sum F_y = 0; \quad 416.20 \sin 45^\circ - A_y = 0 \quad A_y = 294.3 \text{ N} = 294 \text{ N} \]

(Ans.)

**Kinematic.** Using the result of \( \alpha \),

\[ + \sum \omega = \omega_0 + at; \quad 0 = 125 + (-65.4)t \]

\[ t = 1.911 \text{ s} = 1.91 \text{ s} \]

(Ans.)

---

**Ans:**

\( A_x = 294 \text{ N} \)
\( A_y = 294 \text{ N} \)
\( t = 1.91 \text{ s} \)
17–73.

Cable is unwound from a spool supported on small rollers at A and B by exerting a force $T = 300$ N on the cable. Compute the time needed to unravel 5 m of cable from the spool if the spool and cable have a total mass of 600 kg and a radius of gyration of $k_O = 1.2$ m. For the calculation, neglect the mass of the cable being unwound and the mass of the rollers at A and B. The rollers turn with no friction.

**SOLUTION**

$I_O = mk_O^2 = 600(1.2)^2 = 864 \text{ kg \cdot m}^2$

$\zeta + \sum M_O = I_O \alpha; \quad 300(0.8) = 864(\alpha) \quad \alpha = 0.2778 \text{ rad/s}^2$

The angular displacement $\theta = \frac{s}{r} = \frac{5}{0.8} = 6.25 \text{ rad}$.

$$\theta = \theta_0 + \omega_0 r + \frac{1}{2} \alpha t^2$$

$$6.25 = 0 + 0 + \frac{1}{2}(0.2778)t^2$$

$t = 6.71 \text{ s}$  

Ans.
17–74.

The 5-kg cylinder is initially at rest when it is placed in contact with the wall $B$ and the rotor at $A$. If the rotor always maintains a constant clockwise angular velocity $\omega = 6$ rad/s, determine the initial angular acceleration of the cylinder. The coefficient of kinetic friction at the contacting surfaces $B$ and $C$ is $\mu_k = 0.2$.

**SOLUTION**

*Equations of Motion:* The mass moment of inertia of the cylinder about point $O$ is given by $I_O = \frac{1}{2}mr^2 = \frac{1}{2}(5)(0.125^2) = 0.0390625$ kg $\cdot$ m$^2$. Applying Eq. 17–16, we have

\[ \Sigma \sum F_x = m(a_G)_x; \quad N_B + 0.2N_A \cos 45^\circ - N_A \sin 45^\circ = 0 \quad (1) \]

\[ \Sigma \sum F_y = m(a_G)_y; \quad 0.2N_B + 0.2N_A \sin 45^\circ + N_A \cos 45^\circ - 5(9.81) = 0 \quad (2) \]

\[ \zeta \sum M_O = I_O \alpha; \quad 0.2N_A (0.125) - 0.2N_B (0.125) = 0.0390625\alpha \quad (3) \]

Solving Eqs. (1), (2), and (3) yields:

\[ N_A = 51.01 \text{ N} \quad N_B = 28.85 \text{ N} \]

\[ \alpha = 14.2 \text{ rad/s}^2 \quad \text{Ans.} \]
The wheel has a mass of 25 kg and a radius of gyration $k_B = 0.15$ m. It is originally spinning at $\omega = 40$ rad/s. If it is placed on the ground, for which the coefficient of kinetic friction is $\mu_C = 0.5$, determine the time required for the motion to stop. What are the horizontal and vertical components of reaction which the pin at $A$ exerts on $AB$ during this time? Neglect the mass of $AB$.

**SOLUTION**

\[ I_B = mk_B^2 = 25(0.15)^2 = 0.5625 \text{ kg} \cdot \text{m}^2 \]

\[ + \sum F_y = m(a_G)_y; \quad \left( \frac{1}{2} \right) F_{AB} + N_C - 25(9.81) = 0 \tag{1} \]

\[ \sum F_x = m(a_G)_x; \quad 0.5N_C - \left( \frac{1}{2} \right) F_{AB} = 0 \tag{2} \]

\[ \sum M_B = I_B\alpha; \quad 0.5N_C(0.2) = 0.5625(-\alpha) \tag{3} \]

Solving Eqs. (1), (2) and (3) yields:

\[ F_{AB} = 111.48 \text{ N} \quad N_C = 178.4 \text{ N} \]

\[ \alpha = -31.71 \text{ rad/s}^2 \]

\[ A_x = \frac{1}{2} F_{AB} = 0.8(111.48) = 89.2 \text{ N} \quad \text{Ans.} \]

\[ A_y = \frac{3}{2} F_{AB} = 0.6(111.48) = 66.9 \text{ N} \quad \text{Ans.} \]

\[ \omega = \omega_0 + \alpha_c t \]

\[ 0 = 40 + (-31.71) t \]

\[ t = 1.26 \text{ s} \quad \text{Ans.} \]
The 20-kg roll of paper has a radius of gyration \( k_A = 120 \text{ mm} \) about an axis passing through point \( A \). It is pin supported at both ends by two brackets \( AB \). The roll rests on the floor, for which the coefficient of kinetic friction is \( \mu_k = 0.2 \). If a horizontal force \( F = 60 \text{ N} \) is applied to the end of the paper, determine the initial angular acceleration of the roll as the paper unrolls.

**SOLUTION**

**Equations of Motion.** The mass moment of inertia of the paper roll about \( A \) is 
\[ I_A = m k_A^2 = 20(0.12^2) = 0.288 \text{ kg} \cdot \text{m}^2. \] Since it is required to slip at \( C \), the friction is 
\[ F_f = \mu_k N = 0.2 \text{ N}. \]

Referring to the FBD of the paper roll, Fig. 1

\[ \sum F_x = m(a_G) + 0.2 - F_{AB} = 20(0) \]  \hspace{1cm} (1)

\[ \sum F_y = m(a_G) + N - F_{AB} - 20(9.81) = 20(0) \]  \hspace{1cm} (2)

Solving Eqs. (1) and (2)
\[ F_{AB} = 145.94 \text{ N} \]
\[ N = 283.76 \text{ N} \]

Subsequently
\[ \sum M_A = I_A \alpha; \quad 0.2(283.76)(0.3) - 60(0.3) = 0.288(-\alpha) \]
\[ \alpha = 3.3824 \text{ rad/s}^2 = 3.38 \text{ rad/s}^2 \]  \hspace{1cm} Ans.
Disk $D$ turns with a constant clockwise angular velocity of 30 rad/s. Disk $E$ has a weight of 60 lb and is initially at rest when it is brought into contact with $D$. Determine the time required for disk $E$ to attain the same angular velocity as disk $D$. The coefficient of kinetic friction between the two disks is $\mu_k = 0.3$. Neglect the weight of bar $BC$.

**SOLUTION**

**Equations of Motion:** The mass moment of inertia of disk $E$ about point $B$ is given by $I_B = \frac{1}{2} mr^2 = \frac{1}{2} \left( \frac{60}{32.2} \right)(1^2) = 0.9317$ slug $\cdot$ ft$^2$. Applying Eq. 17–16, we have

$$\begin{align*}
\sum F_x &= m(a_O)_x; \\
0.3N - F_{BC} \cos 45^\circ &= 0 \\
\sum F_y &= m(a_O)_y; \\
N - F_{BC} \sin 45^\circ - 60 &= 0 \\
\sum M_O &= I_O \alpha; \\
0.3N(1) &= 0.9317\alpha
\end{align*}$$

Solving Eqs. (1), (2) and (3) yields:

$$F_{BC} = 36.37 \text{ lb} \quad N = 85.71 \text{ lb} \quad \alpha = 27.60 \text{ rad/s}^2$$

**Kinematics:** Applying equation $\omega = \omega_0 + \alpha t$, we have

$$(\omega +) \quad 30 = 0 + 27.60t$$

$$t = 1.09 \text{ s} \quad \text{Ans.}$$
17–78.

Two cylinders $A$ and $B$, having a weight of 10 lb and 5 lb, respectively, are attached to the ends of a cord which passes over a 3-lb pulley (disk). If the cylinders are released from rest, determine their speed in $t = 0.5$ s. The cord does not slip on the pulley. Neglect the mass of the cord. Suggestion: Analyze the “system” consisting of both the cylinders and the pulley.

**SOLUTION**

*Equation of Motion:* The mass moment of inertia of the pulley (disk) about point $O$ is given by $I_O = \frac{1}{2}mr^2 = \frac{1}{2} \left( \frac{3}{32.2} \right) (0.75^2) = 0.02620$ slug·ft$^2$. Here, $a = ar$ or $\alpha = \frac{a}{r} = \frac{a}{0.75}$. Applying Eq. 17–16, we have

$$\zeta + \sum M_O = I_O \alpha; \quad 5(0.75) - 10(0.75) = -0.02620 \left( \frac{a}{0.75} \right)$$

$$- \left[ \left( \frac{5}{32.2} \right) a \right](0.75) - \left[ \left( \frac{10}{32.2} \right) a \right](0.75)$$

$$a = 9.758 \text{ ft/s}^2$$

*Kinematic:* Applying equation $v = v_0 + at$, we have

$$v = 0 + 9.758(0.5) = 4.88 \text{ ft/s}$$

Ans:

$v = 4.88 \text{ ft/s}$
The two blocks $A$ and $B$ have a mass of 5 kg and 10 kg, respectively. If the pulley can be treated as a disk of mass 3 kg and radius 0.15 m, determine the acceleration of block $A$. Neglect the mass of the cord and any slipping on the pulley.

**SOLUTION**

**Kinematics:** Since the pulley rotates about a fixed axis passes through point $O$, its angular acceleration is

$$\alpha = \frac{a}{r} = \frac{a}{0.15} = 6.6667a$$

The mass moment of inertia of the pulley about point $O$ is

$$I_o = \frac{1}{2}Mr^2 = \frac{1}{2}(3)(0.15^2) = 0.03375 \text{ kg} \cdot \text{m}^2$$

**Equation of Motion:** Write the moment equation of motion about point $O$ by referring to the free-body and kinetic diagram of the system shown in Fig. $a$,

$$\zeta + \sum M_o = \sum (M_k)_o; \quad 5(9.81)(0.15) - 10(9.81)(0.15)$$

$$= -0.03375(6.6667a) - 5a(0.15) - 10a(0.15)$$

$$a = 2.973 \text{ m/s}^2 = 2.97 \text{ m/s}^2$$

**Ans.**
*17–80.

The two blocks $A$ and $B$ have a mass $m_A$ and $m_B$, respectively, where $m_B > m_A$. If the pulley can be treated as a disk of mass $M$, determine the acceleration of block $A$. Neglect the mass of the cord and any slipping on the pulley.

SOLUTION

\[ a = \alpha r \]

\[ \zeta + \Sigma M_C = \Sigma (M_k)_C; \quad m_B g(r) - m_A g(r) = \left( \frac{1}{2} Mr^2 \right) \alpha + m_B r^2 \alpha + m_A r^2 \alpha \]

\[ \alpha = \frac{g(m_B - m_A)}{r \left( \frac{1}{2} M + m_B + m_A \right)} \]

\[ a = \frac{g(m_B - m_A)}{\left( \frac{1}{2} M + m_B + m_A \right)} \]

Ans:

\[ a = \frac{g(m_B - m_A)}{\left( \frac{1}{2} M + m_B + m_A \right)} \]
17–81.

Determine the angular acceleration of the 25-kg diving board and the horizontal and vertical components of reaction at the pin A the instant the man jumps off. Assume that the board is uniform and rigid, and that at the instant he jumps off the spring is compressed a maximum amount of 200 mm, $\omega = 0$, and the board is horizontal. Take $k = 7$ kN/m.

**SOLUTION**

\[ \zeta + \sum M_A = I_A \alpha; \quad 1.5(1400 - 245.25) = \left[ \frac{1}{3}(25)(3)^2 \right] \alpha \]

\[ + \sum F_i = m(a_G)_i; \quad 1400 - 245.25 - A_y = 25(1.5\alpha) \]

\[ \sum F_n = m(a_G)_n; \quad A_x = 0 \]

Solving,

\[ A_x = 0 \]

\[ A_y = 289 \text{ N} \]

\[ \alpha = 23.1 \text{ rad/s}^2 \]
The lightweight turbine consists of a rotor which is powered from a torque applied at its center. At the instant the rotor is horizontal it has an angular velocity of 15 rad/s and a clockwise angular acceleration of 8 rad/s². Determine the internal normal force, shear force, and moment at a section through $A$. Assume the rotor is a 50-m-long slender rod, having a mass of 3 kg/m.

**SOLUTION**

\[ \sum F = m(a)_{n}; \quad N_A = 45(15)^2(17.5) = 177\, \text{kN} \]

\[ \sum F_i = m(a)_{i}; \quad V_A + 45(9.81) = 45(8)(17.5) \]

\[ V_A = 5.86 \, \text{kN} \]

\[ \sum M_A = \sum (M_k)_A; \quad M_A + 45(9.81)(7.5) = \left[ \frac{1}{12} (45)(15)^2 \right] (8) + [45(8)(17.5)](7.5) \]

\[ M_A = 50.7 \, \text{kN} \cdot \text{m} \]

**Ans:**

\[ N_A = 177 \, \text{kN} \]

\[ V_A = 5.86 \, \text{kN} \]

\[ M_A = 50.7 \, \text{kN} \cdot \text{m} \]
17–83.

The two-bar assembly is released from rest in the position shown. Determine the initial bending moment at the fixed joint B. Each bar has a mass \( m \) and length \( l \).

**SOLUTION**

**Assembly:**

\[
I_A = \frac{1}{3} ml^2 + \frac{1}{12} (m)(l)^2 + m(l^2 + \left(\frac{l}{2}\right)^2)
\]

\[= 1.667 ml^2\]

\[\zeta + \Sigma M_A = I_A \alpha; \quad mg\left(\frac{l}{2}\right) + mg(l) = (1.667ml^2)\alpha\]

\[\alpha = \frac{0.9 g}{l}\]

**Segment BC:**

\[\zeta + \Sigma M_B = \Sigma (M_k)_{Bi}; \quad M = \left[\frac{1}{12} ml^2\right] \alpha + m(l^2 + \left(\frac{l}{2}\right)^2)^{1/2} \alpha\left(\frac{l/2}{l^2} + \left(\frac{l}{2}\right)^2\right)\left(\frac{l}{2}\right)\]

\[M = \frac{1}{3} ml^2 \alpha = \frac{1}{3} ml^2 \left(\frac{0.9g}{l}\right)\]

\[M = 0.3gml\]

Ans.

Ans:

\[M = 0.3gml\]
The armature (slender rod) $AB$ has a mass of 0.2 kg and can pivot about the pin at $A$. Movement is controlled by the electromagnet $E$, which exerts a horizontal attractive force on the armature at $B$ of $F_B = (0.2(10^{-3})l^2)$ N, where $l$ in meters is the gap between the armature and the magnet at any instant. If the armature lies in the horizontal plane, and is originally at rest, determine the speed of the contact at $B$ the instant $l = 0.01$ m. Originally $l = 0.02$ m.

**SOLUTION**

**Equation of Motion:** The mass moment of inertia of the armature about point $A$ is given by $I_A = I_G + mr^2_G = \frac{1}{12} (0.2)(0.15^2) + 0.2(0.075^2) = 1.50(10^{-3})$ kg⋅m$^2$

Applying Eq. 17–16, we have

$$\zeta + \sum M_A = I_A \ddot{\alpha}, \quad \frac{0.2(10^{-3})}{l^2} (0.15) = 1.50(10^{-3}) \alpha$$

$$\alpha = \frac{0.02}{l^2}$$

**Kinematic:** From the geometry, $l = 0.02 - 0.15\theta$. Then $dl = -0.15d\theta$ or $d\theta = -\frac{dl}{0.15}$. Also, $\omega = \frac{v}{0.15}$ hence $d\omega = \frac{dv}{0.15}$. Substitute into equation $\omega d\omega = \alpha d\theta$, we have

$$\frac{v}{0.15} \left( \frac{dv}{0.15} \right) = \alpha \left( \frac{dl}{0.15} \right)$$

$$vdv = -0.15 \alpha dl$$

$$\int_0^v vdv = \int_{0.02}^{0.01} -0.15 \left( \frac{0.02}{l^2} \right) dl$$

$$v = 0.548 \text{ m/s}$$

**Ans.**
The bar has a weight per length of \( w \). If it is rotating in the vertical plane at a constant rate \( \omega \) about point \( O \), determine the internal normal force, shear force, and moment as a function of \( x \) and \( \theta \).

**SOLUTION**

\[
a = \omega^2 \left( L - \frac{x}{z} \right)^\theta
\]

Forces:

\[
\frac{wx}{g} \omega^2 \left( L - \frac{x}{z} \right)^\theta = N \theta - S \theta + wx \downarrow \quad (1)
\]

Moments:

\[
I\alpha = M - S \left( \frac{x}{2} \right)
\]

\[
O = M - \frac{1}{2} S x \quad (2)
\]

Solving (1) and (2),

\[
N = wx \left[ \frac{\omega^2}{g} \left( L - \frac{x}{2} \right) + \cos \theta \right] \quad \text{Ans.}
\]

\[
V = wx \sin \theta \quad \text{Ans.}
\]

\[
M = \frac{1}{2} wx^2 \sin \theta \quad \text{Ans.}
\]

\[\text{Ans:}\]

\[
N = wx \left[ \frac{\omega^2}{g} \left( L - \frac{x}{2} \right) + \cos \theta \right]
\]

\[
V = wx \sin \theta
\]

\[
M = \frac{1}{2} wx^2 \sin \theta
\]
The 4-kg slender rod is initially supported horizontally by a spring at B and pin at A. Determine the angular acceleration of the rod and the acceleration of the rod’s mass center at the instant the 100-N force is applied.

**SOLUTION**

**Equation of Motion.** The mass moment of inertia of the rod about A is $I_A = \frac{1}{12}(4)(3^2) + 4(1.5^2) = 12.0 \text{ kg} \cdot \text{m}^2$. Initially, the beam is at rest, $\omega = 0$. Thus, $(a_G)_n = \omega^2 r = 0$. Also, $(a_G)_t = \alpha r_G = \alpha(1.5)$. The force developed in the spring before the application of the 100 N force is $F_{sp} = \frac{4(9.81)}{2} = 19.62 \text{ N}$. Referring to the FBD of the rod, Fig. a,

$$\zeta + M_A = I_A \alpha; \quad 19.62(3) - 100(1.5) - 4(9.81)(1.5) = 12.0(-\alpha)$$

$$\alpha = 12.5 \text{ rad/s}$$

Ans.

Then

$$(a_G)_t = 12.5(1.5) = 18.75 \text{ m/s}^2$$

Since $(a_G)_n = 0$. Then

$$(a_G)_t = (a_G)_t = 18.75 \text{ m/s}^2$$

Ans.
17–87.

The 100-kg pendulum has a center of mass at $G$ and a radius of gyration about $G$ of $k_G = 250$ mm. Determine the horizontal and vertical components of reaction on the beam by the pin $A$ and the normal reaction of the roller $B$ at the instant $\theta = 90^\circ$ when the pendulum is rotating at $\omega = 8$ rad/s. Neglect the weight of the beam and the support.

**SOLUTION**

**Equations of Motion:** Since the pendulum rotates about the fixed axis passing through point $C$, $(a_G)_h = ar_G = a(0.75)$ and $(a_G)_v = \omega^2r_G = 8^2(0.75) = 48 \text{ m/s}^2$. Here, the mass moment of inertia of the pendulum about this axis is $I_C = 100(0.25)^2 + 100(0.75^2) = 62.5 \text{ kg} \cdot \text{m}^2$. Writing the moment equation of motion about point $C$ and referring to the free-body diagram of the pendulum, Fig. a, we have

$$z + \sum M_C = I_C\alpha; \quad 0 = 62.5\alpha \quad \alpha = 0$$

Using this result to write the force equations of motion along the $n$ and $t$ axes,

$$\begin{align*}
\sum F_n &= m(a_G)_h; \\
-C_y &= 100[0(0.75)] \\
C_t &= 0
\end{align*}$$

$$+ \sum F_t = m(a_G)_v; \quad C_n - 100(9.81) = 100(48) \quad C_n = 5781 \text{ N}$$

**Equilibrium:** Writing the moment equation of equilibrium about point $A$ and using the free-body diagram of the beam in Fig. b, we have

$$+ \sum M_A = 0; \quad N_B (1.2) - 5781(0.6) = 0 \quad N_B = 2890.5 \text{ N} = 2.89 \text{ kN} \quad \text{Ans.}$$

Using this result to write the force equations of equilibrium along the $x$ and $y$ axes, we have

$$\begin{align*}
\sum F_x &= 0; \\
A_x &= 0 \quad \text{Ans.}
\end{align*}$$

$$\begin{align*}
+ \sum F_y &= 0; \\
A_y + 2890.5 - 5781 &= 0 \quad A_y = 2890.5 \text{ N} = 2.89 \text{ kN} \quad \text{Ans.}
\end{align*}$$

Ans:

$N_B = 2.89 \text{ kN}$

$A_x = 0$

$A_y = 2.89 \text{ kN}$

877
*17–88.

The 100-kg pendulum has a center of mass at $G$ and a radius of gyration about $G$ of $k_G = 250$ mm. Determine the horizontal and vertical components of reaction on the beam by the pin $A$ and the normal reaction of the roller $B$ at the instant $\theta = 0^\circ$ when the pendulum is rotating at $\omega = 4 \text{ rad/s}$. Neglect the weight of the beam and the support.

**SOLUTION**

**Equations of Motion:** Since the pendulum rotates about the fixed axis passing through point $C$, $(a_G)_t = a_G = \alpha(0.75)$ and $(a_G)_n = \omega^2 r_G = 4^2(0.75) = 12 \text{ m/s}^2$. Here, the mass moment of inertia of the pendulum about this axis is $I_C = 100(0.25^2) + 100(0.75)^2 = 62.5 \text{ kg} \cdot \text{m}^2$. Writing the moment equation of motion about point $C$ and referring to the free-body diagram shown in Fig. $a$,

$$\zeta + \sum M_C = I_C \alpha; \quad -100(9.81)(0.75) = -62.5 \alpha \quad \alpha = 11.772 \text{ rad/s}^2$$

Using this result to write the force equations of motion along the $n$ and $t$ axes, we have

$$+ \sum F_t = m(a_G)_t; \quad C_t - 100(9.81) = -100[11.772(0.75)] \quad C_t = 98.1 \text{ N}$$

$$\sum F_n = m(a_G)_n; \quad C_n = 100(12) \quad C_n = 1200 \text{ N}$$

**Equilibrium:** Writing the moment equation of equilibrium about point $A$ and using the free-body diagram of the beam in Fig. $b$,

$$+ \sum M_A = 0; \quad N_B(1.2) - 98.1(0.6) - 1200(1) = 0 \quad N_B = 1049.05 \text{ N} = 1.05 \text{ kN} \quad \text{Ans.}$$

Using this result to write the force equations of equilibrium along the $x$ and $y$ axes, we have

$$\sum F_x = 0; \quad 1200 - A_x = 0 \quad A_x = 1200 \text{ N} = 1.20 \text{ kN} \quad \text{Ans.}$$

$$+ \sum F_y = 0; \quad 1049.05 - 98.1 - A_y = 0 \quad A_y = 950.95 \text{ N} = 951 \text{ N} \quad \text{Ans.}$$

**Ans:**

$N_B = 1.05 \text{ kN}$

$A_x = 1.20 \text{ kN}$

$A_y = 951 \text{ N}$
The “Catherine wheel” is a firework that consists of a coiled tube of powder which is pinned at its center. If the powder burns at a constant rate of 20 g/s such as that the exhaust gases always exert a force having a constant magnitude of 0.3 N, directed tangent to the wheel, determine the angular velocity of the wheel when 75% of the mass is burned off. Initially, the wheel is at rest and has a mass of 100 g and a radius of \( r = 75 \text{ mm} \). For the calculation, consider the wheel to always be a thin disk.

**SOLUTION**

Mass of wheel when 75% of the powder is burned = 0.025 kg

Time to burn off 75% = \( \frac{0.075 \text{ kg}}{0.02 \text{ kg/s}} = 3.75 \text{ s} \)

\[ m(t) = 0.1 - 0.02t \]

Mass of disk per unit area is

\[ \rho_0 = \frac{m}{A} = \frac{0.1 \text{ kg}}{\pi(0.075 \text{ m})^2} = 5.6588 \text{ kg/m}^2 \]

At any time \( t \),

\[ 5.6588 = \frac{0.1 - 0.02t}{\pi r^2} \]

\[ r(t) = \sqrt{\frac{0.1 - 0.02t}{\pi(5.6588)}} \]

\[ 0.3r = \frac{1}{2} mr^2 \alpha \]

\[ \alpha = \frac{6}{mr} = \frac{0.6}{(0.1 - 0.02t)\sqrt{\frac{0.1 - 0.02t}{\pi(5.6588)}}} \]

\[ \alpha = 0.6 \left( \sqrt{\frac{5.6588}{\pi}} \right)[0.1 - 0.02t]^{-\frac{3}{2}} \]

\[ \alpha = 2.530(0.1 - 0.02t)^{-\frac{3}{2}} \]

\[ d\omega = \alpha \, dt \]

\[ \int_0^\omega d\omega = 2.530 \int_0^t [0.1 - 0.02t]^{-\frac{3}{2}} \, dt \]

\[ \omega = 253 \left[ (0.1 - 0.02t)^{-\frac{1}{2}} - 3.162 \right] \]

For \( t = 3.75 \text{ s} \),

\[ \omega = 800 \text{ rad/s} \]

**Ans:**

\[ \omega = 800 \text{ rad/s} \]
17–90.

If the disk in Fig. 17–21 rolls without slipping, show that when moments are summed about the instantaneous center of zero velocity, \( I_C \), it is possible to use the moment equation \( \Sigma M_{IC} = I_{IC} \alpha \), where \( I_{IC} \) represents the moment of inertia of the disk calculated about the instantaneous axis of zero velocity.

**SOLUTION**

\[ \zeta + \Sigma M_{IC} = \Sigma (M_k)_{IC}; \quad \Sigma M_{IC} = I_{IC} \alpha + (ma_G)r \]

Since there is no slipping, \( a_G = \omega r \)

Thus, \( \Sigma M_{IC} = (I_G + mr^2) \alpha \)

By the parallel-axis theorem, the term in parenthesis represents \( I_{IC} \). Thus,

\[ \Sigma M_{IC} = I_{IC} \alpha \]

Q.E.D.

\[
\text{Ans:} \quad \Sigma M_{IC} = I_{IC} \alpha
\]
17–91.

The 20-kg punching bag has a radius of gyration about its center of mass \( G \) of \( k_G = 0.4 \) m. If it is initially at rest and is subjected to a horizontal force \( F = 30 \) N, determine the initial angular acceleration of the bag and the tension in the supporting cable \( AB \).

**SOLUTION**

\[ \sum F_x = m(a_G)_x; \quad 30 = 20(a_G)_x \]

\[ \sum F_y = m(a_G)_y; \quad T - 196.2 = 20(a_G)_y \]

\[ \sum M_G = I_G\alpha; \quad 30(0.6) = 20(0.4)^2\alpha \]

\[ \alpha = 5.62 \text{ rad/s}^2 \]

\[ (a_G)_x = 1.5 \text{ m/s}^2 \]

\[ a_B = a_G + a_{B/G} \]

\[ a_B \hat{i} = (a_G)_x \hat{j} + (a_G)_y \hat{i} - \alpha(0.3) \hat{i} \]

\[ (a_G)_y = 0 \]

Thus,

\[ T = 196 \text{ N} \]

\[ \alpha = 5.62 \text{ rad/s}^2 \]

\[ T = 196 \text{ N} \]
17–92.

The uniform 150-lb beam is initially at rest when the forces are applied to the cables. Determine the magnitude of the acceleration of the mass center and the angular acceleration of the beam at this instant.

**SOLUTION**

*Equations of Motion:* The mass moment of inertia of the beam about its mass center is 
\[ I_G = \frac{1}{12} mf^2 = \frac{1}{12} \left( \frac{150}{32.2} \right) (12^2) = 55.90 \text{ slug} \cdot \text{ft}^2. \]

Thus, the magnitude of \( a_G \) is
\[ a_G = \sqrt{(a_{Gx})^2 + (a_{Gy})^2} = \sqrt{21.47^2 + 26.45^2} = 34.1 \text{ ft/s}^2 \]

\[ \alpha = 7.86 \text{ rad/s}^2 \]

Ans: 
\[ a_G = 34.1 \text{ ft/s}^2 \]
The slender 12-kg bar has a clockwise angular velocity of \( \omega = 2 \text{ rad/s} \) when it is in the position shown. Determine its angular acceleration and the normal reactions of the smooth surface \( A \) and \( B \) at this instant.

**SOLUTION**

**Equations of Motion.** The mass moment of inertia of the rod about its center of gravity \( G \) is \( I_G = \frac{1}{12} m l^2 = \frac{1}{12}(12)(3^2) = 9.00 \text{ kg} \cdot \text{m}^2 \). Referring to the FBD and kinetic diagram of the rod, Fig. a

\[ \sum F_x = m(a_G)_x; \quad N_B = 12(a_G)_x \quad \text{(1)} \]

\[ + \sum F_y = m(a_G)_y; \quad N_A = 12(9.81) - 12(a_G)_y \quad \text{(2)} \]

\[ \sum M_O = (M_k)_O; \quad -12(9.81)(1.5 \cos 60^\circ) = -12(a_G)_x(1.5 \sin 60^\circ) \]

\[ -12(a_G)_y(1.5 \cos 60^\circ) - 9.00 \alpha \]

\[ \sqrt{3}(a_G)_x + (a_G)_y + \alpha = 9.81 \quad \text{(3)} \]

**Kinematics.** Applying the relative acceleration equation relating \( a_G \) and \( a_B \) by referring to Fig. b.

\[ a_G = a_B + \alpha \times r_{G/B} - \omega^2 r_{G/B} \]

\[-(a_G)_i - (a_G)_j = -a_B + (\alpha) \times (-1.5 \cos 60^\circ i - 1.5 \sin 60^\circ j) \]

\[-2 \times (-1.5 \cos 60^\circ i - 1.5 \sin 60^\circ j) \]

\[-(a_G)_i - (a_G)_j = (3 - 0.75 \sqrt{3} \alpha)i + (0.75 \alpha - a_B + 3 \sqrt{3})j \]
Equating \( i \) and \( j \) components,
\[
-(a_G)_x = 3 - 0.75\sqrt{3}\alpha \quad \quad (4)
\]
\[
-(a_G)_y = 0.75\alpha - a_B + 3\sqrt{3} \quad \quad (5)
\]
Also, relate \( \mathbf{a}_B \) and \( \mathbf{a}_A \),
\[
\mathbf{a}_A = \mathbf{a}_B + \alpha \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B}
\]
\[
-a_Ai = -a_Bi + (-\alpha k) \times (-3 \cos 60^\circ i - 3 \sin 60^\circ j)
\]
\[
-2(-3 \cos 60^\circ i - 3 \sin 60^\circ j)
\]
\[
-a_Ai = (-3 \alpha) + (1.5\alpha - a_B + 6\sqrt{3})j
\]
Equating \( j \) components,
\[
0 = 1.5\alpha - a_B + 6\sqrt{3}; \quad a_B = 1.5\alpha + 6\sqrt{3} \quad \quad (6)
\]
Substituting Eq. (6) into (5)
\[
(a_G)_y = 0.75\alpha + 3\sqrt{3} \quad \quad (7)
\]
Substituting Eq. (4) and (7) into (3)
\[
\sqrt{3}(0.75\sqrt{3}\alpha - 3) + 0.75\alpha + 3\sqrt{3} + \alpha = 9.81
\]
\[
\alpha = 2.4525 \text{ rad/s}^2 = 2.45 \text{ rad/s}^2 \quad \quad \text{Ans.}
\]
Substituting this result into Eqs. (4) and (7)
\[
-(a_G)_x = 3 - (0.75\sqrt{3})(2.4525); \quad (a_G)_x = 0.1859 \text{ m/s}^2
\]
\[
(a_G)_y = 0.75(2.4525) + 3\sqrt{3}; \quad (a_G)_y = 7.0355 \text{ m/s}^2
\]
Substituting these results into Eqs. (1) and (2)
\[
N_B = 12(0.1859); \quad N_B = 2.2307 \text{ N} = 2.23 \text{ N}
\]
\[
N_A - 12(9.81) = -12(7.0355); \quad N_A = 33.2937 \text{ N} = 33.3 \text{ N}
\]
\[
\text{Ans:} \quad \alpha = 2.45 \text{ rad/s}^2 \quad \quad N_B = 2.23 \text{ N}
\]
\[
N_A = 33.3 \text{ N}
\]
17–94.

The tire has a weight of 30 lb and a radius of gyration of \( k_G = 0.6 \) ft. If the coefficients of static and kinetic friction between the wheel and the plane are \( \mu_s = 0.2 \) and \( \mu_k = 0.15 \), determine the tire’s angular acceleration as it rolls down the incline. Set \( \theta = 12^\circ \).

**SOLUTION**

\[ +\sum F_x = m(a_G)_x : \quad 30 \sin 12^\circ - F = \left( \frac{30}{32.2} \right) a_G \]

\[ +\sum F_y = m(a_G)_y : \quad N - 30 \cos 12^\circ = 0 \]

\[ \zeta + \sum M_G = I_G \alpha; \quad F(1.25) = \left[ \left( \frac{30}{32.2} \right)(0.6)^2 \right] \alpha \]

Assume the wheel does not slip.

\[ a_G = (1.25) \alpha \]

Solving:

\[ F = 1.17 \text{ lb} \]

\[ N = 29.34 \text{ lb} \]

\[ a_G = 5.44 \text{ ft/s}^2 \]

\[ \alpha = 4.35 \text{ rad/s}^2 \]

\[ F_{\text{max}} = 0.2(29.34) = 5.87 \text{ lb} > 1.17 \text{ lb} \]

**Ans:**

\[ \alpha = 4.32 \text{ rad/s}^2 \]
17–95.

The tire has a weight of 30 lb and a radius of gyration of \( k_G = 0.6 \text{ ft} \). If the coefficients of static and kinetic friction between the wheel and the plane are \( \mu_s = 0.2 \) and \( \mu_k = 0.15 \), determine the maximum angle \( \theta \) of the inclined plane so that the tire rolls without slipping.

**SOLUTION**

Since wheel is on the verge of slipping:

\[
\begin{align*}
\sum F_x &= m(a_G)_x; \quad 30 \sin \theta - 0.2N = \left(\frac{30}{32.2}\right)(1.25a) \\
\sum F_y &= m(a_G)_y; \quad N - 30 \cos \theta = 0 \\
\sum M &= I_G \alpha; \quad 0.2N(1.25) = \left[\left(\frac{30}{32.2}\right)(0.6)^2\right]a
\end{align*}
\]

Substituting Eqs. (2) and (3) into Eq. (1),

\[
30 \sin \theta - 6 \cos \theta = 26.042 \cos \theta
\]

\[
30 \sin \theta = 32.042 \cos \theta
\]

\[
\tan \theta = 1.068
\]

\[
\theta = 46.9^\circ \quad \text{Ans.}
\]
The spool has a mass of 100 kg and a radius of gyration of $k_G = 0.3$ m. If the coefficients of static and kinetic friction at $A$ are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 50$ N.

**SOLUTION**

\[ \sum F_x = m(a_G)_x; \quad 50 + F_A = 100a_G \]
\[ \sum F_y = m(a_G)_y; \quad N_A - 100(9.81) = 0 \]
\[ \sum M_G = I_G \alpha; \quad 50(0.25) - F_A(0.4) = [100(0.3)^2] \alpha \]

Assume no slipping: $a_G = 0.4 \alpha$

\[ \alpha = 1.30 \text{ rad/s}^2 \]
\[ a_G = 0.520 \text{ m/s}^2 \quad N_A = 981 \text{ N} \quad F_A = 2.00 \text{ N} \]

Since $(F_A)_{\text{max}} = 0.2(981) = 196.2 \text{ N} > 2.00 \text{ N} \quad \text{OK}$

\[ \alpha = 1.30 \text{ rad/s}^2 \]
17–97.

Solve Prob. 17–96 if the cord and force $P = 50$ N are directed vertically upwards.

**SOLUTION**

$$\sum F_x = m(a_G)x; \quad F_A = 100a_G$$

$$+\uparrow \sum F_y = m(a_G)y; \quad N_A + 50 - 100(9.81) = 0$$

$$\zeta + \Sigma M_G = I_G \alpha; \quad 50(0.25) - F_A(0.4) = [100(0.3)] \alpha$$

*Assume no slipping: $a_G = 0.4 \alpha$*

$$\alpha = 0.500 \text{ rad/s}^2$$

$$a_G = 0.2 \text{ m/s}^2 \quad N_A = 931 \text{ N} \quad F_A = 20 \text{ N}$$

Since $(F_A)_{max} = 0.2(931) = 186.2 \text{ N} > 20 \text{ N}$

**OK**

$\alpha = 0.500 \text{ rad/s}^2$

Ans:
The spool has a mass of 100 kg and a radius of gyration \( k_G = 0.3 \text{ m} \). If the coefficients of static and kinetic friction at \( A \) are \( \mu_s = 0.2 \) and \( \mu_k = 0.15 \), respectively, determine the angular acceleration of the spool if \( P = 600 \text{ N} \).

**SOLUTION**

\[ \sum F_x = m(a_G)_x \quad \Rightarrow \quad 600 + F_A = 100a_G \]

\[ \sum F_y = m(a_G)_y \quad \Rightarrow \quad N_A - 100(9.81) = 0 \]

\[ \sum M_G = I_G \alpha \quad \Rightarrow \quad 600(0.25) - F_A(0.4) = [100(0.3)] \alpha \]

Assume no slipping: \( a_G = 0.4 \alpha \)

\[ \alpha = 15.6 \text{ rad/s}^2 \]

\[ a_G = 6.24 \text{ m/s}^2 \quad N_A = 981 \text{ N} \quad F_A = 24.0 \text{ N} \]

Since \((F_A)_{\text{max}} = 0.2(981) = 196.2 \text{ N} > 24.0 \text{ N}\)  

\[ \alpha = 15.6 \text{ rad/s}^2 \]

**Ans:**

\[ a = 15.6 \text{ rad/s}^2 \]
17–99.

The 12-kg uniform bar is supported by a roller at $A$. If a horizontal force of $F = 80$ N is applied to the roller, determine the acceleration of the center of the roller at the instant the force is applied. Neglect the weight and the size of the roller.

SOLUTION

Equations of Motion. The mass moment of inertia of the bar about its center of gravity $G$ is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(12)(2^2) = 4.00$ kg·m$^2$. Referring to the FBD and kinetic diagram of the bar, Fig. $a$

\[ \sum F_x = m(a_G)_x; \quad 80 = 12(a_G)_x \quad (a_G)_x = 6.667 \text{ m/s}^2 \to \]

\[ \sum M_A = (\mu_k)_A; \quad 0 = 12(6.667)(1) - 4.00\alpha \quad \alpha = 20.0 \text{ rad/s}^2 \to \]

Kinematic. Since the bar is initially at rest, $\omega = 0$. Applying the relative acceleration equation by referring to Fig. $b$,

\[ a_G = a_A + \alpha \times r_{G/A} = \omega^2 r_{G/A} \]

\[ 6.667\hat{i} + (a_G)\hat{j} = a_A\hat{i} + (-20.0\hat{k}) \times (-\hat{j}) - 0 \]

\[ 6.667\hat{i} + (a_G)\hat{j} = (a_A - 20)\hat{i} \]

Equating $i$ and $j$ components,

\[ 6.667 = a_A - 20; \quad a_A = 26.67 \text{ m/s}^2 = 26.7 \text{ m/s}^2 \to \quad \text{Ans.} \]

\[ (a_G)_y = 0 \]

\[ a_A = 26.7 \text{ m/s}^2 \to \]

Ans: $a_A = 26.7 \text{ m/s}^2$
A force of \( F = 10 \, \text{N} \) is applied to the 10-kg ring as shown. If slipping does not occur, determine the ring's initial angular acceleration, and the acceleration of its mass center, \( G \). Neglect the thickness of the ring.

**SOLUTION**

**Equations of Motion.** The mass moment of inertia of the ring about its center of gravity \( G \) is \( I_G = mr^2 = 10(0.4^2) = 1.60 \, \text{kg} \cdot \text{m}^2 \). Referring to the FBD and kinetic diagram of the ring, Fig. a,

\[
\sum M_C = (\mu_k) \cdot (10 \cos 45^\circ)(0.4 \cos 30^\circ) - (10 \cos 45^\circ)[0.4(1 + \sin 30^\circ)] = -(10a_G)(0.4) - 1.60 \alpha
\]

\[
4a_G + 1.60 \alpha = 1.7932 \tag{1}
\]

**Kinematics.** Since the ring rolls without slipping,

\[
a_G = \alpha r = \alpha (0.4) \tag{2}
\]

Solving Eqs. (1) and (2)

\[
\alpha = 0.5604 \, \text{rad/s}^2 = 0.560 \, \text{rad/s}^2 \quad \text{Ans.}
\]

\[
a_G = 0.2241 \, \text{m/s}^2 = 0.224 \, \text{m/s}^2 \quad \text{Ans.}
\]
17–101.

If the coefficient of static friction at $C$ is $\mu_s = 0.3$, determine the largest force $F$ that can be applied to the 5-kg ring, without causing it to slip. Neglect the thickness of the ring.

**SOLUTION**

**Equations of Motion:** The mass moment of inertia of the ring about its center of gravity $G$ is $I_G = mr^2 = 10(0.4^2) = 1.60 \text{ kg} \cdot \text{m}^2$. Here, it is required that the ring is on the verge of slipping at $C$, $F_f = \mu_s N = 0.3 \text{ N}$. Referring to the FBD and kinetic diagram of the ring, Fig. a

1. $\Sigma F_y = m(a_G) = F \sin 45^\circ + N - 10(9.81) = 10(0)$
2. $\Sigma F_x = m(a_G) = F \cos 45^\circ - 0.3 N = 10a_G$
3. $\Sigma M_G = I_G \alpha = F \sin 15^\circ(0.4) - 0.3 N(0.4) = -1.60 \alpha$

**Kinematics.** Since the ring rolls without slipping,

$$a_G = ar = a(0.4)$$

Solving Eqs. (1) to (4),

$$F = 42.34 \text{ N} = 42.3 \text{ N}$$

$N = 68.16 \text{ N}$

$\alpha = 2.373 \text{ rad/s}^2$

$a_G = 0.9490 \text{ m/s}^2$

**Ans:**

$$F = 42.3 \text{ N}$$
17–102.

The 25-lb slender rod has a length of 6 ft. Using a collar of negligible mass, its end $A$ is confined to move along the smooth circular bar of radius $3\sqrt{2}$ ft. End $B$ rests on the floor, for which the coefficient of kinetic friction is $\mu_B = 0.4$.

If the bar is released from rest when $\theta = 30^\circ$, determine the angular acceleration of the bar at this instant.

**SOLUTION**

\[ \sum F_x = m(a_G)_x; \quad -0.4 N_B + N_A \cos 45^\circ = \frac{25}{32.2} (a_G)_x \]

\[ \sum F_y = m(a_G)_y; \quad N_B - 25 - N_A \sin 45^\circ = -\frac{25}{32.2} (a_G)_y \]

\[ \sum M_G = I_G \alpha; \quad N_B (3 \cos 30^\circ) - 0.4 N_B (3 \sin 30^\circ) + N_A \sin 15^\circ = \frac{1}{12} \left( \frac{25}{32.2} \right) (6)^2 \alpha \]

\[ \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} \]

\[ a_B = a_A + 6\alpha \]

\[ \mathbf{a}_A = 7.34847 \alpha \]

\[ \mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A} \]

\[ (a_G)_x + (a_G)_y = 7.34847 \alpha + 3\alpha \]

\[ (a_G)_x = -5.196\alpha + 1.5\alpha = -3.696\alpha \]

\[ (a_G)_y = 5.196\alpha - 2.598\alpha = 2.598\alpha \]

Solving Eqs. (1)–(5) yields:

\[ N_B = 9.01 \text{ lb} \]

\[ N_A = -11.2 \text{ lb} \]

\[ \alpha = 4.01 \text{ rad/s}^2 \]

**Ans:**

\[ \alpha = 4.01 \text{ rad/s}^2 \]
17–103.

The 15-lb circular plate is suspended from a pin at $A$. If the pin is connected to a track which is given an acceleration $a_A = 5 \text{ ft/s}^2$, determine the horizontal and vertical components of reaction at $A$ and the angular acceleration of the plate. The plate is originally at rest.

**SOLUTION**

\[ \sum F_x = m(a_G)_x; \quad A_x = \frac{15}{32.2}(a_G)_x \]

\[ + \sum F_y = m(a_G)_y; \quad A_y - 15 = \frac{15}{32.2}(a_G)_y \]

\[ \sum M_G = I_G\alpha; \quad A_\alpha = \left[ \frac{1}{2} \left( \frac{15}{32.2} \right) (2)^2 \right] \alpha \]

\[ a_G = a_A + a_{G/A} \]

\[ a_G = 5i - 2\alpha i \]

\[ (+\uparrow) \quad (a_G)_y = 0 \]

\[ (+\downarrow) \quad (a_G)_x = 5 - \alpha \]

Thus,

\[ A_y = 15.0 \text{ lb} \]

\[ A_x = 0.776 \text{ lb} \]

\[ \alpha = 1.67 \text{ rad/s}^2 \]

\[ a_G = (a_G)_x = 1.67 \text{ ft/s}^2 \]
If \( P = 30 \) lb, determine the angular acceleration of the 50-lb roller. Assume the roller to be a uniform cylinder and that no slipping occurs.

**SOLUTION**

*Equations of Motion:* The mass moment of inertia of the roller about its mass center is 
\[
I_G = \frac{1}{2}mr^2 = \frac{1}{2}(\frac{50}{32.2})(1.5^2) = 1.7469 \text{ slug} \cdot \text{ft}^2.
\]
We have

\[
\sum F_x = m(a_G)_x; \quad 30 \cos 30^\circ - F_f = \frac{50}{32.2}a_G
\]

\[
\sum F_y = m(a_G)_y; \quad N - 50 - 30 \sin 30^\circ = 0 \quad N = 65 \text{ lb}
\]

\[
\sum M_G = I_G\alpha; \quad F_f(1.5) = 1.7469\alpha
\]

Since the roller rolls without slipping,

\[
a_G = ar = \alpha(1.5)
\]

Solving Eqs. (1) through (3) yields

\[
\alpha = 7.436 \text{ rad/s}^2 = 7.44 \text{ rad/s}^2
\]

\[
F_f = 8.660 \text{ lb} \quad a_G = 11.15 \text{ ft/s}^2
\]

Ans: \( \alpha = 7.44 \text{ rad/s}^2 \)
17–105.

If the coefficient of static friction between the 50-lb roller and the ground is $\mu_s = 0.25$, determine the maximum force $P$ that can be applied to the handle, so that roller rolls on the ground without slipping. Also, find the angular acceleration of the roller. Assume the roller to be a uniform cylinder.

**SOLUTION**

**Equations of Motion:** The mass moment of inertia of the roller about its mass center is $I_G = \frac{1}{2}mr^2 = \frac{1}{2}(\frac{50}{32.2})(1.5^2) = 1.7469 \text{ slug} \cdot \text{ft}^2$. We have

\begin{align*}
\sum F_x &= m(a_G)_x; \quad P \cos 30^\circ - F_f = \frac{50}{32.2}a_G \\
\sum F_y &= m(a_G)_y; \quad N - P \sin 30^\circ - 50 = 0 \\
\sum M_G &= I_G\alpha; \quad F_f(1.5) = 1.7469\alpha
\end{align*}

Solving Eqs. (1) through (5) yields

\begin{align*}
\alpha &= 18.93 \text{ rad/s}^2 = 18.9 \text{ rad/s}^2 \quad P = 76.37 \text{ lb} = 76.4 \text{ lb} \\
N &= 88.18 \text{ lb} \quad a_G = 28.39 \text{ ft/s}^2 \quad F_f = 22.05 \text{ lb}
\end{align*}

Ans: $\alpha = 18.9 \text{ rad/s}^2$, $P = 76.4 \text{ lb}$
17–106.

The uniform bar of mass $m$ and length $L$ is balanced in the vertical position when the horizontal force $P$ is applied to the roller at $A$. Determine the bar’s initial angular acceleration and the acceleration of its top point $B$.

**SOLUTION**

$\Sigma F = m(a_G); \quad P = ma_G$

$\Sigma M_G = I_G \alpha; \quad P\left(\frac{L}{2}\right) = \left(\frac{1}{12}mL^2\right) \alpha$

$P = \frac{1}{6}mL\alpha$

$\alpha = \frac{6P}{mL}$

$a_G = \frac{P}{m}$

$a_B = a_G + a_{B/G}$

$-a_B \mathbf{i} = -\frac{P}{m} \mathbf{i} + \frac{L}{2} \alpha \mathbf{i}$

$(\perp) \quad a_B = \frac{P}{m} - \frac{L\alpha}{2}$

$= \frac{P}{m} - \frac{L}{2}\left(\frac{6P}{mL}\right)$

$a_B = \frac{2P}{m} = \frac{2P}{m}$

Ans:

$\alpha = \frac{6P}{mL}$

$a_B = \frac{2P}{m}$
17–107.
Solve Prob. 17–106 if the roller is removed and the coefficient of kinetic friction at the ground is \( \mu_k \).

**SOLUTION**

\[ \sum F_x = m(a)_x; \quad P - \mu_k N_A = ma_G \]

\[ \sum M_G = I_\alpha; \quad (P - \mu_k N_A)\frac{L}{2} = \left( \frac{1}{12}mL^2 \right)\alpha \]

\[ \sum F_y = m(a)_y; \quad N_A - mg = 0 \]

Solving,

\[ N_A = mg \]

\[ a_G = \frac{L}{6} \alpha \]

\[ \alpha = \frac{6(P - \mu_k mg)}{mL} \]

\[ a_B = a_G + a_{B/G} \]

\[ (\perp) \quad a_B = -\frac{L}{6} \alpha + \frac{L}{2} \alpha \]

\[ a_B = \frac{L\alpha}{3} \]

\[ a_B = \frac{2(P - \mu_k mg)}{m} \]

\[ \alpha = \frac{6(P - \mu_k mg)}{mL} \]

\[ a_B = \frac{2(P - \mu_k mg)}{m} \]
*17–108.

The semicircular disk having a mass of 10 kg is rotating at \( \omega = 4 \text{ rad/s} \) at the instant \( \theta = 60^\circ \). If the coefficient of static friction at \( A \) is \( \mu_s = 0.5 \), determine if the disk slips at this instant.

**SOLUTION**

**Equations of Motion:** The mass moment of inertia of the semicircular disk about its center of mass is given by \( I_G = \frac{1}{2} (10 \left( 0.4^2 \right) - 10 (0.1698^2) = 0.5118 \text{ kg} \cdot \text{m}^2 \). From the geometry, \( r_{GA} = \sqrt{0.1698^2 + 0.4^2 - 2(0.1698)(0.4) \cos 60^\circ} = 0.3477 \text{ m} \). Also, using law of sines, \( \sin \theta = \sin 60^\circ \frac{0.1698}{0.3477} \Rightarrow \theta = 25.01^\circ \). Applying Eq. 17–16, we have

\[
\zeta + \Sigma M_A = \Sigma (M_k)_A; \quad 10(9.81)(0.1698 \sin 60^\circ) = 0.5118 \alpha \\
+ 10(a_G)_z \cos 25.01^\circ(0.3477) \\
+ 10(a_G)_y \sin 25.01^\circ(0.3477)
\]

Equating \( x \) and \( y \) components, we have

\[
\begin{align*}
\alpha &= 0.3151\alpha - 2.3523 \quad (a_G)_x = 2.012 \text{ m/s}^2 \\
\alpha &= 0.1470\alpha - 1.3581 \quad (a_G)_y = 0.6779 \text{ m/s}^2
\end{align*}
\]

Solving Eqs. (1), (2), (3), (4), and (5) yields:

\[
\alpha = 13.85 \text{ rad/s}^2 \quad (a_G)_x = 2.012 \text{ m/s}^2 \quad (a_G)_y = 0.6779 \text{ m/s}^2 \\
F_f = 20.12 \text{ N} \quad N = 91.32 \text{ N}
\]

Since \( F_f < (F_f)_{\text{max}} = \mu_s N = 0.5(91.32) = 45.66 \text{ N} \), then the semicircular disk does not slip.

**Ans:**

Since \( F_f < (F_f)_{\text{max}} = \mu_s N = 0.5(91.32) = 45.66 \text{ N} \), then the semicircular disk does not slip.
17–109.

The 500-kg concrete culvert has a mean radius of 0.5 m. If the truck has an acceleration of 3 m/s², determine the culvert’s angular acceleration. Assume that the culvert does not slip on the truck bed, and neglect its thickness.

**SOLUTION**

**Equations of Motion:** The mass moment of inertia of the culvert about its mass center is \( I_G = mr^2 = 500(0.5^2) = 125 \text{ kg} \cdot \text{m}^2 \). Writing the moment equation of motion about point \( A \) using Fig. \( a \),

\[
\zeta + \Sigma M_A = \Sigma (M_k)_A; \quad 0 = 125\alpha - 500a_G(0.5) \quad (1)
\]

**Kinematics:** Since the culvert does not slip at \( A \), \( (a_A)_n = 3 \text{ m/s}^2 \). Applying the relative acceleration equation and referring to Fig. \( b \),

\[
a_G = a_A + \alpha \times r_{G/A} - \omega^2 r_{G/A}
\]

\[
a_G = 3i + (a_A)_n j + (\alpha k \times 0.5j) - \omega^2 (0.5j)
\]

\[
a_G = (3 - 0.5\alpha)i + [(a_A)_n - 0.5\omega^2]j
\]

Equating the \( i \) components,

\[
a_G = 3 - 0.5\alpha \quad (2)
\]

Solving Eqs. (1) and (2) yields

\[
a_G = 1.5 \text{ m/s}^2 \rightarrow \alpha = 3 \text{ rad/s}^2
\]

**Ans:**

\[
\alpha = 3 \text{ rad/s}^2
\]

**Ans:**

\[
\alpha = 3 \text{ rad/s}^2
\]
The 15-lb disk rests on the 5-lb plate. A cord is wrapped around the periphery of the disk and attached to the wall at B. If a torque $M = 40$ lb-ft is applied to the disk, determine the angular acceleration of the disk and the time needed for the end C of the plate to travel 3 ft and strike the wall. Assume the disk does not slip on the plate and the plate rests on the surface at D having a coefficient of kinetic friction of $\mu_k = 0.2$. Neglect the mass of the cord.

**SOLUTION**

Disk:

\[ \pm \sum F_x = m(a_G); \quad T - F_p = \frac{15}{32.2} a_G \]

\[ \zeta + \sum M_G = I_G \alpha; \quad -F_p(1.25) + 40 - T(1.25) = \left[ \frac{1}{2} \cdot \frac{15}{32.2} \right]^2 \alpha \]

Plate:

\[ \pm \sum F_x = m(a_G); \quad F_p - 4 = \frac{5}{32.2} a_p \]

\[ a_p = a_G + a_{p/G} \]

\[ (\pm) \quad a_p = a_G + \alpha(1.25) \]

\[ a_G = \alpha(1.25) \]

Thus

\[ a_p = 2.5 \alpha \]

Solving,

\[ F_p = 9.65 \text{ lb} \]

\[ a_p = 36.367 \text{ ft/s}^2 \]

\[ \alpha = 14.5 \text{ rad/s}^2 \]

\[ T = 18.1 \text{ lb} \]

\[ (\pm) \quad s = s_0 + v_0 t + \frac{1}{2} a t^2 \]

\[ 3 = 0 + 0 + \frac{1}{2} (36.367) t^2 \]

\[ t = 0.406 \text{ s} \]

Ans:

\[ \alpha = 14.5 \text{ rad/s}^2 \]

\[ t = 0.406 \text{ s} \]
17–111.

The semicircular disk having a mass of 10 kg is rotating at $\omega = 4 \text{ rad/s}$ at the instant $\theta = 60^\circ$. If the coefficient of static friction at $A$ is $\mu_s = 0.5$, determine if the disk slips at this instant.

**SOLUTION**

For roll $A$.

$$\ddot{\alpha} + \sum M_A = I_A \alpha; \quad T(0.09) = \frac{1}{2}(8)(0.09)^2 \alpha_A \quad (1)$$

For roll $B$

$$\ddot{\alpha} + \sum M_O = \Sigma (M_k)_O; \quad 8(9.81)(0.09) = \frac{1}{2}(8)(0.09)^2 \alpha_B + 8a_B(0.09) \quad (2)$$

$$+ \sum F_x = m(a_O)_x; \quad T - 8(9.81) = -8a_B \quad (3)$$

Kinematics:

$$\begin{align*}
a_B &= a_O + (a_{B/O})_h + (a_{B/O})_n \\
\begin{bmatrix} a_B \\ a_B \end{bmatrix} &= \begin{bmatrix} a_O \\ a_B(0.09) \end{bmatrix} + [0] \\
\begin{bmatrix} a_O \\ a_B \end{bmatrix} &= a_O + 0.09a_B \quad (4)
\end{align*}$$

also,

$$\begin{align*}
\begin{bmatrix} a_O \\ a_B \end{bmatrix} &= a_A(0.09) \quad (5)
\end{align*}$$

Solving Eqs. (1)–(5) yields:

$$\begin{align*}
\alpha_A &= 43.6 \text{ rad/s}^2 \quad \text{Ans.} \\
\alpha_B &= 43.6 \text{ rad/s}^2 \quad \text{Ans.} \\
T &= 15.7 \text{ N} \quad \text{Ans.} \\
a_B &= 7.85 \text{ m/s}^2 \\
a_O &= 3.92 \text{ m/s}^2
\end{align*}$$

Ans:
The disk does not slip.
The circular concrete culvert rolls with an angular velocity of \( \omega = 0.5 \text{ rad/s} \) when the man is at the position shown. At this instant the center of gravity of the culvert and the man is located at point \( G \), and the radius of gyration about \( G \) is \( k_G = 3.5 \text{ ft} \). Determine the angular acceleration of the culvert. The combined weight of the culvert and the man is 500 lb. Assume that the culvert rolls without slipping, and the man does not move within the culvert.

**SOLUTIONS**

**Equations of Motion:** The mass moment of inertia of the system about its mass center is \( I_G = mk_G^2 = \frac{500}{32.2} (3.5^2) = 190.22 \text{ slug} \cdot \text{ft}^2 \). Writing the moment equation of motion about point \( A \), Fig. a,

\[
\sum M_A = \sum (M_k)_A; \quad -500(0.5) = -\frac{500}{32.2} (a_G)_x(4) - \frac{500}{32.2} (a_G)_y(0.5) - 190.22\alpha \quad (1)
\]

**Kinematics:** Since the culvert rolls without slipping,

\[
a_0 = \alpha r = \alpha(4) \rightarrow
\]

Applying the relative acceleration equation and referring to Fig. b,

\[
(a_G)_i - (a_G)_j = 4\alpha i + (-\alpha k) \times (0.5i) - (0.5^2)(0.5i)
\]

\[
(a_G)_i - (a_G)_j = (4\alpha - 0.125)i - 0.5\alpha j
\]

Equation the \( i \) and \( j \) components,

\[
(a_G)_x = 4\alpha - 0.125 \quad (2)
\]

\[
(a_G)_y = 0.5\alpha \quad (3)
\]

Substituting Eqs. (2) and (3) into Eq. (1),

\[
-500(0.5) = -\frac{500}{32.2} (4\alpha - 0.125)(4) - \frac{500}{32.2} (0.5\alpha)(0.5) - 190.22\alpha
\]

\[
\alpha = 0.582 \text{ rad/s}^2
\]

**Ans:**

\[
\alpha = 0.582 \text{ rad/s}^2
\]
The uniform disk of mass $m$ is rotating with an angular velocity of $\omega_0$ when it is placed on the floor. Determine the initial angular acceleration of the disk and the acceleration of its mass center. The coefficient of kinetic friction between the disk and the floor is $\mu_k$.

**SOLUTION**

*Equations of Motion.* Since the disk slips, the frictional force is $F_f = \mu_k N$. The mass moment of inertia of the disk about its mass center is $I_G = \frac{1}{2}mr^2$. We have

$$+ \sum F_y = m(a_G)_y; \quad N - mg = 0 \quad N = mg$$
$$\sum F_x = m(a_G)_x; \quad \mu_k (mg) = ma_G \quad a_G = \mu_k g \quad \text{Ans.}$$
$$\sum M_G = I_G \alpha; \quad -\mu_k (mg)r = \left(\frac{1}{2}mr^2\right) \alpha \quad \alpha = \frac{2\mu_k g}{r} \quad \text{Ans.}$$

**Ans:**

$$a_G = \mu_k g \quad \alpha = \frac{2\mu_k g}{r}$$
The uniform disk of mass \( m \) is rotating with an angular velocity of \( \omega_0 \) when it is placed on the floor. Determine the time before it starts to roll without slipping. What is the angular velocity of the disk at this instant? The coefficient of kinetic friction between the disk and the floor is \( \mu_k \).

**SOLUTION**

**Equations of Motion:** Since the disk slips, the frictional force is \( F_f = \mu_k N \). The mass moment of inertia of the disk about its mass center is \( I_G = \frac{1}{2}mr^2 \).

\[
\begin{align*}
+ \sum F_y &= m(a_G)_y; \quad N - mg = 0 \\
\sum N &= mG = ma_G \quad a_G = \mu_k g \\
+ \sum M &= I_G\alpha; \quad \mu_k(mg)r = -\left(\frac{1}{2}mr^2\right)\alpha \quad \alpha = \frac{2\mu_k g}{r}
\end{align*}
\]

**Kinematics:** At the instant when the disk rolls without slipping, \( v_G = \omega r \). Thus,

\[
\begin{align*}
(1) \quad v_G &= (v_G)_0 + a_G t \\
\omega r &= 0 + \mu_k gt \\
t &= \frac{\omega r}{\mu_k g}
\end{align*}
\]

and

\[
\omega = \omega_0 + at \quad (2)
\]

Solving Eqs. (1) and (2) yields

\[
\begin{align*}
\omega &= \frac{1}{3}\omega_0 \\
t &= \frac{\omega_0 r}{3\mu_k g} \quad \text{Ans.}
\end{align*}
\]
17–115.

A cord is wrapped around each of the two 10-kg disks. If they are released from rest, determine the angular acceleration of each disk and the tension in the cord \( C \). Neglect the mass of the cord.

**SOLUTION**

For \( A \):

\[
\sum \tau = \sum I \alpha_A; \quad T(0.09) = \frac{1}{2}(10)(0.09)^2 \alpha_A
\]

For \( B \):

\[
\sum \tau = \sum I \alpha_B; \quad T(0.09) = \frac{1}{2}(10)(0.09)^2 \alpha_B
\]

\[+ \sum F_y = m(a_B)_y; \quad 10(9.81) - T = 10a_B\]

\[a_B = a_P + (a_{B/P})_h + (a_{B/P})_n\]

\[(+ \downarrow) a_B = 0.09\alpha_A + 0.09\alpha_B + 0\]

Solving,

\[a_B = 7.85 \text{ m/s}^2\]

\[\alpha_A = 43.6 \text{ rad/s}^2\]

\[\alpha_B = 43.6 \text{ rad/s}^2\]

\[T = 19.6 \text{ N}\]

\[A_y = 10(9.81) + 19.62\]

\[= 118 \text{ N}\]
*17–116.

The disk of mass \( m \) and radius \( r \) rolls without slipping on the circular path. Determine the normal force which the path exerts on the disk and the disk’s angular acceleration if at the instant shown the disk has an angular velocity of \( \omega \).

**SOLUTION**

*Equation of Motion:* The mass moment of inertia of the disk about its center of mass is given by \( I_G = \frac{1}{2} mr^2 \). Applying Eq. 17–16, we have

\[
\zeta + \sum M_A = \sum (M_k)_A; \quad mg \sin \theta(r) = \left( \frac{1}{2} mr^2 \right) \alpha + m(a_G)_l(r) \tag{1}
\]

\[
\sum F_n = m(a_G)_n; \quad N - mg \cos \theta = m(a_G)_n \tag{2}
\]

*Kinematics:* Since the semicircular disk does not slip at \( A \), then \( v_G = \omega r \) and \( (a_G)_l = \alpha r \). Substitute \((a_G)_l = \alpha r\) into Eq. [1] yields

\[
mg \sin \theta(r) = \left( \frac{1}{2} mr^2 \right) \alpha + m(\alpha r)(r)
\]

\[
\alpha = \frac{2g}{3r} \sin \theta \quad \text{Ans.}
\]

Also, the center of the mass for the disk moves around a circular path having a radius of \( \rho = R - r \). Thus, \((a_G)_n = \frac{v_G^2}{\rho} = \frac{\omega^2 r^2}{R - r}\). Substitute into Eq. [2] yields

\[
N - mg \cos \theta = m\left( \frac{\omega^2 r^2}{R - r} \right)
\]

\[
N = m\left( \frac{\omega^2 r^2}{R - r} + g \cos \theta \right) \quad \text{Ans.}
\]

\[
\alpha = \frac{2g}{3r} \sin \theta
\]

\[
N = m\left( \frac{\omega^2 r^2}{R - r} + g \cos \theta \right)
\]
17–117.
The uniform beam has a weight \( W \). If it is originally at rest while being supported at \( A \) and \( B \) by cables, determine the tension in cable \( A \) if cable \( B \) suddenly fails. Assume the beam is a slender rod.

**SOLUTION**

\[ + \Sigma F_y = m(a_G); \quad T_A - W = -\frac{W}{g} a_G \]

\[ \zeta + \Sigma M_A = I_\alpha; \quad W \left( \frac{L}{4} \right) = \left[ \frac{1}{12} \left( \frac{W}{g} \right) L^2 \right] a + \frac{W}{g} \left( \frac{L}{4} \right) \left( \frac{L}{4} \right) \]

\[ 1 = \frac{1}{8} \left( \frac{L}{4} + \frac{L}{3} \right) a \]

Since \( a_G = a \left( \frac{L}{4} \right) \).

\[ a = \frac{12}{7} \left( \frac{g}{L} \right) \]

\[ T_A = W - \frac{W}{g} \left( \frac{L}{4} \right) = W - \frac{W}{g} \left( \frac{12}{7} \right) \left( \frac{g}{L} \right) \left( \frac{L}{4} \right) \]

\[ T_A = \frac{4}{7} W \quad \text{Ans.} \]

Also,

\[ + \Sigma F_y = m(a_G); \quad T_A - W = -\frac{W}{g} a_G \]

\[ \zeta + \Sigma M_G = I_G \alpha; \quad T_A \left( \frac{L}{4} \right) = \left[ \frac{1}{12} \left( \frac{W}{g} \right) L^2 \right] a \]

Since \( a_G = \frac{L}{4} \alpha \)

\[ T_A = \frac{1}{3} \left( \frac{W}{g} \right) L \alpha \]

\[ \frac{1}{3} \left( \frac{W}{g} \right) L \alpha - W = -\frac{W}{g} \left( \frac{L}{4} \right) \alpha \]

\[ \alpha = \frac{12}{7} \left( \frac{g}{L} \right) \]

\[ T_A = \frac{1}{3} \left( \frac{W}{g} \right) L \left( \frac{12}{7} \right) \left( \frac{g}{L} \right) \]

\[ T_A = \frac{4}{7} W \quad \text{Ans.} \]
17–118.

The 500-lb beam is supported at A and B when it is subjected to a force of 1000 lb as shown. If the pin support at A suddenly fails, determine the beam's initial angular acceleration and the force of the roller support on the beam. For the calculation, assume that the beam is a slender rod so that its thickness can be neglected.

**SOLUTION**

\[ \sum F_x = m(a_G)_x; \quad 1000 \left( \frac{4}{5} \right) = \frac{500}{32.2} (a_G)_x \]

\[ \sum F_y = m(a_G)_y; \quad 1000 \left( \frac{3}{5} \right) + 500 - B_y = \frac{500}{32.2} (a_G)_y \]

\[ \sum M_B = \sum (M_k)_B; \quad 500(3) + 1000 \left( \frac{3}{5} \right) (8) = \frac{500}{32.2} (a_G)_y (3) + \left[ \frac{1}{12} \left( \frac{500}{32.2} \right) (10)^2 \right] \alpha \]

\[ a_B = a_G + a_{BG} \]

\[- a_B l = -(a_G)_x i - (a_G)_y j + \alpha(3) j \]

\[ (+ \downarrow) \quad (a_G)_y = \alpha(3) \]

\[ \alpha = 23.4 \text{ rad/s}^2 \quad \text{Ans.} \]

\[ B_y = 9.62 \text{ lb} \quad \text{Ans.} \]

\( B_y > 0 \) means that the beam stays in contact with the roller support.
17–119.

The solid ball of radius $r$ and mass $m$ rolls without slipping down the $60^\circ$ trough. Determine its angular acceleration.

**SOLUTION**

\[ d = r \sin 30^\circ = \frac{r}{2} \]

\[ \Sigma M_{a\rightarrow d} = \Sigma (M_k)_{a\rightarrow d}; \quad mg \sin 45^\circ \left( \frac{r}{2} \right) = \left[ \frac{2}{5} mr^2 + m \left( \frac{r}{2} \right)^2 \right] \alpha \]

\[ \alpha = \frac{10g}{13 \sqrt{2} r} \]

**Ans:**

\[ \alpha = \frac{10g}{13 \sqrt{2} r} \]
By pressing down with the finger at B, a thin ring having a mass \( m \) is given an initial velocity \( v_0 \) and a backspin \( \omega_0 \) when the finger is released. If the coefficient of kinetic friction between the table and the ring is \( \mu_k \), determine the distance the ring travels forward before backsptinning stops.

**SOLUTION**

\[ N_A - mg = 0 \]

\[ \sum F_y = ma_G \]

\[ \mu_k (mg) = m(a_G) \]

\[ a_G = \frac{\mu_k g}{r} \]

\[ \sum M_G = I_G \alpha \]

\[ \mu_k (mg)r = mr^2 \alpha \]

\[ \alpha = \frac{\mu_k g}{r} \]

\[ \omega = \omega_0 + \alpha \cdot t \]

\[ 0 = \omega_0 - \left( \frac{\mu_k g}{r} \right) t \]

\[ t = \frac{\omega_0 r}{\mu_k g} \]

\[ s = s_0 + v_0 t + \frac{1}{2}a_c t^2 \]

\[ s = 0 + v_0 \left( \frac{\omega_0 r}{\mu_k g} \right) - \left( \frac{1}{2} \right) \left( \frac{\omega_0^2 r^2}{\mu_k^2 g^2} \right) \]

\[ s = \left( \frac{\omega_0 r}{\mu_k g} \right) \left( v_0 - \frac{1}{2} \omega_0 r \right) \]

\[ s = \left( \frac{\omega_0 r}{\mu_k g} \right) \left( v_0 - \frac{1}{2} \omega_0 r \right) \]

\[ s = \left( \frac{\omega_0 r}{\mu_k g} \right) \left( v_0 - \frac{1}{2} \omega_0 r \right) \]