3–1.

The members of a truss are pin connected at joint \( O \). Determine the magnitudes of \( F_1 \) and \( F_2 \) for equilibrium. Set \( \theta = 60^\circ \).

**SOLUTION**

\[ \sum F_x = 0; \quad F_2 \sin 70^\circ + F_1 \cos 60^\circ - 5 \cos 30^\circ - \frac{4}{5} (7) = 0 \]
\[ 0.9397F_2 + 0.5F_1 = 9.930 \]

\[ \sum F_y = 0; \quad F_2 \cos 70^\circ + 5 \sin 30^\circ - F_1 \sin 60^\circ - \frac{3}{5} (7) = 0 \]
\[ 0.3420F_2 - 0.8660F_1 = 1.7 \]

Solving:

\[ F_2 = 9.60 \text{ kN} \]
\[ F_1 = 1.83 \text{ kN} \]

\[ \text{Ans:} \]
\[ F_2 = 9.60 \text{ kN} \]
\[ F_1 = 1.83 \text{ kN} \]
3–2.

The members of a truss are pin connected at joint $O$. Determine the magnitude of $F_1$ and its angle $\theta$ for equilibrium. Set $F_2 = 6 \text{ kN}$.

**SOLUTION**

\[ \sum F_x = 0; \quad 6 \sin 70^\circ + F_1 \cos \theta - 5 \cos 30^\circ - \frac{4}{5}(7) = 0 \]

\[ F_1 \cos \theta = 4.2920 \]

\[ + \sum F_y = 0; \quad 6 \cos 70^\circ + 5 \sin 30^\circ - F_1 \sin \theta - \frac{3}{5}(7) = 0 \]

\[ F_1 \sin \theta = 0.3521 \]

Solving:

\[ \theta = 4.69^\circ \]

\[ F_1 = 4.31 \text{ kN} \]
3–3.
Determine the magnitude and direction of $\mathbf{F}$ so that the particle is in equilibrium.

**SOLUTION**

_Equations of Equilibrium._ Referring to the FBD shown in Fig. a,

$$
\begin{align*}
\sum F_x &= 0; \quad F \sin \theta + 5 - 4 \cos 60^\circ - 8 \cos 30^\circ = 0 \quad F \sin \theta = 3.9282 \\
\sum F_y &= 0; \quad 8 \sin 30^\circ - 4 \sin 60^\circ - F \cos \theta = 0 \quad F \cos \theta = 0.5359
\end{align*}
$$

Divide Eq (1) by (2),

$$
\frac{\sin \theta}{\cos \theta} = 7.3301
$$

Realizing that $\tan \theta = \frac{\sin \theta}{\cos \theta}$, then

$$
\tan \theta = 7.3301 \quad \theta = 82.23^\circ = 82.2^\circ
$$

Substitute this result into Eq. (1),

$$
\begin{align*}
F \sin 82.23^\circ &= 3.9282 \\
F &= 3.9646 \text{ kN} = 3.96 \text{ kN}
\end{align*}
$$
3–4.

The bearing consists of rollers, symmetrically confined within the housing. The bottom one is subjected to a 125-N force at its contact A due to the load on the shaft. Determine the normal reactions $N_B$ and $N_C$ on the bearing at its contact points B and C for equilibrium.

**SOLUTION**

\[ \sum F_y = 0; \quad 125 - N_C \cos 40° = 0 \]

\[ N_C = 163.176 = 163 \text{ N} \quad \text{Ans.} \]

\[ \sum F_x = 0; \quad N_B - 163.176 \sin 40° = 0 \]

\[ N_B = 105 \text{ N} \quad \text{Ans.} \]

\[ N_B = 105 \text{ N} \]

\[ N_C = 163 \text{ N} \]
3–5.

The members of a truss are connected to the gusset plate. If the forces are concurrent at point O, determine the magnitudes of \( F \) and \( T \) for equilibrium. Take \( \theta = 90^\circ \).

**SOLUTION**

\[
\phi = 90^\circ - \tan^{-1}\left(\frac{3}{4}\right) = 53.13^\circ
\]

\[
\Rightarrow \sum F_x = 0; \quad T \cos 53.13^\circ - F \left(\frac{4}{5}\right) = 0
\]

\[
\Rightarrow \sum F_y = 0; \quad 9 - T \sin 53.13^\circ - F \left(\frac{3}{5}\right) = 0
\]

Solving,

\[
T = 7.20 \text{ kN}
\]

\[
F = 5.40 \text{ kN}
\]

Ans:

\[
T = 7.20 \text{ kN}
\]

\[
F = 5.40 \text{ kN}
\]
The gusset plate is subjected to the forces of three members. Determine the tension force in member \( C \) and its angle \( \theta \) for equilibrium. The forces are concurrent at point \( O \). Take \( F = 8 \text{kN} \).

\section*{SOLUTION}

\[ \sum F_x = 0; \quad T \cos \phi - 8 \left( \frac{4}{5} \right) = 0 \tag{1} \]
\[ + \sum F_y = 0; \quad 9 - 8 \left( \frac{3}{5} \right) - T \sin \phi = 0 \tag{2} \]

Rearrange then divide Eq. (1) into Eq. (2):

\begin{align*}
\tan \phi &= 0.656, \quad \phi = 33.27^\circ \\
T &= 7.66 \text{kN} \\
\theta &= \phi + \tan^{-1} \left( \frac{3}{4} \right) = 70.1^\circ
\end{align*}

\text{Ans:} \quad T = 7.66 \text{kN} \\
\text{Ans:} \quad \theta = 70.1^\circ
3–7.

The man attempts to pull down the tree using the cable and small pulley arrangement shown. If the tension in AB is 60 lb, determine the tension in cable CAD and the angle \( \theta \) which the cable makes at the pulley.

**SOLUTION**

\[ \sum F_x = 0; \quad 60 \cos 10^\circ - T - T \cos \theta = 0 \]
\[ \sum F_y = 0; \quad T \sin \theta - 60 \sin 10^\circ = 0 \]

Thus,

\[ T(1 + \cos \theta) = 60 \cos 10^\circ \]
\[ T(2 \cos^2 \frac{\theta}{2}) = 60 \cos 10^\circ \quad (1) \]
\[ 2T \sin \frac{\theta}{2} \cos \frac{\theta}{2} = 60 \sin 10^\circ \quad (2) \]

Divide Eq.(2) by Eq.(1)

\[ \tan \frac{\theta}{2} = \tan 10^\circ \]

\[ \theta = 20^\circ \quad \text{Ans.} \]

\[ T = 30.5 \text{ lb} \quad \text{Ans.} \]
*3–8.

The cords $ABC$ and $BD$ can each support a maximum load of 100 lb. Determine the maximum weight of the crate, and the angle $\theta$ for equilibrium.

**SOLUTION**

*Equations of Equilibrium.* Assume that for equilibrium, the tension along the length of rope $ABC$ is constant. Assuming that the tension in cable $BD$ reaches the limit first. Then, $T_{BD} = 100$ lb. Referring to the FBD shown in Fig. 1,

\[ \sum F_x = 0; \quad W \left( \frac{5}{13} \right) - 100 \cos \theta = 0 \]

\[ 100 \cos \theta = \frac{5W}{13} \]  \hspace{1cm} (1)

\[ \sum F_y = 0; \quad 100 \sin \theta - W - W \left( \frac{12}{13} \right) = 0 \]

\[ 100 \sin \theta = \frac{25}{13}W \]  \hspace{1cm} (2)

Divide Eq. (2) by (1),

\[ \frac{\sin \theta}{\cos \theta} = 5 \]

Realizing that $\tan \theta = \frac{\sin \theta}{\cos \theta}$

\[ \tan \theta = 5 \]

\[ \theta = 78.69^\circ = 78.7^\circ \]  \hspace{1cm} Ans.

Substitute this result into Eq. (1),

\[ 100 \cos 78.69^\circ = \frac{5}{13}W \]

\[ W = 50.99 \text{ lb} = 51.0 \text{ lb} < 100 \text{ lb} \]  \hspace{1cm} (O.K)  \hspace{1cm} Ans.

Ans:

$\theta = 78.7^\circ$

$W = 51.0 \text{ lb}$
3–9.

Determine the maximum force \( F \) that can be supported in the position shown if each chain can support a maximum tension of 600 lb before it fails.

**SOLUTION**

*Equations of Equilibrium.* Referring to the FBD shown in Fig. a,

\[
\begin{align*}
\sum F_y &= 0; \quad T_{AB} \left(\frac{4}{5}\right) - F \sin 30^\circ = 0 \quad T_{AB} = 0.625 F \\
\sum F_x &= 0; \quad T_{AC} + 0.625 F \left(\frac{3}{5}\right) - F \cos 30^\circ = 0 \quad T_{AC} = 0.4910 F
\end{align*}
\]

Since chain \( AB \) is subjected to a higher tension, its tension will reach the limit first. Thus,

\[ T_{AB} = 600; \quad 0.625 F = 600 \]

\[ F = 960 \text{ lb} \quad \text{Ans.} \]
3–10.
The block has a weight of 20 lb and is being hoisted at uniform velocity. Determine the angle $\theta$ for equilibrium and the force in cord $AB$.

**SOLUTION**

*Equations of Equilibrium.* Assume that for equilibrium, the tension along the length of cord $CAD$ is constant. Thus, $F = 20$ lb. Referring to the FBD shown in Fig. a,

$$\downarrow \Sigma F_x = 0; \quad 20 \sin \theta - T_{AB} \sin 20^\circ = 0$$

$$T_{AB} = \frac{20 \sin \theta}{\sin 20^\circ}$$

$$+ \Sigma F_y = 0; \quad T_{AB} \cos 20^\circ - 20 \cos \theta - 20 = 0$$

Substitute Eq (1) into (2),

$$\frac{20 \sin \theta}{\sin 20^\circ} \cos 20^\circ - 20 \cos \theta = 20$$

$$\sin \theta \cos 20^\circ - \cos \theta \sin 20^\circ = \sin 20^\circ$$

Realizing that $\sin (\theta - 20^\circ) = \sin \theta \cos 20^\circ - \cos \theta \sin 20^\circ$, then

$$\sin (\theta - 20^\circ) = \sin 20^\circ$$

$$\theta - 20^\circ = 20^\circ$$

$$\theta = 40^\circ$$

Substitute this result into Eq (1)

$$T_{AB} = \frac{20 \sin 40^\circ}{\sin 20^\circ} = 37.59 \text{ lb} = 37.6 \text{ lb}$$

Ans:

$\theta = 40^\circ$

$T_{AB} = 37.6 \text{ lb}$
3–11. Determine the maximum weight $W$ of the block that can be suspended in the position shown if cords $AB$ and $CAD$ can each support a maximum tension of 80 lb. Also, what is the angle $\theta$ for equilibrium?

**SOLUTION**

*Equations of Equilibrium.* Assume that for equilibrium, the tension along the length of cord $CAD$ is constant. Thus, $F = W$. Assuming that the tension in cord $AB$ reaches the limit first, then $T_{AB} = 80$ lb. Referring to the $FBD$ shown in Fig. a,

\[ \pm \sum F_x = 0; \quad W \sin \theta - 80 \sin 20^\circ = 0 \]
\[ W = \frac{80 \sin 20^\circ}{\sin \theta} \quad (1) \]

\[ \pm \sum F_y = 0; \quad 80 \cos 20^\circ - W - W \cos \theta = 0 \]
\[ W = \frac{80 \cos 20^\circ}{1 + \cos \theta} \quad (2) \]

Equating Eqs (1) and (2),

\[ \frac{80 \sin 20^\circ}{\sin \theta} = \frac{80 \cos 20^\circ}{1 + \cos \theta} \]

\[ \sin \theta \cos 20^\circ - \cos \theta \sin 20^\circ = \sin 20^\circ \]

Realizing then $\sin (\theta - 20^\circ) = \sin \theta \cos 20^\circ - \cos \theta \sin 20^\circ$, then

\[ \sin (\theta - 20^\circ) = \sin 20^\circ \]

\[ \theta - 20^\circ = 20^\circ \]

\[ \theta = 40^\circ \]

Ans. 

Substitute this result into Eq (1)

\[ W = \frac{80 \sin 20^\circ}{\sin 40^\circ} = 42.56 \text{ lb} = 42.6 \text{ lb} < 80 \text{ lb} \quad (\text{O.K}) \]

Ans.
3–12.

The lift sling is used to hoist a container having a mass of 500 kg. Determine the force in each of the cables AB and AC as a function of $\theta$. If the maximum tension allowed in each cable is 5 kN, determine the shortest lengths of cables AB and AC that can be used for the lift. The center of gravity of the container is located at G.

**SOLUTION**

**Free-Body Diagram:** By observation, the force $F_1$ has to support the entire weight of the container. Thus, $F_1 = 500(9.81) = 4905$ N.

**Equations of Equilibrium:**

$$\sum F_x = 0; \quad F_{AC} \cos \theta - F_{AB} \cos \theta = 0 \quad F_{AC} = F_{AB} = F$$

$$\sum F_y = 0; \quad 4905 - 2F \sin \theta = 0 \quad F = \{2452.5 \cos \theta\} \text{ N}$$

Thus,

$$F_{AC} = F_{AB} = F = \{2.45 \cos \theta\} \text{ kN} \quad \text{Ans.}$$

If the maximum allowable tension in the cable is 5 kN, then

$$2452.5 \cos \theta = 5000$$

$$\theta = 29.37^\circ$$

From the geometry, $l = \frac{1.5}{\cos \theta}$ and $\theta = 29.37^\circ$. Therefore

$$l = \frac{1.5}{\cos 29.37^\circ} = 1.72 \text{ m} \quad \text{Ans.}$$

Ans:

$$F_{AC} = \{2.45 \cos \theta\} \text{ kN}$$

$$l = 1.72 \text{ m}$$
A nuclear-reactor vessel has a weight of \(500 \times 10^3\) lb. Determine the horizontal compressive force that the spreader bar \(AB\) exerts on point \(A\) and the force that each cable segment \(CA\) and \(AD\) exert on this point while the vessel is hoisted upward at constant velocity.

**SOLUTION**

At point \(C\):

\[
\begin{align*}
\sum F_x &= 0; \\
F_{CB} \cos 30^\circ - F_{CA} \cos 30^\circ &= 0 \\
F_{CB} &= F_{CA} \\
+\sum F_y &= 0; \\
500(10^3) - F_{CA} \sin 30^\circ - F_{CB} \sin 30^\circ &= 0 \\
500(10^3) - 2F_{CA} \sin 30^\circ &= 0 \\
F_{CA} &= 500(10^3) \text{ lb} \\
\end{align*}
\]

At point \(A\):

\[
\begin{align*}
\sum F_x &= 0; \\
500(10^3) \cos 30^\circ - F_{AB} &= 0 \\
F_{AB} &= 433(10^3) \text{ lb} \\
+\sum F_y &= 0; \\
500(10^3) \sin 30^\circ - F_{AD} &= 0 \\
F_{AD} &= 500(10^3) \sin 30^\circ \\
F_{AD} &= 250(10^3) \text{ lb} \\
\end{align*}
\]
3–14.

Determine the stretch in each spring for equilibrium of the 2-kg block. The springs are shown in the equilibrium position.

**SOLUTION**

\[ F_{AD} = 2(9.81) = x_{AD}(40) \quad x_{AD} = 0.4905 \text{ m} \]

\[ \sum F_x = 0; \quad F_{AB} \left( \frac{4}{5} \right) - F_{AC} \left( \frac{1}{\sqrt{2}} \right) = 0 \]

\[ \sum F_y = 0; \quad F_{AC} \left( \frac{1}{\sqrt{2}} \right) + F_{AB} \left( \frac{3}{5} \right) - 2(9.81) = 0 \]

\[ F_{AC} = 15.86 \text{ N} \]

\[ x_{AC} = \frac{15.86}{20} = 0.793 \text{ m} \]

\[ F_{AB} = 14.01 \text{ N} \]

\[ x_{AB} = \frac{14.01}{30} = 0.467 \text{ m} \]

\[ x_{AD} = 0.4905 \text{ m} \]

\[ x_{AC} = 0.793 \text{ m} \]

\[ x_{AB} = 0.467 \text{ m} \]
3–15.

The unstretched length of spring \( AB \) is 3 m. If the block is held in the equilibrium position shown, determine the mass of the block at \( D \).

**SOLUTION**

\[ F = kx = 30(5 - 3) = 60 \text{ N} \]

\[ \sum F_x = 0; \quad T \cos 45^\circ - 60 \left( \frac{4}{5} \right) = 0 \]

\[ T = 67.88 \text{ N} \]

\[ \sum F_y = 0; \quad -W + 67.88 \sin 45^\circ + 60 \left( \frac{3}{5} \right) = 0 \]

\[ W = 84 \text{ N} \]

\[ m = \frac{84}{9.81} = 8.56 \text{ kg} \]

**Ans:**

\[ m = 8.56 \text{ kg} \]
*3–16.

Determine the mass of each of the two cylinders if they cause a sag of \( s = 0.5 \) m when suspended from the rings at \( A \) and \( B \). Note that \( s = 0 \) when the cylinders are removed.

**SOLUTION**

\[ T_{AC} = 100 \text{ N/m} \times (2.828 - 2.5) = 32.84 \text{ N} \]

\[ + \sum F_y = 0; \quad 32.84 \sin 45^\circ - m(9.81) = 0 \]

\[ m = 2.37 \text{ kg} \]
3–17.  
Determine the stiffness $k_T$ of the single spring such that the force $F$ will stretch it by the same amount $s$ as the force $F$ stretches the two springs. Express $k_T$ in terms of stiffness $k_1$ and $k_2$ of the two springs.

SOLUTION

\[ F = ks \]
\[ s = s_1 + s_2 \]
\[ s = \frac{F}{k_T} = \frac{F}{k_1} + \frac{F}{k_2} \]
\[ \frac{1}{k_T} = \frac{1}{k_1} + \frac{1}{k_2} \]

Ans:

\[ \frac{1}{k_T} = \frac{1}{k_1} + \frac{1}{k_2} \]
3–18.

If the spring $DB$ has an unstretched length of 2 m, determine the stiffness of the spring to hold the 40-kg crate in the position shown.

**SOLUTION**

*Equations of Equilibrium.* Referring to the FBD shown in Fig. a,

\[ \sum_{\text{y}} \mathbf{F} = 0; \quad T_{BD}(\frac{3}{\sqrt{13}}) - T_{CD}(\frac{1}{\sqrt{2}}) = 0 \]

\[ \sum_{\text{x}} \mathbf{F} = 0; \quad T_{BD}(\frac{2}{\sqrt{13}}) + T_{CD}(\frac{1}{\sqrt{2}}) - 40(9.81) = 0 \]

Solving Eqs (1) and (2)

\[ T_{BD} = 282.96 \text{ N} \quad T_{CD} = 332.96 \text{ N} \]

The stretched length of the spring is

\[ l = \sqrt{3^2 + 2^2} = \sqrt{13} \text{ m} \]

Then, \( x = l - l_0 = (\sqrt{13} - 2) \text{ m} \). Thus,

\[ F_{sp} = kx; \quad 282.96 = k(\sqrt{13} - 2) \]

\[ k = 176.24 \text{ N/m} = 176 \text{ N/m} \]

**Ans.:**

\[ k = 176 \text{ N/m} \]
3–19.

Determine the unstretched length of $DB$ to hold the 40-kg crate in the position shown. Take $k = 180 \text{ N/m}$.

**SOLUTION**

*Equations of Equilibrium.* Referring to the FBD shown in Fig. a,

$$\sum \Sigma F_j = 0; \quad T_{BD}\left(\frac{3}{\sqrt{13}}\right) - T_{CD}\left(\frac{1}{\sqrt{2}}\right) = 0$$

$$4\Sigma F_j = 0; \quad T_{BD}\left(\frac{2}{\sqrt{13}}\right) + T_{CD}\left(\frac{1}{\sqrt{2}}\right) - 40(9.81) = 0$$

Solving Eqs (1) and (2)

$$T_{BD} = 282.96 \text{ N} \quad T_{CD} = 332.96 \text{ N}$$

The stretched length of the spring is

$$l = \sqrt{3^2 + 2^2} = \sqrt{13} \text{ m}$$

Then, $x = l - l_0 = \sqrt{13} - l_0$. Thus

$$F_{sp} = kx; \quad 282.96 = 180(\sqrt{13} - l_0)$$

$$l_0 = 2.034 \text{ m} = 2.03 \text{ m}$$

Ans.

$$l_0 = 2.03 \text{ m}$$

A vertical force $P = 10 \text{ lb}$ is applied to the ends of the 2-ft cord $AB$ and spring $AC$. If the spring has an unstretched length of 2 ft, determine the angle $\theta$ for equilibrium. Take $k = 15 \text{ lb/ft}$.

**SOLUTION**

→ $\sum F_x = 0; \quad F_x \cos \phi - T \cos \theta = 0$

$+ \sum F_y = 0; \quad T \sin \theta + F_s \sin \phi - 10 = 0$

$s = \sqrt{(4)^2 + (2)^2} - 2(4)(2) \cos \theta - 2 = 2\sqrt{5} - 4 \cos \theta - 2$

$F_s = ks = 2k(\sqrt{5} - 4 \cos \theta - 1)$

From Eq. (1): $T = F_s\left(\frac{\cos \phi}{\cos \theta}\right)$

$T = 2k(\sqrt{5} - 4 \cos \theta - 1)\left(\frac{2 - \cos \theta}{\sqrt{5} - 4 \cos \theta}\right)\left(\frac{1}{\cos \theta}\right)$

From Eq. (2):

$2k\left(\sqrt{5} - 4 \cos \theta - 1\right)(2 - \cos \theta)\tan \theta + 2k\left(\sqrt{5} - 4 \cos \theta - 1\right)2 \sin \theta = 10$

$\frac{(\sqrt{5} - 4 \cos \theta - 1)}{\sqrt{5} - 4 \cos \theta}(2 \tan \theta - \sin \theta + \sin \theta) = \frac{10}{2k}$

$\tan \theta \frac{(\sqrt{5} - 4 \cos \theta - 1)}{\sqrt{5} - 4 \cos \theta} = \frac{10}{4k}$

Set $k = 15 \text{ lb/ft}$

Solving for $\theta$ by trial and error,

$\theta = 35.0^\circ \quad \text{Ans.}$
3–21.

Determine the unstretched length of spring $AC$ if a force $P = 80$ lb causes the angle $\theta = 60^\circ$ for equilibrium. Cord $AB$ is 2 ft long. Take $k = 50$ lb/ft.

**SOLUTION**

$$l = \sqrt{4^2 + 2^2 - 2(2)(4) \cos 60^\circ}$$

$$l = \sqrt{12}$$

$$\frac{\sqrt{12}}{\sin 60^\circ} = \frac{2}{\sin \phi}$$

$$\phi = \sin^{-1}\left(\frac{2 \sin 60^\circ}{\sqrt{12}}\right) = 30^\circ$$

$$+ \Sigma F_x = 0; \quad T \sin 60^\circ + F_x \sin 30^\circ - 80 = 0$$

$$\Rightarrow \Sigma F_y = 0; \quad -T \cos 60^\circ + F_y \cos 30^\circ = 0$$

Solving for $F_y$,

$$F_y = 40 \text{ lb}$$

$$F_x = kx$$

$$40 = 50(\sqrt{12} - l') \quad l = \sqrt{12} - \frac{40}{50} = 2.66 \text{ ft}$$

**Ans:**

$$l = 2.66 \text{ ft}$$
3–22.

The springs BA and BC each have a stiffness of 500 N/m and an unstretched length of 3 m. Determine the horizontal force $F$ applied to the cord which is attached to the small ring $B$ so that the displacement of the ring from the wall is $d = 1.5$ m.

**SOLUTION**

\[ \sum F_x = 0; \quad \frac{1.5}{\sqrt{11.25}} (T)(2) - F = 0 \]

\[ T = ks = 500(\sqrt{3^2 + (1.5)^2} - 3) = 177.05 \text{ N} \]

\[ F = 158 \text{ N} \]

Ans: $F = 158$ N
3–23.

The springs $BA$ and $BC$ each have a stiffness of 500 N/m and an unstretched length of 3 m. Determine the displacement $d$ of the cord from the wall when a force $F = 175$ N is applied to the cord.

**SOLUTION**

$$
\sum F_x = 0; \quad 175 = 2T \sin \theta
$$

$$
T \sin \theta = 87.5
$$

$$
T \left[ \frac{d}{\sqrt{3^2 + d^2}} \right] = 87.5
$$

$$
T = ks = 500(\sqrt{3^2 + d^2} - 3)
$$

$$
d \left( 1 - \frac{3}{\sqrt{9 + d^2}} \right) = 0.175
$$

By trial and error:

$$
d = 1.56 \text{ m}
$$

Ans.

\[d = 1.56 \text{ m}\]
*3–24.

Determine the distances $x$ and $y$ for equilibrium if $F_1 = 800$ N and $F_2 = 1000$ N.

**SOLUTION**

*Equations of Equilibrium.* The tension throughout rope $ABCD$ is constant, that is $F_1 = 800$ N. Referring to the FBD shown in Fig. a,

\[ +\Sigma F_y = 0; \quad 800 \sin \phi - 800 \sin \theta = 0 \quad \phi = 0 \]

\[ -\Sigma F_x = 0; \quad 1000 - 2[800 \cos \theta] = 0 \quad \theta = 51.32^\circ \]

Referring to the geometry shown in Fig. b,

$y = 2$ m \hspace{2cm} \text{Ans.}

and

\[ \frac{2}{x} = \tan 51.32^\circ; \quad x = 1.601 \text{ m} = 1.60 \text{ m} \]

\text{Ans.:}

$y = 2$ m

$x = 1.60$ m
3–25. Determine the magnitude of $F_1$ and the distance $y$ if $x = 1.5$ m and $F_2 = 1000$ N.

**SOLUTION**

*Equations of Equilibrium.* The tension throughout rope $ABCD$ is constant, that is $F_1$. Referring to the FBD shown in Fig. a,

$$+\Sigma F_y = 0; \quad F_1 \left( \frac{y}{\sqrt{y^2 + 1.5^2}} \right) - F_1 \left( \frac{2}{2.5} \right) = 0$$

$$\frac{y}{\sqrt{y^2 + 1.5^2}} = \frac{2}{2.5}$$

$$y = 2 \text{ m}$$

$$\Sigma F_x = 0; \quad 1000 - 2 \left[ F_1 \left( \frac{1.5}{2.5} \right) \right] = 0$$

$$F_1 = 833.33 \text{ N} = 833 \text{ N}$$

Ans. $y = 2 \text{ m}$

Ans. $F_1 = 833 \text{ N}$
The 30-kg pipe is supported at $A$ by a system of five cords. Determine the force in each cord for equilibrium.

**SOLUTION**

**At $H$:**

$\Sigma F_y = 0; \quad T_{HA} - 30(9.81) = 0$

$T_{HA} = 294 \text{ N}$

**At $A$:**

$\Sigma F_y = 0; \quad T_{AB} \sin 60^\circ - 30(9.81) = 0$

$T_{AB} = 339.83 = 340 \text{ N}$

$\Sigma F_x = 0; \quad T_{AE} - 339.83 \cos 60^\circ = 0$

$T_{AE} = 170 \text{ N}$

**At $B$:**

$\Sigma F_y = 0; \quad T_{BD} \left( \frac{3}{5} \right) - 339.83 \sin 60^\circ = 0$

$T_{BD} = 490.50 = 490 \text{ N}$

$\Sigma F_x = 0; \quad 490.50 \left( \frac{4}{5} \right) + 339.83 \cos 60^\circ - T_{BC} = 0$

$T_{BC} = 562 \text{ N}$

**Ans:**

$T_{HA} = 294 \text{ N}$

$T_{AB} = 340 \text{ N}$

$T_{AE} = 170 \text{ N}$

$T_{BD} = 490 \text{ N}$

$T_{BC} = 562 \text{ N}$
3–27.

Each cord can sustain a maximum tension of 500 N. Determine the largest mass of pipe that can be supported.

**SOLUTION**

At \( H \):

\[ \sum F_y = 0; \quad F_{HA} = W \]

At \( A \):

\[ \sum F_y = 0; \quad F_{AB} \sin 60^\circ - W = 0 \]

\[ F_{AB} = 1.1547 \ W \]

\[ \sum F_x = 0; \quad F_{AE} - (1.1547 \ W) \cos 60^\circ = 0 \]

\[ F_{AE} = 0.5774 \ W \]

At \( B \):

\[ \sum F_y = 0; \quad F_{BD} \left( \frac{3}{5} \right) - (1.1547 \ \cos 30^\circ)W = 0 \]

\[ F_{BD} = 1.667 \ W \]

\[ \sum F_x = 0; \quad -F_{BC} + 1.667 \ W \left( \frac{4}{5} \right) + 1.1547 \ \sin 30^\circ = 0 \]

\[ F_{BC} = 1.9107 \ W \]

By comparison, cord \( BC \) carries the largest load. Thus

\[ 500 = 1.9107 \ W \]

\[ W = 261.69 \ N \]

\[ m = \frac{261.69}{9.81} = 26.7 \ kg \]

\[ \text{Ans:} \quad m = 26.7 \ kg \]
*3–28.

The street-lights at $A$ and $B$ are suspended from the two poles as shown. If each light has a weight of 50 lb, determine the tension in each of the three supporting cables and the required height $h$ of the pole $DE$ so that cable $AB$ is horizontal.

**SOLUTION**

At point $B$:

\[ \Sigma F_y = 0; \quad \frac{1}{\sqrt{2}} F_{BC} - 50 = 0 \]

\[ F_{BC} = 70.71 = 70.7 \text{ lb} \]

\[ \Sigma F_x = 0; \quad \frac{1}{\sqrt{2}} (70.71) - F_{AB} = 0 \]

\[ F_{AB} = 50 \text{ lb} \]

At point $A$:

\[ \Sigma F_y = 0; \quad 50 - F_{AD} \cos \theta = 0 \]

\[ \theta = 45^\circ \]

\[ F_{AD} = 70.7 \text{ lb} \]

\[ h = 18 + 5 = 23 \text{ ft} \]
Determine the tension developed in each cord required for equilibrium of the 20-kg lamp.

**SOLUTION**

**Equations of Equilibrium:** Applying the equations of equilibrium along the $x$ and $y$ axes to the free-body diagram of joint $D$ shown in Fig. $a$, we have

- $\sum F_x = 0$; $F_{DE} \sin 30^\circ - 20(9.81) = 0$  \[ F_{DE} = 392.4 \text{ N} = 392 \text{ N} \quad \text{Ans.} \]
- $\sum F_y = 0$; $392.4 \cos 30^\circ - F_{CD} = 0$ \[ F_{CD} = 339.83 \text{ N} = 340 \text{ N} \quad \text{Ans.} \]

Using the result $F_{CD} = 339.83 \text{ N}$ and applying the equations of equilibrium along the $x$ and $y$ axes to the free-body diagram of joint $D$ shown in Fig. $b$, we have

- $\sum F_x = 0$; $339.83 - F_{CA} \left( \frac{3}{5} \right) - F_{CD} \cos 45^\circ = 0$ \[ (1) \]
- $\sum F_y = 0$; $F_{CA} \left( \frac{4}{5} \right) - F_{CB} \sin 45^\circ = 0$ \[ (2) \]

Solving Eqs. (1) and (2), yields

- $F_{CB} = 275 \text{ N}$ \[ F_{CA} = 243 \text{ N} \quad \text{Ans.} \]
Determine the maximum mass of the lamp that the cord system can support so that no single cord develops a tension exceeding 400 N.

**SOLUTION**

*Equations of Equilibrium:* Applying the equations of equilibrium along the $x$ and $y$ axes to the free-body diagram of joint $D$ shown in Fig. a, we have

\[ + \sum F_y = 0; \quad F_{DE} \sin 30^\circ - m(9.81) = 0 \quad F_{DE} = 19.62m \]

\[ \sum F_x = 0; \quad 19.62m \cos 30^\circ - F_{CD} = 0 \quad F_{CD} = 16.99m \]

Using the result $F_{CD} = 16.99m$ and applying the equations of equilibrium along the $x$ and $y$ axes to the free-body diagram of joint $D$ shown in Fig. b, we have

\[ \sum F_x = 0; \quad 16.99m - F_{CA} \left( \frac{3}{5} \right) - F_{CD} \cos 45^\circ = 0 \quad (1) \]

\[ + \sum F_y = 0; \quad F_{CA} \left( \frac{4}{5} \right) - F_{CB} \sin 45^\circ = 0 \quad (2) \]

Solving Eqs. (1) and (2), yields

\[ F_{CB} = 13.73m \quad F_{CA} = 12.14m \]

Notice that cord $DE$ is subjected to the greatest tensile force, and so it will achieve the maximum allowable tensile force first. Thus

\[ F_{DE} = 400 = 19.62m \]

\[ m = 20.4 \text{ kg} \quad \text{Ans.} \]
Blocks $D$ and $E$ have a mass of 4 kg and 6 kg, respectively. If $x = 2\, \text{m}$ determine the force $F$ and the sag $s$ for equilibrium.

**SOLUTION**

*Equations of Equilibrium.* Referring to the geometry shown in Fig. $a$,

$$
\cos \phi = \frac{s}{\sqrt{s^2 + 2^2}} \quad \sin \phi = \frac{2}{\sqrt{s^2 + 2^2}}
$$

$$
\cos \theta = \frac{s}{\sqrt{s^2 + 4^2}} \quad \sin \theta = \frac{4}{\sqrt{s^2 + 4^2}}
$$

Referring to the FBD shown in Fig. $b$,

$$
\Sigma F_x = 0; \quad 6(9.81)\left(\frac{2}{\sqrt{s^2 + 2^2}}\right) - 4(9.81)\left(\frac{4}{\sqrt{s^2 + 4^2}}\right) = 0
$$

$$
s = 3.381 \, \text{m} = 3.38 \, \text{m}
$$

Ans.

$$
\Sigma F_y = 0; \quad 6(9.81)\left(\frac{3.381}{\sqrt{3.381^2 + 2^2}}\right) + 4(9.81)\left(\frac{3.381}{\sqrt{3.381^2 + 4^2}}\right) - F = 0
$$

$$
F = 75.99 \, \text{N} = 76.0 \, \text{N}
$$

Ans.

Ans:

$s = 3.38 \, \text{m}$

$F = 76.0 \, \text{N}$
*3–32.

Blocks $D$ and $E$ have a mass of 4 kg and 6 kg, respectively. If $F = 80$ N, determine the sag $s$ and distance $x$ for equilibrium.

**SOLUTION**

*Equations of Equilibrium.* Referring to the FBD shown in Fig. $a$,

\[ \sum F_x = 0; \quad 6(9.81) \sin \phi - 4(9.81) \sin \theta = 0 \]

\[ \sin \phi = \frac{2}{3} \sin \theta \]

\[ \sum F_y = 0; \quad 6(9.81) \cos \phi + 4(9.81) \cos \theta - 80 = 0 \]

\[ 3 \cos \phi + 2 \cos \theta = 4.0775 \]

Using Eq (1), the geometry shown in Fig. $b$ can be constructed. Thus

\[ \cos \phi = \frac{\sqrt{9 - 4 \sin^2 \theta}}{3} \]

Substitute this result into Eq. (2),

\[ 3 \left( \frac{\sqrt{9 - 4 \sin^2 \theta}}{3} \right) + 2 \cos \theta = 4.0775 \]

\[ \sqrt{9 - 4 \sin^2 \theta} = 4.0775 - 2 \cos \theta \]

\[ 9 - 4 \sin^2 \theta = 4 \cos^2 \theta - 16.310 \cos \theta + 16.6258 \]

\[ 16.310 \cos \theta = 4(\cos^2 \theta + \sin^2 \theta) + 7.6258 \]

Here, $\cos^2 \theta + \sin^2 \theta = 1$. Then

\[ \cos \theta = 0.7128 \quad \theta = 44.54^\circ \]

Substitute this result into Eq (1)

\[ \sin \phi = \frac{2}{3} \sin 44.54^\circ \quad \phi = 27.88^\circ \]

From Fig. $c$, \( \frac{6 - x}{s} = \tan 44.54^\circ \) and \( \frac{x}{s} = \tan 27.88^\circ \).

So then,

\[ \frac{6 - x}{s} + \frac{x}{s} = \tan 44.54^\circ + \tan 27.88^\circ \]

\[ \frac{6}{s} = 1.5129 \]

\[ s = 3.9659 \text{ m} = 3.97 \text{ m} \]

\[ x = 3.9659 \tan 27.88^\circ \]

\[ = 2.0978 \text{ m} = 2.10 \text{ m} \]

**Ans.**

\[ s = 3.97 \text{ m} \]

\[ x = 2.10 \text{ m} \]
3–33.

The lamp has a weight of 15 lb and is supported by the six cords connected together as shown. Determine the tension in each cord and the angle \( \theta \) for equilibrium. Cord BC is horizontal.

**SOLUTION**

*Equations of Equilibrium.* Considering the equilibrium of Joint A by referring to its FBD shown in Fig. a,

\[
\sum F_x = 0; \quad TA_C \cos 45^\circ - TAB \cos 60^\circ = 0
\]

\[
\sum F_y = 0; \quad TA_C \sin 45^\circ + TAB \sin 60^\circ - 15 = 0
\]

Solving Eqs (1) and (2) yield

\[
TAB = 10.98 = 11.0 lb \quad TA_C = 7.764 lb = 7.76 lb
\]

Ans.

Then, joint B by referring to its FBD shown in Fig. b

\[
\sum F_y = 0; \quad TBE \sin 30^\circ - 10.98 \sin 60^\circ = 0 \quad TBE = 19.02 lb = 19.0 lb
\]

\[
\sum F_x = 0; \quad TBC + 10.98 \cos 60^\circ - 19.02 \cos 30^\circ = 0
\]

\[
TBC = 10.98 lb = 11.0 lb
\]

Ans.

Finally joint C by referring to its FBD shown in Fig. c

\[
\sum F_x = 0; \quad TCD \cos \theta - 10.98 - 7.764 \cos 45^\circ = 0
\]

\[
TCD \cos \theta = 16.4711
\]

(3)

\[
\sum F_y = 0; \quad TCD \sin \theta - 7.764 \sin 45^\circ = 0
\]

\[
TCD \sin \theta = 5.4904
\]

(4)

Divided Eq (4) by (3)

\[
\tan \theta = 0.3333 \quad \theta = 18.43^\circ = 18.4^\circ
\]

Ans.

Substitute this result into Eq (3)

\[
TCD \cos 18.43^\circ = 16.4711 \quad TCD = 17.36 lb = 17.4 lb
\]

Ans.
3–34. Each cord can sustain a maximum tension of 20 lb. Determine the largest weight of the lamp that can be supported. Also, determine \( \theta \) of cord DC for equilibrium.

**SOLUTION**

**Equations of Equilibrium.** Considering the equilibrium of Joint A by referring to its FBD shown in Fig. a,

\[
\begin{align*}
\sum F_x &= 0; \quad T_{AC} \cos 45^\circ - T_{AB} \cos 60^\circ = 0 \quad (1) \\
\sum F_y &= 0; \quad T_{AC} \sin 45^\circ - T_{AB} \sin 60^\circ - W = 0 \quad (2)
\end{align*}
\]

Solving Eqs (1) and (2) yield

\[
T_{AB} = 0.7321 \ W \quad T_{AC} = 0.5176 \ W
\]

Then, joint B by referring to its FBD shown in Fig. b,

\[
\begin{align*}
\sum F_x &= 0; \quad T_{BE} \sin 30^\circ - 0.7321 \ W \sin 60^\circ = 0 \quad T_{BE} = 1.2679 \ W \\
\sum F_y &= 0; \quad T_{BC} + 0.7321 \ W \cos 60^\circ - 1.2679 \ W \cos 30^\circ = 0 \\
T_{BC} &= 0.7321 \ W
\end{align*}
\]

Finally, joint C by referring to its FBD shown in Fig. c,

\[
\begin{align*}
\sum F_x &= 0; \quad T_{CD} \cos \theta - 0.7321 \ W - 0.5176 \ W \cos 45^\circ = 0 \\
T_{CD} \cos \theta &= 1.0981 \ W \quad (3) \\
\sum F_y &= 0; \quad T_{CD} \sin \theta - 0.5176 \ W \sin 45^\circ = 0 \\
T_{CD} \sin \theta &= 0.3660 \ W \quad (4)
\end{align*}
\]

Divided Eq (4) by (3)

\[
\tan \theta = 0.3333 \quad \theta = 18.43^\circ = 18.4^\circ \quad \text{Ans.}
\]

Substitute this result into Eq (3),

\[
T_{CD} \cos 18.43^\circ = 1.0981 \ W \quad T_{CD} = 1.1575 \ W
\]

Here cord BE is subjected to the largest tension. Therefore, its tension will reach the limit first, that is \( T_{BE} = 20 \) lb. Then

\[
20 = 1.2679 \ W; \quad W = 15.77 \ lb = 15.8 \ lb \quad \text{Ans.}
\]
3–35.

The ring of negligible size is subjected to a vertical force of 200 lb. Determine the required length \( l \) of cord AC such that the tension acting in AC is 160 lb. Also, what is the force in cord AB? *Hint:* Use the equilibrium condition to determine the required angle \( \theta \) for attachment, then determine \( l \) using trigonometry applied to triangle \( ABC \).

**SOLUTION**

\[ \sum F_x = 0; \quad F_{AB} \cos 40^\circ - 160 \cos \theta = 0 \]
\[ \sum F_y = 0; \quad 160 \sin \theta + F_{AB} \sin 40^\circ - 200 = 0 \]

Thus,
\[ \sin \theta + 0.8391 \cos \theta = 1.25 \]

Solving by trial and error,
\[ \theta = 33.25^\circ \]
\[ F_{AB} = 175 \text{ lb} \quad \text{Ans.} \]
\[ \frac{2}{\sin 33.25^\circ} = \frac{l}{\sin 40^\circ} \]
\[ l = 2.34 \text{ ft} \quad \text{Ans.} \]

Also,
\[ \theta = 66.75^\circ \]
\[ F_{AB} = 82.4 \text{ lb} \quad \text{Ans.} \]
\[ \frac{2}{\sin 66.75^\circ} = \frac{l}{\sin 40^\circ} \]
\[ l = 1.40 \text{ ft} \quad \text{Ans.} \]
Cable $ABC$ has a length of 5 m. Determine the position $x$ and the tension developed in $ABC$ required for equilibrium of the 100-kg sack. Neglect the size of the pulley at $B$.

**SOLUTION**

**Equations of Equilibrium:** Since cable $ABC$ passes over the smooth pulley at $B$, the tension in the cable is constant throughout its entire length. Applying the equation of equilibrium along the $y$ axis to the free-body diagram in Fig. a, we have

$$+ \uparrow \sum F_y = 0; \quad 2T \sin \phi - 100(9.81) = 0$$

(1)

**Geometry:** Referring to Fig. b, we can write

$$\frac{3.5 - x}{\cos \phi} + \frac{x}{\cos \phi} = 5$$

$$\phi = \cos^{-1} \left( \frac{3.5}{5} \right) = 45.57^\circ$$

Also,

$$x \tan 45.57^\circ + 0.75 = (3.5 - x) \tan 45.57^\circ$$

$$x = 1.38 \text{ m}$$

Ans.

Substituting $\phi = 45.57^\circ$ into Eq. (1), yields

$$T = 687 \text{ N}$$

Ans.
3–37.
A 4-kg sphere rests on the smooth parabolic surface. Determine the normal force it exerts on the surface and the mass \( m_B \) of block \( B \) needed to hold it in the equilibrium position shown.

**SOLUTION**

*Geometry:* The angle \( \theta \) which the surface make with the horizontal is to be determined first.

\[
\tan \theta \bigg|_{x=0.4 \text{ m}} = \frac{dy}{dx} \bigg|_{x=0.4 \text{ m}} = 5.0 \frac{m}{x} \bigg|_{x=0.4 \text{ m}} = 2.00
\]

\( \theta = 63.43^\circ \)

*Free Body Diagram:* The tension in the cord is the same throughout the cord and is equal to the weight of block \( B \), \( W_B = m_B (9.81) \).

*Equations of Equilibrium:*

\[
\sum F_x = 0; \quad m_B (9.81) \cos 60^\circ - N \sin 63.43^\circ = 0
\]

\[N = 5.4840 m_B \quad [1]\]

\[
\sum F_y = 0; \quad m_B (9.81) \sin 60^\circ + N \cos 63.43^\circ - 39.24 = 0
\]

\[8.4957 m_B + 0.4472 N = 39.24 \quad [2]\]

Solving Eqs. [1] and [2] yields

\( m_B = 3.58 \text{ kg} \quad N = 19.7 \text{ N} \)

Ans.
3–38.

Determine the forces in cables $AC$ and $AB$ needed to hold the 20-kg ball $D$ in equilibrium. Take $F = 300$ N and $d = 1$ m.

**SOLUTION**

*Equations of Equilibrium:*

\[ \begin{align*}
\sum F_x &= 0; \quad 300 - F_{AB} \left( \frac{4}{\sqrt{41}} \right) - F_{AC} \left( \frac{2}{\sqrt{5}} \right) = 0 \\
&\quad 0.6247F_{AB} + 0.8944F_{AC} = 300 \\
\sum F_y &= 0; \quad F_{AB} \left( \frac{5}{\sqrt{41}} \right) + F_{AC} \left( \frac{1}{\sqrt{5}} \right) - 196.2 = 0 \\
&\quad 0.7809F_{AB} + 0.4472F_{AC} = 196.2 \tag{1} \tag{2}
\end{align*} \]

Solving Eqs. (1) and (2) yields

\[ F_{AB} = 98.6 \text{ N} \quad F_{AC} = 267 \text{ N} \]

Ans.
3–39.

The ball $D$ has a mass of 20 kg. If a force of $F = 100$ N is applied horizontally to the ring at $A$, determine the largest dimension $d$ so that the force in cable $AC$ is zero.

**SOLUTION**

**Equations of Equilibrium:**

\[ \sum F_x = 0; \quad 100 - F_{AB} \cos \theta = 0 \quad F_{AB} \cos \theta = 100 \quad (1) \]
\[ \sum F_y = 0; \quad F_{AB} \sin \theta - 196.2 = 0 \quad F_{AB} \sin \theta = 196.2 \quad (2) \]

Solving Eqs. (1) and (2) yields

\[ \theta = 62.99^\circ \quad F_{AB} = 220.21 \text{ N} \]

From the geometry,

\[ d + 1.5 = 2 \tan 62.99^\circ \]
\[ d = 2.42 \text{ m} \]

**Ans.**

\[ d = 2.42 \text{ m} \]
*3–40.

The 200-lb uniform tank is suspended by means of a 6-ft-long cable, which is attached to the sides of the tank and passes over the small pulley located at O. If the cable can be attached at either points A and B, or C and D, determine which attachment produces the least amount of tension in the cable. What is this tension?

**SOLUTION**

**Free-Body Diagram:** By observation, the force \( F \) has to support the entire weight of the tank. Thus, \( F = 200 \text{ lb} \). The tension in cable \( AOB \) or \( COD \) is the same throughout the cable.

**Equations of Equilibrium:**

\[
\begin{align*}
\sum F_x &= 0; \quad T \cos \theta - T \cos \theta = 0 \quad (\text{Satisfied!}) \\
\sum F_y &= 0; \quad 200 - 2T \sin \theta = 0 \quad T = \frac{100}{\sin \theta} \quad (1)
\end{align*}
\]

From the function obtained above, one realizes that in order to produce the least amount of tension in the cable, \( \sin \theta \) hence \( \theta \) must be as great as possible. Since the attachment of the cable to point C and D produces a greater \( \theta \left( \theta = \cos^{-1} \frac{1}{3} = 70.53^\circ \right) \) as compared to the attachment of the cable to points A and B \( \left( \theta = \cos^{-1} \frac{2}{3} = 48.19^\circ \right) \), the attachment of the cable to point C and D will produce the least amount of tension in the cable. Ans.

Thus,

\[
T = \frac{100}{\sin 70.53^\circ} = 106 \text{ lb}
\]

**Ans:**

\[
T = 106 \text{ lb}
\]
The single elastic cord $ABC$ is used to support the 40-lb load. Determine the position $x$ and the tension in the cord that is required for equilibrium. The cord passes through the smooth ring at $B$ and has an unstretched length of 6 ft and stiffness of $k = 50$ lb/ft.

**SOLUTION**

*Equations of Equilibrium:* Since elastic cord $ABC$ passes over the smooth ring at $B$, the tension in the cord is constant throughout its entire length. Applying the equation of equilibrium along the $y$ axis to the free-body diagram in Fig. a, we have

$$+\sum F_y = 0; \quad 2T \sin \phi - 40 = 0 \quad (1)$$

*Geometry:* Referring to Fig. (b), the stretched length of cord $ABC$ is

$$l_{ABC} = \frac{x}{\cos \phi} + \frac{5 - x}{\cos \phi} = \frac{5}{\cos \phi} \quad (2)$$

Also,

$$x \tan \phi + 1 = (5 - x) \tan \phi$$

$$x = \frac{5 \tan \phi - 1}{2 \tan \phi} \quad (3)$$

*Spring Force Formula:* Applying the spring force formula using Eq. (2), we obtain

$$F_{sp} = k(l_{ABC} - l_0)$$

$$T = 50 \left[ \frac{5}{\cos \phi} - 6 \right] \quad (4)$$

Substituting Eq. (4) into Eq. (1) yields

$$5 \tan \phi - 6 \sin \phi = 0.4$$

Solving the above equation by trial and error

$$\phi = 40.86^\circ$$

Substituting $\phi = 40.86^\circ$ into Eqs. (1) and (3) yields

$$T = 30.6 \text{ lb} \quad x = 1.92 \text{ ft}$$

Ans.

$$T = 30.6 \text{ lb}$$

$$x = 1.92 \text{ ft}$$
3–42.

A “scale” is constructed with a 4-ft-long cord and the 10-lb block \( D \). The cord is fixed to a pin at \( A \) and passes over two small pulleys at \( B \) and \( C \). Determine the weight of the suspended block at \( B \) if the system is in equilibrium when \( s = 1.5 \) ft.

**SOLUTION**

**Free-Body Diagram:** The tension force in the cord is the same throughout the cord, that is, 10 lb. From the geometry,

\[
\theta = \sin^{-1}\left(\frac{0.5}{1.25}\right) = 23.58^\circ
\]

**Equations of Equilibrium:**

\[
\begin{align*}
\sum F_x &= 0; \quad 10 \sin 23.58^\circ - 10 \sin 23.58^\circ = 0 \quad \text{(Satisfied!)} \\
\sum F_y &= 0; \quad 2(10) \cos 23.58^\circ - W_B = 0
\end{align*}
\]

\[W_B = 18.3 \text{ lb}\]

**Ans:**

\[W_B = 18.3 \text{ lb}\]
3–43.

The three cables are used to support the 40-kg flowerpot. Determine the force developed in each cable for equilibrium.

**SOLUTION**

*Equations of Equilibrium.* Referring to the FBD shown in Fig. a,

\[ \sum F_z = 0; \quad F_{AD} \left( \frac{1.5}{\sqrt{1.5^2 + 2^2 + 1.5^2}} \right) - 40(9.81) = 0 \]

\[ F_{AD} = \frac{762.69 N}{763 N} \quad \text{Ans.} \]

Using this result,

\[ \sum F_x = 0; \quad F_{AC} - 762.69 \left( \frac{1.5}{\sqrt{1.5^2 + 2^2 + 1.5^2}} \right) = 0 \]

\[ F_{AC} = 392.4 N = 392 N \quad \text{Ans.} \]

\[ \sum F_y = 0; \quad F_{AB} - 762.69 \left( \frac{2}{\sqrt{1.5^2 + 2^2 + 1.5^2}} \right) = 0 \]

\[ F_{AB} = 523.2 N = 523 N \quad \text{Ans.} \]
*3–44.

Determine the magnitudes of $F_1$, $F_2$, and $F_3$ for equilibrium of the particle.

**SOLUTION**

_Equations of Equilibrium._ Referring to the FBD shown,

\[ \sum F_y = 0; \quad 10 \left( \frac{24}{25} \right) - 4 \cos 30^\circ - F_2 \cos 30^\circ = 0 \quad F_2 = 7.085 \text{ kN} = 7.09 \text{ kN} \quad \text{Ans.} \]

\[ \sum F_x = 0; \quad F_1 - 4 \sin 30^\circ - 10 \left( \frac{7}{25} \right) = 0 \quad F_1 = 4.80 \text{ kN} \quad \text{Ans.} \]

Using the result of $F_2 = 7.085$ kN,

\[ \sum F_z = 0; \quad 7.085 \sin 30^\circ - F_3 = 0 \quad F_3 = 3.543 \text{ kN} = 3.54 \text{ kN} \quad \text{Ans.} \]
3–45.

If the bucket and its contents have a total weight of 20 lb, determine the force in the supporting cables $DA$, $DB$, and $DC$.

**SOLUTION**

$$\mathbf{u}_{DA} = \left[ \frac{3}{4.5} \mathbf{i} - \frac{1.5}{4.5} \mathbf{j} + \frac{3}{4.5} \mathbf{k} \right]$$

$$\mathbf{u}_{DC} = \left[ -\frac{1.5}{3.5} \mathbf{i} + \frac{1}{3.5} \mathbf{j} + \frac{3}{3.5} \mathbf{k} \right]$$

$$\Sigma F_x = 0; \quad \frac{3}{4.5} F_{DA} - \frac{1.5}{3.5} F_{DC} = 0$$

$$\Sigma F_y = 0; \quad -\frac{1.5}{4.5} F_{DA} - F_{DB} + \frac{1}{3.5} F_{DC} = 0$$

$$\Sigma F_z = 0; \quad \frac{3}{4.5} F_{DA} + \frac{3}{3.5} F_{DC} - 20 = 0$$

$F_{DA} = 10.0 \text{ lb}$

$F_{DB} = 1.11 \text{ lb}$

$F_{DC} = 15.6 \text{ lb}$

Ans:

$F_{DA} = 10.0 \text{ lb}$

$F_{DB} = 1.11 \text{ lb}$

$F_{DC} = 15.6 \text{ lb}$
SOLUTION

Cartesian Vector Notation:

\[ \mathbf{F}_{OC} = F_{OC} \left( \frac{6i + 4j + 12k}{\sqrt{6^2 + 4^2 + 12^2}} \right) = \frac{3}{7}F_{OC}i + \frac{2}{7}F_{OC}j + \frac{6}{7}F_{OC}k \]

\[ \mathbf{F}_{OA} = -F_{OA}\mathbf{j} \quad \mathbf{F}_{OB} = -F_{OB}\mathbf{i} \]

\[ \mathbf{F} = \{-196.2\mathbf{k}\} \text{ N} \]

Equations of Equilibrium:

\[ \sum \mathbf{F} = \mathbf{0}; \quad \mathbf{F}_{OC} + \mathbf{F}_{OA} + \mathbf{F}_{OB} + \mathbf{F} = \mathbf{0} \]

\[ \left( \frac{3}{7}F_{OC} - F_{OB} \right)i + \left( \frac{2}{7}F_{OC} - F_{OA} \right)j + \left( \frac{6}{7}F_{OC} - 196.2 \right)k = 0 \]

Equating \(i\), \(j\), and \(k\) components, we have

\[ \frac{3}{7}F_{OC} - F_{OB} = 0 \quad (1) \]

\[ \frac{2}{7}F_{OC} - F_{OA} = 0 \quad (2) \]

\[ \frac{6}{7}F_{OC} - 196.2 = 0 \quad (3) \]

Solving Eqs. (1), (2) and (3) yields

\[ F_{OC} = 228.9 \text{ N} \quad F_{OB} = 98.1 \text{ N} \quad F_{OA} = 65.4 \text{ N} \]

Spring Elongation: Using spring formula, Eq. 3–2, the spring elongation is

\[ s = \frac{F}{k} \]

\[ s_{OB} = \frac{98.1}{300} = 0.327 \text{ m} = 327 \text{ mm} \quad \text{Ans.} \]

\[ s_{OA} = \frac{65.4}{300} = 0.218 \text{ m} = 218 \text{ mm} \quad \text{Ans.} \]
3–47.

Determine the force in each cable needed to support the 20-kg flowerpot.

SOLUTION

Equations of Equilibrium.

\[ \sum F_z = 0; \quad F_{AB} \left( \frac{6}{\sqrt{45}} \right) - 20(9.81) = 0 \quad \Rightarrow \quad F_{AB} = 219.36 \text{ N} = 219 \text{ N} \quad \text{Ans.} \]

\[ \sum F_x = 0; \quad F_{AC} \left( \frac{2}{\sqrt{20}} \right) - F_{AB} \left( \frac{2}{\sqrt{20}} \right) = 0 \quad \Rightarrow \quad F_{AC} = F_{AD} = F \]

Using the results of \( F_{AB} = 219.36 \text{ N} \) and \( F_{AC} = F_{AD} = F \),

\[ \sum F_y = 0; \quad 2 \left( F \left( \frac{4}{\sqrt{20}} \right) \right) - 219.36 \left( \frac{3}{\sqrt{45}} \right) = 0 \]

\[ F_{AC} = F_{AD} = F = 54.84 \text{ N} = 54.8 \text{ N} \quad \text{Ans.} \]
Determine the tension in the cables in order to support the 100-kg crate in the equilibrium position shown.

SOLUTION

**Force Vectors:** We can express each of the forces on the free-body diagram shown in Fig. (a) in Cartesian vector form as

\[ \mathbf{F}_{AB} = F_{AB} \mathbf{i} \]
\[ \mathbf{F}_{AC} = -F_{AC} \mathbf{j} \]
\[ \mathbf{F}_{AD} = F_{AD} \left[ \frac{(-2 \mathbf{i} + (2-0) \mathbf{j} + (1-0) \mathbf{k})}{\sqrt{(-2)^2 + (2-0)^2 + (1-0)^2}} \right] = \frac{2}{3} F_{AD} \mathbf{i} + \frac{2}{3} F_{AD} \mathbf{j} + \frac{1}{3} F_{AD} \mathbf{k} \]

\[ \mathbf{W} = [-100(9.81)] \mathbf{k} \mathbf{N} = [-981 \mathbf{k}] \mathbf{N} \]

**Equations of Equilibrium:** Equilibrium requires

\[ \Sigma \mathbf{F} = 0; \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = 0 \]
\[ F_{AB} \mathbf{i} + (-F_{AC} \mathbf{j}) + \left( -\frac{2}{3} F_{AD} \mathbf{i} + \frac{2}{3} F_{AD} \mathbf{j} + \frac{1}{3} F_{AD} \mathbf{k} \right) + (-981 \mathbf{k}) = 0 \]

\[ \left( F_{AB} - \frac{2}{3} F_{AD} \right) \mathbf{i} + \left( -F_{AC} + \frac{2}{3} F_{AD} \right) \mathbf{j} + \left( \frac{1}{3} F_{AD} - 981 \right) \mathbf{k} = 0 \]

Equating the \( \mathbf{i}, \mathbf{j}, \) and \( \mathbf{k} \) components yields

\[ F_{AB} - \frac{2}{3} F_{AD} = 0 \]
\[ -F_{AC} + \frac{2}{3} F_{AD} = 0 \]
\[ \frac{1}{3} F_{AD} - 981 = 0 \]

Solving Eqs. (1) through (3) yields

\[ F_{AD} = 2943 \text{ N} = 2.94 \text{ kN} \quad \text{Ans.} \]
\[ F_{AB} = F_{AC} = 1962 \text{ N} = 1.96 \text{ kN} \quad \text{Ans.} \]
3–49.

Determine the maximum mass of the crate so that the tension developed in any cable does not exceed 3 kN.

SOLUTION

**Force Vectors:** We can express each of the forces on the free-body diagram shown in Fig. (a) in Cartesian vector form as

\[ \mathbf{F}_{AB} = F_{AB} \mathbf{i} \]

\[ \mathbf{F}_{AC} = -F_{AC} \mathbf{j} \]

\[ \mathbf{F}_{AD} = F_{AD} \left[ \frac{-(2-0) \mathbf{i}+2-0 \mathbf{j}+(1-0) \mathbf{k}}{\sqrt{(-2-0)^2 + (2-0)^2 + (1-0)^2}} \right] = -\frac{2}{3}F_{AD} \mathbf{i} + \frac{2}{3}F_{AD} \mathbf{j} + \frac{1}{3}F_{AD} \mathbf{k} \]

\[ \mathbf{W} = -m(9.81) \mathbf{k} \]

**Equations of Equilibrium:** Equilibrium requires

\[ \sum \mathbf{F} = \mathbf{0} \]

\[ F_{AB} \mathbf{i} + (-F_{AC}) \mathbf{j} + \left( -\frac{2}{3}F_{AD} \mathbf{i} + \frac{2}{3}F_{AD} \mathbf{j} + \frac{1}{3}F_{AD} \mathbf{k} \right) + [-m(9.81) \mathbf{k}] = \mathbf{0} \]

\[ \left( F_{AB} - \frac{2}{3}F_{AD} \right) \mathbf{i} + \left( -F_{AC} + \frac{2}{3}F_{AD} \right) \mathbf{j} + \left( \frac{1}{3}F_{AD} - 9.81m \right) \mathbf{k} = \mathbf{0} \]

Equating the \( \mathbf{i}, \mathbf{j}, \) and \( \mathbf{k} \) components yields

\[ F_{AB} - \frac{2}{3}F_{AD} = 0 \]  \hspace{1cm} (1)

\[ -F_{AC} + \frac{2}{3}F_{AD} = 0 \]  \hspace{1cm} (2)

\[ \frac{1}{3}F_{AD} - 9.81m = 0 \]  \hspace{1cm} (3)

When cable \( AD \) is subjected to maximum tension, \( F_{AD} = 3000 \) N. Thus, by substituting this value into Eqs. (1) through (3), we have

\[ F_{AB} = F_{AC} = 2000 \text{ N} \]

\[ m = 102 \text{ kg} \]

Ans.
3–50. Determine the force in each cable if \( F = 500 \text{ lb} \).

**SOLUTION**

*Equations of Equilibrium.* Referring to the FBD shown in Fig. a,

\[
\sum F_x = 0; \quad F_{AB} \left( \frac{3}{7} \right) - F_{AC} \left( \frac{3}{\sqrt{46}} \right) - F_{AD} \left( \frac{2}{7} \right) = 0
\]  

\( \text{(1)} \)

\[
\sum F_y = 0; \quad -F_{AB} \left( \frac{2}{7} \right) - F_{AC} \left( \frac{1}{\sqrt{46}} \right) + F_{AD} \left( \frac{3}{7} \right) = 0
\]  

\( \text{(2)} \)

\[
\sum F_z = 0; \quad -F_{AB} \left( \frac{6}{7} \right) - F_{AC} \left( \frac{6}{\sqrt{46}} \right) - F_{AD} \left( \frac{6}{7} \right) + 500 = 0
\]  

\( \text{(3)} \)

Solving Eqs (1), (2) and (3)

\[
F_{AC} = 113.04 \text{ lb} = 113 \text{ lb} \quad \text{Ans.}
\]

\[
F_{AB} = 256.67 \text{ lb} = 257 \text{ lb} \quad \text{Ans.}
\]

\[
F_{AD} = 210 \text{ lb} \quad \text{Ans.}
\]
3–51.

Determine the greatest force \( F \) that can be applied to the ring if each cable can support a maximum force of 800 lb.

**SOLUTION**

*Equations of Equilibrium.* Referring to the FBD shown in Fig. a,

\[
\sum F_x = 0; \quad F_{AB} \left( \frac{3}{7} \right) - F_{AC} \left( \frac{3}{46} \right) - F_{AD} \left( \frac{2}{7} \right) = 0 \tag{1}
\]

\[
\sum F_y = 0; \quad -F_{AB} \left( \frac{2}{7} \right) - F_{AC} \left( \frac{1}{46} \right) + F_{AD} \left( \frac{3}{7} \right) = 0 \tag{2}
\]

\[
\sum F_z = 0; \quad -F_{AB} \left( \frac{6}{7} \right) - F_{AC} \left( \frac{6}{46} \right) - F_{AD} \left( \frac{6}{7} \right) + F = 0 \tag{3}
\]

Solving Eqs (1), (2) and (3)

\[
F_{AC} = 0.2261 \ F \quad F_{AB} = 0.5133 \ F \quad F_{AD} = 0.42 \ F
\]

Since cable \( AB \) is subjected to the greatest tension, its tension will reach the limit first that is \( F_{AB} = 800 \) lb. Then

\[
800 = 0.5133 \ F
\]

\[
F = 1558.44 \text{ lb} = 1558 \text{ lb}
\]  

Ans.
Determine the tension developed in cables $AB$ and $AC$ and the force developed along strut $AD$ for equilibrium of the 400-lb crate.

**SOLUTION**

**Force Vectors:** We can express each of the forces on the free-body diagram shown in Fig. $a$ in Cartesian vector form as

$$
\mathbf{F}_{AB} = F_{AB} \left[ \frac{-2 - 0}{}i + \frac{-6 - 0}{}j + \frac{1.5 - 0}{}k \right] = -\frac{4}{13} F_{AB} i - \frac{12}{13} F_{AB} j + \frac{3}{13} F_{AB} k
$$

$$
\mathbf{F}_{AC} = F_{AC} \left[ \frac{2 - 0}{}i + \frac{-6 - 0}{}j + \frac{3 - 0}{}k \right] = \frac{2}{7} F_{AC} i - \frac{6}{7} F_{AC} j + \frac{3}{7} F_{AC} k
$$

$$
\mathbf{F}_{AD} = F_{AD} \left[ \frac{0 - 0}{}i + \frac{0 - (-6)}{}j + \frac{0 - (-2.5)}{}k \right] = \frac{12}{13} F_{AD} j + \frac{5}{13} F_{AD} k
$$

$$
\mathbf{W} = \{-400k\} \text{ lb}
$$

**Equations of Equilibrium:** Equilibrium requires

$$
\sum \mathbf{F} = 0: \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = 0
$$

$$
\left( -\frac{4}{13} F_{AB} i - \frac{12}{13} F_{AB} j + \frac{3}{13} F_{AB} k \right) + \left( \frac{2}{7} F_{AC} i - \frac{6}{7} F_{AC} j + \frac{3}{7} F_{AC} k \right) + \left( \frac{12}{13} F_{AD} j + \frac{5}{13} F_{AD} k \right) + \{-400k\} = 0
$$

$$
\left( -\frac{4}{13} F_{AB} + \frac{2}{7} F_{AC} \right) i + \left( -\frac{12}{13} F_{AB} - \frac{6}{7} F_{AC} + \frac{12}{13} F_{AD} \right) j + \left( \frac{3}{13} F_{AB} + \frac{3}{7} F_{AC} + \frac{5}{13} F_{AD} - 400 \right) k = 0
$$

Equating the $i$, $j$, and $k$ components yields

1. \[-\frac{4}{13} F_{AB} + \frac{2}{7} F_{AC} = 0\]
2. \[-\frac{12}{13} F_{AB} - \frac{6}{7} F_{AC} + \frac{12}{13} F_{AD} = 0\]
3. \[\frac{3}{13} F_{AB} + \frac{3}{7} F_{AC} + \frac{5}{13} F_{AD} - 400 = 0\]

Solving Eqs. (1) through (3) yields

$$
F_{AB} = 274 \text{ lb} \quad \text{Ans.}
$$

$$
F_{AC} = 295 \text{ lb} \quad \text{Ans.}
$$

$$
F_{AD} = 547 \text{ lb} \quad \text{Ans.}
$$

**Ans:**

$$
F_{AB} = 274 \text{ lb}
$$

$$
F_{AC} = 295 \text{ lb}
$$

$$
F_{AD} = 547 \text{ lb}
$$
3–53.

If the tension developed in each of the cables cannot exceed 300 lb, determine the largest weight of the crate that can be supported. Also, what is the force developed along strut $AD$?

**SOLUTION**

**Force Vectors:** We can express each of the forces on the free-body diagram shown in Fig. a in Cartesian vector form as

\[
F_{AB} = F_{AB} \left[ \frac{(-2 - 0)i + (-6 - 0)j + (1.5 - 0)k}{\sqrt{(-2 - 0)^2 + (-6 - 0)^2 + (1.5 - 0)^2}} \right] = \frac{-4}{13} F_{AB}i - \frac{12}{13} F_{AB}j + \frac{3}{13} F_{AB}k
\]

\[
F_{AC} = F_{AC} \left[ \frac{(2 - 0)i + (-6 - 0)j + (3 - 0)k}{\sqrt{(2 - 0)^2 + (-6 - 0)^2 + (3 - 0)^2}} \right] = \frac{2}{7} F_{AC}i - \frac{6}{7} F_{AC}j + \frac{3}{7} F_{AC}k
\]

\[
F_{AD} = F_{AD} \left[ \frac{(0 - 0)i + [0 - (-6)]j + [0 - (-2.5)]k}{\sqrt{(0 - 0)^2 + [0 - (-6)]^2 + [0 - (-2.5)]^2}} \right] = \frac{12}{13} F_{AD}j + \frac{5}{13} F_{AD}k
\]

\[
W = -Wk
\]

**Equations of Equilibrium:** Equilibrium requires

\[
\sum F = 0; \quad F_{AB} + F_{AC} + F_{AD} + W = 0
\]

\[
\left( \frac{-4}{13} F_{AB}i - \frac{12}{13} F_{AB}j + \frac{3}{13} F_{AB}k \right) + \left( \frac{2}{7} F_{AC}i - \frac{6}{7} F_{AC}j + \frac{3}{7} F_{AC}k \right) + \left( \frac{12}{13} F_{AD}j + \frac{5}{13} F_{AD}k \right) + (-Wk) = 0
\]

Evaluating the $i$, $j$, and $k$ components yields

\[
\frac{-4}{13} F_{AB} + \frac{2}{7} F_{AC} = 0 \quad \text{(1)}
\]

\[
\frac{-12}{13} F_{AB} - \frac{6}{7} F_{AC} + \frac{12}{13} F_{AD} = 0 \quad \text{(2)}
\]

\[
\frac{3}{13} F_{AB} + \frac{3}{7} F_{AC} + \frac{5}{13} F_{AD} - W = 0 \quad \text{(3)}
\]

Let us assume that cable $AC$ achieves maximum tension first. Substituting $F_{AC} = 300$ lb into Eqs. (1) through (3) and solving, yields

\[
F_{AB} = 278.57 \text{ lb} \quad F_{AD} = 557 \text{ lb} \quad W = 407 \text{ lb}
\]

Ans.

Since $F_{AB} = 278.57$ lb < 300 lb, our assumption is correct.
3–54.
Determine the tension developed in each cable for equilibrium of the 300-lb crate.

**SOLUTION**

**Force Vectors:** We can express each of the forces shown in Fig. a in Cartesian vector form as

\[ \mathbf{F}_{AB} = \mathbf{F}_{AB} \left[ \frac{-30\mathbf{i} + (-60\mathbf{j} + (20\mathbf{k})}{\sqrt{(-30)^2 + (-60)^2 + (20)^2}} \right] = \frac{-3}{7} \mathbf{F}_{AB} \mathbf{i} - \frac{6}{7} \mathbf{F}_{AB} \mathbf{j} + \frac{2}{7} \mathbf{F}_{AB} \mathbf{k} \]

\[ \mathbf{F}_{AC} = \mathbf{F}_{AC} \left[ \frac{20\mathbf{i} + (-60\mathbf{j} + (30\mathbf{k})}{\sqrt{(20)^2 + (-60)^2 + (30)^2}} \right] = \frac{2}{7} \mathbf{F}_{AC} \mathbf{i} - \frac{6}{7} \mathbf{F}_{AC} \mathbf{j} + \frac{3}{7} \mathbf{F}_{AC} \mathbf{k} \]

\[ \mathbf{F}_{AD} = \mathbf{F}_{AD} \left[ \frac{(0-0)(\mathbf{i} + (30\mathbf{j} + (40\mathbf{k})}{\sqrt{(0-0)^2 + (30)^2 + (40)^2}} \right] = \frac{3}{5} \mathbf{F}_{AD} \mathbf{j} + \frac{4}{5} \mathbf{F}_{AD} \mathbf{k} \]

\[ \mathbf{W} = [-300\mathbf{k}] \text{ lb} \]

**Equations of Equilibrium:** Equilibrium requires

\[ \sum \mathbf{F} = 0; \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = 0 \]

\[ \left( \frac{-3}{7} \mathbf{F}_{AB} \mathbf{i} - \frac{6}{7} \mathbf{F}_{AB} \mathbf{j} + \frac{2}{7} \mathbf{F}_{AB} \mathbf{k} \right) + \left( \frac{2}{7} \mathbf{F}_{AC} \mathbf{i} - \frac{6}{7} \mathbf{F}_{AC} \mathbf{j} + \frac{3}{7} \mathbf{F}_{AC} \mathbf{k} \right) + \left( \frac{3}{5} \mathbf{F}_{AD} \mathbf{j} + \frac{4}{5} \mathbf{F}_{AD} \mathbf{k} \right) + (-300\mathbf{k}) = 0 \]

Equating the \( \mathbf{i}, \mathbf{j}, \text{ and } \mathbf{k} \) components yields

\[ \frac{-3}{7} F_{AB} + \frac{2}{7} F_{AC} = 0 \quad (1) \]

\[ \frac{-6}{7} F_{AB} - \frac{6}{7} F_{AC} + \frac{3}{5} F_{AD} = 0 \quad (2) \]

\[ \frac{2}{7} F_{AB} + \frac{3}{7} F_{AC} + \frac{4}{5} F_{AD} - 300 = 0 \quad (3) \]

Solving Eqs. (1) through (3) yields

\[ F_{AB} = 79.2 \text{ lb} \quad F_{AC} = 119 \text{ lb} \quad F_{AD} = 283 \text{ lb} \]
Determine the maximum weight of the crate that can be suspended from cables $AB$, $AC$, and $AD$ so that the tension developed in any one of the cables does not exceed 250 lb.

**SOLUTION**

**Force Vectors:** We can express each of the forces shown in Fig. $a$ in Cartesian vector form as

$$F_{AB} = F_{AB} \begin{bmatrix} (-3 - 0)i + (-6 - 0)j + (2 - 0)k \end{bmatrix} = \frac{-3}{7} F_{AB}i - \frac{6}{7} F_{AB}j + \frac{2}{7} F_{AB}k$$

$$F_{AC} = F_{AC} \begin{bmatrix} (2 - 0)i + (-6 - 0)j + (3 - 0)k \end{bmatrix} = \frac{2}{7} F_{AC}i - \frac{6}{7} F_{AC}j + \frac{3}{7} F_{AC}k$$

$$F_{AD} = F_{AD} \begin{bmatrix} (0 - 0)i + (3 - 0)j + (4 - 0)k \end{bmatrix} = \frac{3}{5} F_{AD}i + \frac{4}{5} F_{AD}k$$

$$W = -W_C k$$

**Equations of Equilibrium:** Equilibrium requires

$$\sum \mathbf{F} = 0: \quad F_{AB} + F_{AC} + F_{AD} + W = 0$$

$$\left( -\frac{3}{7} F_{AB}i - \frac{6}{7} F_{AC}j + \frac{2}{7} F_{AD}k \right) + \left( \frac{2}{7} F_{AC}i - \frac{6}{7} F_{AC}j + \frac{3}{7} F_{AC}k \right) + \left( \frac{3}{5} F_{AD}i + \frac{4}{5} F_{AD}k \right) + (-W_Ck) = 0$$

$$\left( -\frac{3}{7} F_{AB}i + \frac{2}{7} F_{AC}j \right) + \left( \frac{6}{7} F_{AB}i - \frac{6}{7} F_{AC}j + \frac{3}{5} F_{AD}k \right) + \left( \frac{2}{7} F_{AB}i + \frac{3}{7} F_{AC}j + \frac{4}{5} F_{AD}k - W_C k \right) = 0$$

Equating the $i$, $j$, and $k$ components yields

1. $$-\frac{3}{7} F_{AB} + \frac{2}{7} F_{AC} = 0$$
2. $$-\frac{6}{7} F_{AB} - \frac{6}{7} F_{AC} + \frac{3}{5} F_{AD} = 0$$
3. $$\frac{2}{7} F_{AB} + \frac{3}{7} F_{AC} + \frac{4}{5} F_{AD} - W_C = 0$$

Assuming that cable $AD$ achieves maximum tension first, substituting $F_{AD} = 250$ lb into Eqs. (2) and (3), and solving Eqs. (1) through (3) yields

$$F_{AB} = 70 \text{ lb} \quad F_{AC} = 105 \text{ lb}$$

$$W_C = 265 \text{ lb} \quad \text{Ans.}$$

Since $F_{AB} = 70 \text{ lb} < 250 \text{ lb}$ and $F_{AC} = 105 \text{ lb}$, the above assumption is correct.
3–56.

The 25-kg flowerpot is supported at A by the three cords. Determine the force acting in each cord for equilibrium.

**SOLUTION**

\[ F_{AD} = F_{AD} (\sin 30^\circ \mathbf{i} - \cos 30^\circ \sin 60^\circ \mathbf{j} + \cos 30^\circ \cos 60^\circ \mathbf{k}) \]

\[ = 0.5F_{AD} \mathbf{i} - 0.75F_{AD} \mathbf{j} + 0.4330F_{AD} \mathbf{k} \]

\[ F_{AC} = F_{AC} (\sin 30^\circ \mathbf{i} - \cos 30^\circ \sin 60^\circ \mathbf{j} + \cos 30^\circ \cos 60^\circ \mathbf{k}) \]

\[ = -0.5F_{AC} \mathbf{i} - 0.75F_{AC} \mathbf{j} + 0.4330F_{AC} \mathbf{k} \]

\[ F_{AB} = F_{AB} (\sin 45^\circ \mathbf{j} + \cos 45^\circ \mathbf{k}) = 0.7071F_{AB} \mathbf{j} + 0.7071F_{AB} \mathbf{k} \]

\[ \mathbf{F} = -25(9.81) \mathbf{k} = [-245.25 \mathbf{k}] \text{N} \]

\[ \sum \mathbf{F} = 0; \quad F_{AD} + F_{AB} + F_{AC} + \mathbf{F} = 0 \]

\[ (0.5F_{AD} \mathbf{i} - 0.75F_{AD} \mathbf{j}) + 0.4330F_{AD} \mathbf{k} + (0.7071F_{AB} \mathbf{j} + 0.7071F_{AB} \mathbf{k}) \]

\[ + (-0.5F_{AC} \mathbf{i} - 0.75F_{AC} \mathbf{j} + 0.4330F_{AC} \mathbf{k}) + (-245.25 \mathbf{k}) = 0 \]

\[ (0.5F_{AD} - 0.5F_{AC}) \mathbf{i} + (-0.75F_{AD} + 0.7071F_{AB} - 0.75F_{AC}) \mathbf{j} \]

\[ + (0.4330F_{AD} + 0.7071F_{AB} + 0.4330F_{AC} - 245.25) \mathbf{k} = 0 \]

Thus,

\[ \sum F_x = 0; \quad 0.5F_{AD} - 0.5F_{AC} = 0 \] \[ \text{[1]} \]

\[ \sum F_y = 0; \quad -0.75F_{AD} + 0.7071F_{AB} - 0.75F_{AC} = 0 \] \[ \text{[2]} \]

\[ \sum F_z = 0; \quad 0.4330F_{AD} + 0.7071F_{AB} + 0.4330F_{AC} - 245.25 = 0 \] \[ \text{[3]} \]

Solving Eqs. [1], [2], and [3] yields:

\[ F_{AD} = F_{AC} = 104 \text{ N} \quad F_{AB} = 220 \text{ N} \]

Ans.
If each cord can sustain a maximum tension of 50 N before it fails, determine the greatest weight of the flowerpot the cords can support.

**SOLUTION**

\[ \mathbf{F}_{AD} = F_{AD} (\sin 30^\circ \mathbf{i} - \cos 30^\circ \sin 60^\circ \mathbf{j} + \cos 30^\circ \cos 60^\circ \mathbf{k}) \]
\[ = 0.5F_{AD} \mathbf{i} - 0.75F_{AD} \mathbf{j} + 0.4330F_{AD} \mathbf{k} \]

\[ \mathbf{F}_{AC} = F_{AC} (-\sin 30^\circ \mathbf{i} - \cos 30^\circ \sin 60^\circ \mathbf{j} + \cos 30^\circ \cos 60^\circ \mathbf{k}) \]
\[ = -0.5F_{AC} \mathbf{i} - 0.75F_{AC} \mathbf{j} + 0.4330F_{AC} \mathbf{k} \]

\[ \mathbf{F}_{AB} = F_{AB} (\sin 45^\circ \mathbf{j} + \cos 45^\circ \mathbf{k}) = 0.7071F_{AB} \mathbf{j} + 0.7071F_{AB} \mathbf{k} \]

\[ \mathbf{W} = -W \mathbf{k} \]

\[ \sum F_x = 0; \quad 0.5F_{AD} - 0.5F_{AC} = 0 \]

\[ F_{AD} = F_{AC} \]  \hspace{1cm} (1)

\[ \sum F_y = 0; \quad -0.75F_{AD} + 0.7071F_{AB} - 0.75F_{AC} = 0 \]

\[ 0.7071F_{AB} = 1.5F_{AC} \]  \hspace{1cm} (2)

\[ \sum F_z = 0; \quad 0.4330F_{AD} + 0.7071F_{AB} + 0.4330F_{AC} - W = 0 \]

\[ 0.8660F_{AC} + 1.5F_{AC} - W = 0 \]

\[ 2.366F_{AC} = W \]

Assume \( F_{AC} = 50 \text{ N} \) then
\[ F_{AB} = \frac{1.5(50)}{0.7071} = 106.07 \text{ N} > 50 \text{ N} \text{ (N. G!)} \]

Assume \( F_{AB} = 50 \text{ N} \). Then
\[ F_{AC} = \frac{0.7071(50)}{1.5} = 23.57 \text{ N} < 50 \text{ N} \text{ (O. K!)} \]

Thus,
\[ W = 2.366(23.57) = 55.767 = 55.8 \text{ N} \]

**Ans:**

\[ W = 55.8 \text{ N} \]
3–58.
Determine the tension developed in the three cables required to support the traffic light, which has a mass of 15 kg. Take $h = 4$ m.

**SOLUTION**

\[ \mathbf{u}_{AB} = \left\{ \frac{3}{5} \mathbf{i} + \frac{4}{5} \mathbf{j} \right\} \]

\[ \mathbf{u}_{AC} = \left\{ \frac{-6}{7} \mathbf{i} - \frac{3}{7} \mathbf{j} + \frac{2}{7} \mathbf{k} \right\} \]

\[ \mathbf{u}_{AD} = \left\{ \frac{4}{5} \mathbf{i} - \frac{3}{5} \mathbf{j} \right\} \]

\[ \Sigma F_x = 0; \quad \frac{3}{5} F_{AB} - \frac{6}{7} F_{AC} + \frac{4}{5} F_{AD} = 0 \]

\[ \Sigma F_y = 0; \quad \frac{4}{5} F_{AB} - \frac{3}{7} F_{AC} - \frac{3}{5} F_{AD} = 0 \]

\[ \Sigma F_z = 0; \quad \frac{2}{7} F_{AC} - 15(9.81) = 0 \]

\[ F_{AB} = 441 \text{ N} \]

\[ F_{AC} = 515 \text{ N} \]

\[ F_{AD} = 221 \text{ N} \]

**Ans:**

\[ F_{AB} = 441 \text{ N} \]

\[ F_{AC} = 515 \text{ N} \]

\[ F_{AD} = 221 \text{ N} \]
Determine the tension developed in the three cables required to support the traffic light, which has a mass of 20 kg. Take $h = 3.5$ m.

**SOLUTION**

$$u_{AB} = \frac{3i + 4j + 0.5k}{\sqrt{3^2 + 4^2 + (0.5)^2}} = \frac{3i + 4j + 0.5k}{\sqrt{25.25}}$$

$$u_{AC} = \frac{-6i - 3j + 2.5k}{\sqrt{(-6)^2 + (-3)^2 + 2.5^2}} = \frac{-6i - 3j + 2.5k}{\sqrt{51.25}}$$

$$u_{AD} = \frac{4i - 3j + 0.5k}{\sqrt{4^2 + (-3)^2 + 0.5^2}} = \frac{4i - 3j + 0.5k}{\sqrt{25.25}}$$

$$\Sigma F_x = 0; \quad \frac{3}{\sqrt{25.25}} F_{AB} - \frac{6}{\sqrt{51.25}} F_{AC} + \frac{4}{\sqrt{25.25}} F_{AD} = 0$$

$$\Sigma F_y = 0; \quad \frac{4}{\sqrt{25.25}} F_{AB} - \frac{3}{\sqrt{51.25}} F_{AC} - \frac{3}{\sqrt{25.25}} F_{AD} = 0$$

$$\Sigma F_z = 0; \quad \frac{0.5}{\sqrt{25.25}} F_{AB} + \frac{2.5}{\sqrt{51.25}} F_{AC} + \frac{0.5}{\sqrt{25.25}} F_{AD} - 20(9.81) = 0$$

Solving,

$$F_{AB} = 348 \text{ N} \quad \text{Ans.}$$

$$F_{AC} = 413 \text{ N} \quad \text{Ans.}$$

$$F_{AD} = 174 \text{ N} \quad \text{Ans.}$$

Ans:

$$F_{AB} = 348 \text{ N}$$

$$F_{AC} = 413 \text{ N}$$

$$F_{AD} = 174 \text{ N}$$
The 800-lb cylinder is supported by three chains as shown. Determine the force in each chain for equilibrium. Take \( d = 1 \) ft.

**SOLUTION**

\[
\mathbf{F}_{AD} = \mathbf{F}_{AD}\left(\frac{-1\mathbf{j} + 1\mathbf{k}}{\sqrt{(-1)^2 + 1^2}}\right) = -0.7071\mathbf{F}_{AD}\mathbf{j} + 0.7071\mathbf{F}_{AD}\mathbf{k}
\]

\[
\mathbf{F}_{AC} = \mathbf{F}_{AC}\left(\frac{1\mathbf{i} + 1\mathbf{k}}{\sqrt{1^2 + 1^2}}\right) = 0.7071\mathbf{F}_{AC}\mathbf{i} + 0.7071\mathbf{F}_{AC}\mathbf{k}
\]

\[
\mathbf{F}_{AB} = \mathbf{F}_{AB}\left(\frac{-0.7071\mathbf{i} + 0.7071\mathbf{j} + 1\mathbf{k}}{\sqrt{(-0.7071)^2 + 0.7071^2 + 1^2}}\right)
\]

\[
= -0.5\mathbf{F}_{AB}\mathbf{i} + 0.5\mathbf{F}_{AB}\mathbf{j} + 0.7071\mathbf{F}_{AB}\mathbf{k}
\]

\[
\mathbf{F} = [-800\mathbf{k}] \text{lb}
\]

\[
\sum \mathbf{F} = \mathbf{0}; \quad \mathbf{F}_{AD} + \mathbf{F}_{AC} + \mathbf{F}_{AB} + \mathbf{F} = \mathbf{0}
\]

\[
(-0.7071\mathbf{F}_{AD}\mathbf{j} + 0.7071\mathbf{F}_{AD}\mathbf{k}) + (0.7071\mathbf{F}_{AC}\mathbf{i} + 0.7071\mathbf{F}_{AC}\mathbf{k})
\]

\[
+ (-0.5\mathbf{F}_{AB}\mathbf{i} + 0.5\mathbf{F}_{AB}\mathbf{j} + 0.7071\mathbf{F}_{AB}\mathbf{k}) + (-800\mathbf{k}) = \mathbf{0}
\]

\[
(0.7071\mathbf{F}_{AC} - 0.5\mathbf{F}_{AB})\mathbf{i} + (-0.7071\mathbf{F}_{AD} + 0.5\mathbf{F}_{AB})\mathbf{j}
\]

\[
+ (0.7071\mathbf{F}_{AD} + 0.7071\mathbf{F}_{AC} + 0.7071\mathbf{F}_{AB} - 800)\mathbf{k} = \mathbf{0}
\]

\[
\sum F_x = 0; \quad 0.7071\mathbf{F}_{AC} - 0.5\mathbf{F}_{AB} = 0 \quad \text{(1)}
\]

\[
\sum F_y = 0; \quad -0.7071\mathbf{F}_{AD} + 0.5\mathbf{F}_{AB} = 0 \quad \text{(2)}
\]

\[
\sum F_z = 0; \quad 0.7071\mathbf{F}_{AD} + 0.7071\mathbf{F}_{AC} + 0.7071\mathbf{F}_{AB} - 800 = 0 \quad \text{(3)}
\]

Solving Eqs. (1), (2), and (3) yields:

\[
\mathbf{F}_{AB} = 469 \text{ lb} \quad \mathbf{F}_{AC} = \mathbf{F}_{AD} = 331 \text{ lb} \quad \text{Ans.}
\]

*Ans:*

\[
\mathbf{F}_{AB} = 469 \text{ lb} \quad \mathbf{F}_{AC} = \mathbf{F}_{AD} = 331 \text{ lb}
\]
SOLUTION

Equations of Equilibrium. Referring to the FBD shown in Fig. a,

\[ \Sigma F_x = 0; \quad F_{AB}(\frac{4}{\sqrt{37}}) - F_{AC}(\frac{2}{\sqrt{38}}) - F_{AD}(\frac{4}{\sqrt{66}}) = 0 \]  
\[ \Sigma F_y = 0; \quad F_{AB}(\frac{4}{\sqrt{37}}) + F_{AC}(\frac{3}{\sqrt{38}}) - F_{AD}(\frac{5}{\sqrt{66}}) = 0 \]  
\[ \Sigma F_z = 0; \quad -F_{AB}(\frac{5}{\sqrt{37}}) - F_{AC}(\frac{5}{\sqrt{38}}) - F_{AD}(\frac{5}{\sqrt{66}}) + 800 = 0 \]  

Solving Eqs (1), (2) and (3)

\[ F_{AC} = 85.77 \text{ N} = 85.8 \text{ N} \]  
\[ F_{AB} = 577.73 \text{ N} = 578 \text{ N} \]  
\[ F_{AD} = 565.15 \text{ N} = 565 \text{ N} \]  

Ans.

Ans.

Ans.
3–62. If the maximum force in each rod can not exceed 1500 N, determine the greatest mass of the crate that can be supported.

**SOLUTION**

*Equations of Equilibrium.* Referring to the FBD shown in Fig. a,

\[ \Sigma F_x = 0; \quad F_{OA}\left(\frac{2}{\sqrt{14}}\right) - F_{OC}\left(\frac{3}{\sqrt{22}}\right) + F_{OB}\left(\frac{1}{3}\right) = 0 \]  
\[ \Sigma F_y = 0;\quad -F_{OA}\left(\frac{3}{\sqrt{14}}\right) + F_{OC}\left(\frac{2}{\sqrt{22}}\right) + F_{OB}\left(\frac{2}{3}\right) = 0 \]  
\[ \Sigma F_z = 0;\quad F_{OA}\left(\frac{1}{\sqrt{14}}\right) + F_{OC}\left(\frac{3}{\sqrt{22}}\right) - F_{OB}\left(\frac{2}{3}\right) - m(9.81) = 0 \]

Solving Eqs (1), (2) and (3),

\[ F_{OC} = 16.95m \quad F_{OA} = 15.46m \quad F_{OB} = 7.745m \]

Since link OC subjected to the greatest force, it will reach the limiting force first, that is \( F_{OC} = 1500 \) N. Then

\[ 1500 = 16.95m \]
\[ m = 88.48 \text{ kg} = 88.5 \text{ kg} \]

*Ans.*

\[ m = 88.5 \text{ kg} \]
3–63.

The crate has a mass of 130 kg. Determine the tension developed in each cable for equilibrium.

SOLUTION

Equations of Equilibrium. Referring to the FBD shown in Fig. a,

\[
\sum F_x = 0; \quad F_{AD} \left( \frac{2}{\sqrt{6}} \right) - F_{BD} \left( \frac{2}{\sqrt{6}} \right) - F_{CD} \left( \frac{2}{3} \right) = 0 \quad (1)
\]

\[
\sum F_y = 0; \quad -F_{AD} \left( \frac{1}{\sqrt{6}} \right) - F_{BD} \left( \frac{1}{\sqrt{6}} \right) + F_{CD} \left( \frac{2}{3} \right) = 0 \quad (2)
\]

\[
\sum F_z = 0; \quad F_{AD} \left( \frac{1}{\sqrt{6}} \right) + F_{BD} \left( \frac{1}{\sqrt{6}} \right) + F_{CD} \left( \frac{1}{3} \right) - 130(9.81) = 0 \quad (3)
\]

Solving Eqs (1), (2) and (3)

\[
F_{AD} = 1561.92 \text{ N} = 1.56 \text{ kN} \quad \text{Ans.}
\]

\[
F_{BD} = 520.64 \text{ N} = 521 \text{ N} \quad \text{Ans.}
\]

\[
F_{CD} = 1275.3 \text{ N} = 1.28 \text{ kN} \quad \text{Ans.}
\]
If cable $AD$ is tightened by a turnbuckle and develops a tension of 1300 lb, determine the tension developed in cables $AB$ and $AC$ and the force developed along the antenna tower $AE$ at point $A$.

**SOLUTION**

**Force Vectors:** We can express each of the forces on the free-body diagram shown in Fig. a in Cartesian vector form as

$$
\mathbf{F}_{AB} = 2354 \text{ lb} = 2.35 \text{ kip}
$$

$$
\mathbf{F}_{AC} = 538 \text{ lb}
$$

$$
\mathbf{F}_{AB} = 808 \text{ lb}
$$

Equations of Equilibrium: Equilibrium requires

$$
g \mathbf{F} = 0: \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F}_{AE} = 0
$$

$$
\left(\frac{2}{7} \mathbf{F}_{AB} \mathbf{i} + \frac{3}{7} \mathbf{F}_{AB} \mathbf{j} - \frac{6}{7} \mathbf{F}_{AB} \mathbf{k}\right) + \left(\frac{3}{7} \mathbf{F}_{AC} \mathbf{i} - \frac{2}{7} \mathbf{F}_{AC} \mathbf{j} - \frac{6}{7} \mathbf{F}_{AC} \mathbf{k}\right) + \left(500 \mathbf{j} - 1200 \mathbf{k}\right) = 0
$$

$$
\left(\frac{2}{7} \mathbf{F}_{AB} - \frac{3}{7} \mathbf{F}_{AC}\right) \mathbf{i} + \left(\frac{3}{7} \mathbf{F}_{AB} - \frac{2}{7} \mathbf{F}_{AC} + 500\right) \mathbf{j} + \left(\frac{6}{7} \mathbf{F}_{AB} - \frac{6}{7} \mathbf{F}_{AC} + \mathbf{F}_{AE} - 1200\right) \mathbf{k} = 0
$$

Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components yields

$$
\frac{2}{7} \mathbf{F}_{AB} - \frac{3}{7} \mathbf{F}_{AC} = 0
$$

$$
-\frac{3}{7} \mathbf{F}_{AB} - \frac{2}{7} \mathbf{F}_{AC} + 500 = 0
$$

$$
-\frac{6}{7} \mathbf{F}_{AB} - \frac{6}{7} \mathbf{F}_{AC} + \mathbf{F}_{AE} - 1200 = 0
$$

Solving Eqs. (1) through (3) yields

$F_{AB} = 808 \text{ lb}$

$F_{AC} = 538 \text{ lb}$

$F_{AE} = 2354 \text{ lb} = 2.35 \text{ kip}$

Ans:

$F_{AB} = 808 \text{ lb}$

$F_{AC} = 538 \text{ lb}$

$F_{AE} = 2.35 \text{ kip}$
If the tension developed in either cable $AB$ or $AC$ cannot exceed 1000 lb, determine the maximum tension that can be developed in cable $AD$ when it is tightened by the turnbuckle. Also, what is the force developed along the antenna tower at point $A$?

**SOLUTION**

**Force Vectors:** We can express each of the forces on the free-body diagram shown in Fig. 3 in Cartesian vector form as

$$
F_{AB} = F_{AB}\left(\frac{(10 - 0)i + (-15 - 0)j + (-30 - 0)k}{\sqrt{(10 - 0)^2 + (-15 - 0)^2 + (-30 - 0)^2}}\right) = \frac{2}{7}F_{AB}i - \frac{3}{7}F_{AB}j - \frac{6}{7}F_{AB}k
$$

$$
F_{AC} = F_{AC}\left(\frac{(-15 - 0)i + (-10 - 0)j + (-30 - 0)k}{\sqrt{(-15 - 0)^2 + (-10 - 0)^2 + (-30 - 0)^2}}\right) = -\frac{3}{7}F_{AC}i + \frac{2}{7}F_{AC}j - \frac{6}{7}F_{AC}k
$$

$$
F_{AD} = F\left(\frac{(0 - 0)i + (12.5 - 0)j + (-30 - 0)k}{\sqrt{(0 - 0)^2 + (12.5 - 0)^2 + (-30 - 0)^2}}\right) = \frac{5}{13}Fj - \frac{12}{13}Fk
$$

$$
F_{AE} = F_{AE}k
$$

**Equations of Equilibrium:** Equilibrium requires

$$
g F = 0: \quad F_{AB} + F_{AC} + F_{AD} + F_{AE} = 0
$$

$$
\left(\frac{2}{7}F_{AB}i - \frac{3}{7}F_{AB}j - \frac{6}{7}F_{AB}k\right) + \left(-\frac{3}{7}F_{AC}i + \frac{2}{7}F_{AC}j - \frac{6}{7}F_{AC}k\right) + \left(\frac{5}{13}Fj - \frac{12}{13}Fk\right) + F_{AE}k = 0
$$

Equating the $i$, $j$, and $k$ components yields

$$
\frac{2}{7}F_{AB} - \frac{3}{7}F_{AC} = 0 \quad (1)
$$

$$
-\frac{3}{7}F_{AB} + \frac{2}{7}F_{AC} + \frac{5}{13}F = 0 \quad (2)
$$

$$
-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} - \frac{12}{13}F + F_{AE} = 0 \quad (3)
$$

Let us assume that cable $AB$ achieves maximum tension first. Substituting $F_{AB} = 1000$ lb into Eqs. (1) through (3) and solving yields

$$
F_{AC} = 666.67 \text{ lb}
$$

$$
F_{AE} = 2914 \text{ lb} = 2.91 \text{ kip}
$$

$$
F = 1610 \text{ lb} = 1.61 \text{ kip}
$$

**Ans.**

Since $F_{AC} = 666.67 \text{ lb} < 1000$ lb, our assumption is correct.
Determine the tension developed in cables $AB$, $AC$, and $AD$ required for equilibrium of the 300-lb crate.

**SOLUTION**

**Force Vectors:** We can express each of the forces on the free-body diagram shown in Fig. (a) in Cartesian vector form as

\[ F_{AB} = F_{AB} \left[ \frac{(-2 - 0)\mathbf{i} + (1 - 0)\mathbf{j} + (2 - 0)\mathbf{k}}{\sqrt{(-2 - 0)^2 + (1 - 0)^2 + (2 - 0)^2}} \right] = \frac{2}{3} F_{AB} \mathbf{i} + \frac{1}{3} F_{AB} \mathbf{j} + \frac{2}{3} F_{AB} \mathbf{k} \]

\[ F_{AC} = F_{AC} \left[ \frac{(-2 - 0)\mathbf{i} + (-2 - 0)\mathbf{j} + (1 - 0)\mathbf{k}}{\sqrt{(-2 - 0)^2 + (-2 - 0)^2 + (1 - 0)^2}} \right] = -\frac{2}{3} F_{AC} \mathbf{i} - \frac{2}{3} F_{AC} \mathbf{j} + \frac{1}{3} F_{AC} \mathbf{k} \]

\[ F_{AD} = F_{AD} \mathbf{i} \]

\[ W = [-300\mathbf{k}] \text{ lb} \]

**Equations of Equilibrium:** Equilibrium requires

\[ \Sigma \mathbf{F} = 0; \quad F_{AB} + F_{AC} + F_{AD} + W = 0 \]

\[ \left( \frac{2}{3} F_{AB} \mathbf{i} + \frac{1}{3} F_{AB} \mathbf{j} + \frac{2}{3} F_{AB} \mathbf{k} \right) + \left( -\frac{2}{3} F_{AC} \mathbf{i} - \frac{2}{3} F_{AC} \mathbf{j} + \frac{1}{3} F_{AC} \mathbf{k} \right) + F_{AD} \mathbf{i} + (-300\mathbf{k}) = 0 \]

\[ \left( \frac{2}{3} F_{AB} - \frac{2}{3} F_{AC} + F_{AD} \right) \mathbf{i} + \left( \frac{1}{3} F_{AB} - \frac{2}{3} F_{AC} \right) \mathbf{j} + \left( \frac{2}{3} F_{AB} + \frac{1}{3} F_{AC} - 300 \right) \mathbf{k} = 0 \]

Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components yields

\[ -\frac{2}{3} F_{AB} - \frac{2}{3} F_{AC} + F_{AD} = 0 \]  \hspace{1cm} (1)

\[ \frac{1}{3} F_{AB} - \frac{2}{3} F_{AC} = 0 \]  \hspace{1cm} (2)

\[ \frac{2}{3} F_{AB} + \frac{1}{3} F_{AC} - 300 = 0 \]  \hspace{1cm} (3)

Solving Eqs. (1) through (3) yields

\[ F_{AB} = 360 \text{ lb} \]

\[ F_{AC} = 180 \text{ lb} \]

\[ F_{AD} = 360 \text{ lb} \]

**Ans:**

\[ F_{AB} = 360 \text{ lb} \]

\[ F_{AC} = 180 \text{ lb} \]

\[ F_{AD} = 360 \text{ lb} \]
3–67.

Determine the maximum weight of the crate so that the tension developed in any cable does not exceed 450 lb.

**SOLUTION**

**Force Vectors:** We can express each of the forces on the free-body diagram shown in Fig. (a) in Cartesian vector form as

\[
\mathbf{F}_{AB} = \frac{\mathbf{F}_{AB}}{\sqrt{(-2-0)^2 + (1-0)^2 + (2-0)^2}} = \frac{2}{3} \mathbf{F}_{AB} \mathbf{i} + \frac{1}{3} \mathbf{F}_{AB} \mathbf{j} + \frac{2}{3} \mathbf{F}_{AB} \mathbf{k}
\]

\[
\mathbf{F}_{AC} = \frac{\mathbf{F}_{AC}}{\sqrt{(-2-0)^2 + (-2-0)^2 + (1-0)^2}} = -\frac{2}{3} \mathbf{F}_{AC} \mathbf{i} - \frac{2}{3} \mathbf{F}_{AC} \mathbf{j} + \frac{1}{3} \mathbf{F}_{AC} \mathbf{k}
\]

\[
\mathbf{F}_{AD} = \mathbf{F}_{AD} \mathbf{i}
\]

\[
\mathbf{W} = -W \mathbf{k}
\]

**Equations of Equilibrium:** Equilibrium requires

\[
\sum \mathbf{F} = 0; \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}
\]

\[
\left(\frac{2}{3} \mathbf{F}_{AB} \mathbf{i} + \frac{1}{3} \mathbf{F}_{AB} \mathbf{j} + \frac{2}{3} \mathbf{F}_{AB} \mathbf{k}\right) + \left(-\frac{2}{3} \mathbf{F}_{AC} \mathbf{i} - \frac{2}{3} \mathbf{F}_{AC} \mathbf{j} + \frac{1}{3} \mathbf{F}_{AC} \mathbf{k}\right) + \mathbf{F}_{AD} \mathbf{i} + (-W \mathbf{k}) = \mathbf{0}
\]

\[
\left(\frac{2}{3} \mathbf{F}_{AB} - \frac{2}{3} \mathbf{F}_{AC} + \mathbf{F}_{AD}\right) \mathbf{i} + \left(\frac{1}{3} \mathbf{F}_{AB} - \frac{2}{3} \mathbf{F}_{AC}\right) \mathbf{j} + \left(\frac{2}{3} \mathbf{F}_{AB} + \frac{1}{3} \mathbf{F}_{AC} - W\right) \mathbf{k} = \mathbf{0}
\]

Equating the \(\mathbf{i}, \mathbf{j}, \text{and } \mathbf{k}\) components yields

\[
\frac{2}{3} \mathbf{F}_{AB} - \frac{2}{3} \mathbf{F}_{AC} + \mathbf{F}_{AD} = 0 \quad \text{(1)}
\]

\[
\frac{1}{3} \mathbf{F}_{AB} - \frac{2}{3} \mathbf{F}_{AC} = 0 \quad \text{(2)}
\]

\[
\frac{2}{3} \mathbf{F}_{AB} + \frac{1}{3} \mathbf{F}_{AC} - W = 0 \quad \text{(3)}
\]

Let us assume that cable \(AB\) achieves maximum tension first. Substituting \(\mathbf{F}_{AB} = 450 \text{ lb}\) into Eqs. (1) through (3) and solving, yields

\[
\mathbf{F}_{AC} = 225 \text{ lb} \quad \mathbf{F}_{AD} = 450 \text{ lb} \quad \mathbf{W} = 375 \text{ lb}
\]

Ans.:

\[
\mathbf{W} = 375 \text{ lb}
\]