4–1. 

If \( \mathbf{A} \), \( \mathbf{B} \), and \( \mathbf{D} \) are given vectors, prove the distributive law for the vector cross product, i.e., 

\[
\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D}).
\]

**SOLUTION**

Consider the three vectors; with \( \mathbf{A} \) vertical.

Note \( obd \) is perpendicular to \( \mathbf{A} \).

\[
\begin{align*}
    od &= |\mathbf{A} \times (\mathbf{B} + \mathbf{D})| = |\mathbf{A}||\mathbf{B} + \mathbf{D}| \sin \theta_3 \\
    ob &= |\mathbf{A} \times \mathbf{B}| = |\mathbf{A}||\mathbf{B}| \sin \theta_1 \\
    bd &= |\mathbf{A} \times \mathbf{D}| = |\mathbf{A}||\mathbf{D}| \sin \theta_2
\end{align*}
\]

Also, these three cross products all lie in the plane \( obd \) since they are all perpendicular to \( \mathbf{A} \). As noted the magnitude of each cross product is proportional to the length of each side of the triangle.

The three vector cross products also form a closed triangle \( o'b'd' \) which is similar to triangle \( obd \). Thus from the figure,

\[
\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})
\]

Note also,

\[
\begin{align*}
    \mathbf{A} &= A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k} \\
    \mathbf{B} &= B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k} \\
    \mathbf{D} &= D_x \mathbf{i} + D_y \mathbf{j} + D_z \mathbf{k}
\end{align*}
\]

\[
\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = \begin{vmatrix}
    \mathbf{i} & \mathbf{j} & \mathbf{k} \\
    A_x & A_y & A_z \\
    B_x + D_x & B_y + D_y & B_z + D_z
\end{vmatrix}
\]

\[
= [A_y (B_z + D_z) - A_z (B_y + D_y)]\mathbf{i} \\
- [A_x (B_z + D_z) - A_z (B_x + D_x)]\mathbf{j} \\
+ [A_x (B_y + D_y) - A_y (B_x + D_x)]\mathbf{k}
\]

\[
= [(A_x B_z - A_z B_y)\mathbf{i} - (A_x B_z - A_z B_y)]\mathbf{j} + (A_x B_y - A_y B_x)\mathbf{k}
\]

\[
+ [(A_x D_z - A_z D_y)\mathbf{i} - (A_x D_z - A_z D_y)]\mathbf{j} + (A_x D_y - A_y D_x)\mathbf{k}
\]

\[
= \begin{vmatrix}
    \mathbf{i} & \mathbf{j} & \mathbf{k} \\
    A_x & A_y & A_z \\
    B_x & B_y & B_z
\end{vmatrix}
+ \begin{vmatrix}
    \mathbf{i} & \mathbf{j} & \mathbf{k} \\
    A_x & A_y & A_z \\
    D_x & D_y & D_z
\end{vmatrix}
\]

\[
= (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})
\]

(QED)
Prove the triple scalar product identity
\[ \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}. \]

**SOLUTION**

As shown in the figure
\[ \text{Area} = B(C \sin \theta) = |B \times C| \]
Thus,
\[ \text{Volume of parallelepiped is } |B \times C||h| \]
But,
\[ |h| = |\mathbf{A} \cdot \mathbf{u}_{|B \times C|}| = \left| \mathbf{A} \cdot \frac{(B \times C)}{|B \times C|} \right| \]
Thus,
\[ \text{Volume} = |\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})| \]

Since \( |(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}| \) represents this same volume then
\[ \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} \quad \text{(QED)} \]

Also,
\[ LHS = \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) \]
\[ = (A_x i + A_y j + A_z k) \cdot \begin{vmatrix} i & j & k \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} \]
\[ = A_x (B_y C_z - B_z C_y) - A_y (B_z C_x - B_x C_z) + A_z (B_x C_y - B_y C_x) \]
\[ = A_x B_y C_z - A_x B_z C_y - A_y B_x C_z + A_y B_z C_x + A_z B_x C_y - A_z B_y C_x \]

\[ RHS = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} \]
\[ = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \cdot (C_x i + C_y j + C_z k) \]
\[ = (A_y B_z - A_z B_y) - C_z (A_x B_z - A_z B_x) + C_y (A_x B_y - A_y B_x) \]
\[ = A_x B_y C_z - A_x B_z C_y - A_y B_x C_z + A_y B_z C_x + A_z B_x C_y - A_z B_y C_x \]

Thus, \( LHS = RHS \)
\[ \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} \quad \text{(QED)} \]
4–3.

Given the three nonzero vectors \( \mathbf{A} \), \( \mathbf{B} \), and \( \mathbf{C} \), show that if \( \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0 \), the three vectors must lie in the same plane.

**SOLUTION**

Consider,

\[
|\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})| = |\mathbf{A}| |\mathbf{B} \times \mathbf{C}| \cos \theta \\
= (|\mathbf{A}| \cos \theta)|\mathbf{B} \times \mathbf{C}| \\
= |h| |\mathbf{B} \times \mathbf{C}| \\
= BC |h| \sin \phi \\
= \text{volume of parallelepiped}.
\]

If \( \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0 \), then the volume equals zero, so that \( \mathbf{A} \), \( \mathbf{B} \), and \( \mathbf{C} \) are coplanar.
*4-4.*

Determine the moment about point $A$ of each of the three forces acting on the beam.

**SOLUTION**

$\zeta + (M_{F_1})_A = -375(8)$

$$= -3000 \text{ lb} \cdot \text{ ft} = 3.00 \text{ kip} \cdot \text{ ft} \ (\text{Clockwise})$$

$\zeta + (M_{F_2})_A = -500 \left( \frac{4}{5} \right) (14)$

$$= -5600 \text{ lb} \cdot \text{ ft} = 5.60 \text{ kip} \cdot \text{ ft} \ (\text{Clockwise})$$

$\zeta + (M_{F_3})_A = -160(\cos 30^\circ)(19) + 160 \sin 30^\circ (0.5)$

$$= -2593 \text{ lb} \cdot \text{ ft} = 2.59 \text{ kip} \cdot \text{ ft} \ (\text{Clockwise})$$

**Ans:**

$(M_{F_1})_A = 3.00 \text{ kip} \cdot \text{ ft} \ (\text{Clockwise})$

$(M_{F_2})_A = 5.60 \text{ kip} \cdot \text{ ft} \ (\text{Clockwise})$

$(M_{F_3})_A = 2.59 \text{ kip} \cdot \text{ ft} \ (\text{Clockwise})$
4–5.

Determine the moment about point B of each of the three forces acting on the beam.

**SOLUTION**

\[
\sum + (M_{F_1})_B = 375(11) \]

\[
= 4125 \text{ lb} \cdot \text{ft} = 4.125 \text{ kip} \cdot \text{ft} \quad (\text{Counterclockwise})
\]

\[
\sum + (M_{F_2})_B = 500 \left( \frac{-4}{5} \right)(5) \]

\[
= 2000 \text{ lb} \cdot \text{ft} = 2.00 \text{ kip} \cdot \text{ft} \quad (\text{Counterclockwise})
\]

\[
\sum + (M_{F_3})_B = 160 \sin 30^\circ (0.5) - 160 \cos 30^\circ (0) \]

\[
= 40.0 \text{ lb} \cdot \text{ft} \quad (\text{Counterclockwise})
\]

**Ans:**

\[
(M_{F_1})_B = 4.125 \text{ kip} \cdot \text{ft} \]

\[
(M_{F_2})_B = 2.00 \text{ kip} \cdot \text{ft} \]

\[
(M_{F_3})_B = 40.0 \text{ lb} \cdot \text{ft} \]
4–6.

The crowbar is subjected to a vertical force of $P = 25$ lb at the grip, whereas it takes a force of $F = 155$ lb at the claw to pull the nail out. Find the moment of each force about point $A$ and determine if $P$ is sufficient to pull out the nail. The crowbar contacts the board at point $A$.

**SOLUTION**

\[ \zeta + M_P = 25(14 \cos 20^\circ + 1.5 \sin 20^\circ) = 341 \text{ in} \cdot \text{lb (Counterclockwise)} \]

\[ \zeta + M_F = 155 \sin 60^\circ(3) = 403 \text{ in} \cdot \text{lb (Clockwise)} \]

Since $M_F > M_P$, $P = 25$ lb is *not sufficient* to pull out the nail. Ans.
4-7.

Determine the moment of each of the three forces about point $A$.

**SOLUTION**

The moment arm measured perpendicular to each force from point $A$ is

$$d_1 = 2 \sin 60^\circ = 1.732 \text{ m}$$
$$d_2 = 5 \sin 60^\circ = 4.330 \text{ m}$$
$$d_3 = 2 \sin 53.13^\circ = 1.60 \text{ m}$$

Using each force where $M_A = Fd$, we have

$$\zeta + (M_{F_1})_A = -250(1.732)$$
$$\zeta + (M_{F_1})_A = -433 \text{ N} \cdot \text{ m} = 433 \text{ N} \cdot \text{ m} \ (Clockwise)$$

$$\zeta + (M_{F_2})_A = -300(4.330)$$
$$\zeta + (M_{F_2})_A = -1299 \text{ N} \cdot \text{ m} = 1.30 \text{ kN} \cdot \text{ m} \ (Clockwise)$$

$$\zeta + (M_{F_3})_A = -500(1.60)$$
$$\zeta + (M_{F_3})_A = -800 \text{ N} \cdot \text{ m} = 800 \text{ N} \cdot \text{ m} \ (Clockwise)$$

**Ans:**

$$(M_{F_1})_A = 433 \text{ N} \cdot \text{ m} \ (Clockwise)$$
$$(M_{F_2})_A = 1.30 \text{ kN} \cdot \text{ m} \ (Clockwise)$$
$$(M_{F_3})_A = 800 \text{ N} \cdot \text{ m}$$
*4–8.

Determine the moment of each of the three forces about point $B$.

**SOLUTION**

The forces are resolved into horizontal and vertical components as shown in Fig. $a$.

For $F_1$,

$$\zeta + M_B = 250 \cos 30^\circ (3) - 250 \sin 30^\circ (4)$$

$$= 149.51 \text{ N} \cdot \text{m} = 150 \text{ N} \cdot \text{m}$$

Ans.

For $F_2$,

$$\zeta + M_B = 300 \sin 60^\circ (0) + 300 \cos 60^\circ (4)$$

$$= 600 \text{ N} \cdot \text{m}$$

Ans.

Since the line of action of $F_3$ passes through $B$, its moment about point $B$ is zero. Thus

$$M_B = 0$$

Ans.
4-9. Determine the moment of each force about the bolt located at A. Take \( F_B = 40 \text{ lb}, F_C = 50 \text{ lb}. \)

**SOLUTION**

\[
\begin{align*}
  M_B &= 40 \cos 25^\circ (2.5) = 90.6 \text{ lb} \cdot \text{ft} \\
  M_C &= 50 \cos 30^\circ (3.25) = 141 \text{ lb} \cdot \text{ft}
\end{align*}
\]

Ans.

\[
\begin{align*}
  M_B &= 90.6 \text{ lb} \cdot \text{ft} \\
  M_C &= 141 \text{ lb} \cdot \text{ft}
\end{align*}
\]
4–10.

If $F_B = 30$ lb and $F_C = 45$ lb, determine the resultant moment about the bolt located at $A$.

**SOLUTION**

\[
\sum \mathbf{F}_\text{net} + M_A = 30 \cos 25^\circ (2.5) + 45 \cos 30^\circ (3.25)
\]

\[
= 195 \text{ lb} \cdot \text{ft}
\]

**Ans:**

\[M_A = 195 \text{ lb} \cdot \text{ft}\]
4–11.

The towline exerts a force of \( P = 6 \text{kN} \) at the end of the 8-m-long crane boom. If \( \theta = 30^\circ \), determine the placement \( x \) of the hook at \( B \) so that this force creates a maximum moment about point \( O \). What is this moment?

**SOLUTION**

In order to produce the maximum moment about point \( O \), \( P \) must act perpendicular to the boom’s axis \( OA \) as shown in Fig. a. Thus

\[
\zeta + (M_O)_{\text{max}} = 6 \times (8) = 48.0 \text{kN} \cdot \text{m} \text{ (counterclockwise)}
\]

Referring to the geometry of Fig. a,

\[
x = x' + x'' = \frac{8}{\cos 30^\circ} + \tan 30^\circ = 9.814 \text{ m} = 9.81 \text{ m}
\]

**Ans:**

\((M_O)_{\text{max}} = 48.0 \text{kN} \cdot \text{m} \)

\(x = 9.81 \text{ m} \)
The towline exerts a force of \( P = 6 \text{ kN} \) at the end of the 8-m-long crane boom. If \( x = 10 \text{ m} \), determine the position \( \theta \) of the boom so that this force creates a maximum moment about point \( O \). What is this moment?

**SOLUTION**

In order to produce the maximum moment about point \( O \), \( P \) must act perpendicular to the boom’s axis \( OA \) as shown in Fig. a. Thus,

\[
\zeta + (M_O)_{\text{max}} = 6 \times 8 = 48.0 \text{ kN} \cdot \text{m (counterclockwise)}
\]

Referring to the geometry of Fig. a,

\[
x = x' + x^*; \quad 10 = \frac{8}{\cos \theta} + \tan \theta \]
\[
x = x' + x^*; \quad 10 = \frac{8}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \]
\[
10 \cos \theta - \sin \theta = 8
\]
\[
\frac{10}{\sqrt{101}} \cos \theta - \frac{1}{\sqrt{101}} \sin \theta = \frac{8}{\sqrt{101}}
\]

From the geometry shown in Fig. b,

\[
\alpha = \tan^{-1}\left(\frac{1}{10}\right) = 5.711^\circ
\]
\[
\sin \alpha = \frac{1}{\sqrt{101}} \quad \cos \alpha = \frac{10}{\sqrt{101}}
\]

Then Eq (1) becomes

\[
\cos \theta \cos 5.711^\circ - \sin \theta \sin 5.711^\circ = \frac{8}{\sqrt{101}}
\]

Referring that \( \cos (\theta + 5.711^\circ) = \cos \theta \cos 5.711^\circ - \sin \theta \sin 5.711^\circ \)

\[
\cos (\theta + 5.711^\circ) = \frac{8}{\sqrt{101}}
\]
\[
\theta + 5.711^\circ = 37.247^\circ
\]
\[
\theta = 31.54^\circ = 31.5^\circ
\]

Ans:

\( (M_O)_{\text{max}} = 48.0 \text{ kN} \cdot \text{m (counterclockwise)} \)
\( \theta = 31.5^\circ \)
4–13.

The 20-N horizontal force acts on the handle of the socket wrench. What is the moment of this force about point B. Specify the coordinate direction angles $\alpha$, $\beta$, $\gamma$ of the moment axis.

**SOLUTION**

**Force Vector And Position Vector.** Referring to Fig. a,

\[ F = 20 \sin 60^\circ \hat{i} - \cos 60^\circ \hat{j} = [17.32 \hat{i} - 10 \hat{j}] \text{ N} \]

\[ \mathbf{r}_{BA} = [-0.01 \hat{i} + 0.2 \hat{j}] \text{ m} \]

**Moment of Force F about point B.**

\[
\mathbf{M}_B = \mathbf{r}_{BA} \times \mathbf{F} = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
-0.01 & 0.2 & 0 \\
17.32 & -10 & 0
\end{vmatrix}
\]

\[
= [-3.364 \hat{k}] \text{ N} \cdot \text{m}
\]

\[
= [-3.36 \hat{k}] \text{ N} \cdot \text{m}
\]

Ans.

Here the unit vector for $\mathbf{M}_B$ is $\mathbf{u} = -\hat{k}$. Thus, the coordinate direction angles of $\mathbf{M}_B$ are

\[ \alpha = \cos^{-1} 0 = 90^\circ \]

\[ \beta = \cos^{-1} 0 = 90^\circ \]

\[ \gamma = \cos^{-1} (-1) = 108^\circ \]

Ans.

Ans: $\mathbf{M}_B = [-3.36 \hat{k}] \text{ N} \cdot \text{m}$

$\alpha = 90^\circ$

$\beta = 90^\circ$

$\gamma = 180^\circ$
4–14.

The 20-N horizontal force acts on the handle of the socket wrench. Determine the moment of this force about point O. Specify the coordinate direction angles \( \alpha, \beta, \gamma \) of the moment axis.

**SOLUTION**

**Force Vector and Position Vector.** Referring to Fig. 4–a,

\[
\mathbf{F} = 20 \left( \sin 60^\circ \hat{i} - \cos 60^\circ \hat{j} \right) = 17.32 \hat{i} - 10 \hat{j} \text{ N}
\]

\[
\mathbf{r}_{OA} = \{-0.01 \hat{i} + 0.2 \hat{j} + 0.05 \hat{k}\} \text{ m}
\]

**Moment of \( \mathbf{F} \) About point O.**

\[
\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}
\]

\[
= \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
-0.01 & 0.2 & 0.05 \\
17.32 & -10 & 0 \\
\end{vmatrix}
\]

\[
= \{0.5 \hat{i} + 0.866 \hat{j} - 3.3641 \hat{k}\} \text{ N} \cdot \text{m}
\]

\[
= \{0.5 \hat{i} + 0.866 \hat{j} - 3.36 \hat{k}\} \text{ N} \cdot \text{m}
\]

Ans.

The magnitude of \( \mathbf{M}_O \) is

\[
M_O = \sqrt{(M_O)^2_x + (M_O)^2_y + (M_O)^2_z} = \sqrt{0.5^2 + 0.866^2 + (-3.3641)^2}
\]

\[
= 3.5096 \text{ N} \cdot \text{m}
\]

Thus, the coordinate direction angles of \( \mathbf{M}_O \) are

\[
\alpha = \cos^{-1} \left( \frac{(M_O)_x}{M_O} \right) = \cos^{-1} \left( \frac{0.5}{3.5096} \right) = 81.8^\circ = 81.8^\circ
\]

Ans.

\[
\beta = \cos^{-1} \left( \frac{(M_O)_y}{M_O} \right) = \cos^{-1} \left( \frac{0.8660}{3.5096} \right) = 75.7^\circ = 75.7^\circ
\]

Ans.

\[
\gamma = \cos^{-1} \left( \frac{(M_O)_z}{M_O} \right) = \cos^{-1} \left( \frac{-3.3641}{3.5096} \right) = 163.45^\circ = 163^\circ
\]

Ans.

Ans:

\[
\mathbf{M}_O = \{0.5 \hat{i} + 0.866 \hat{j} - 3.36 \hat{k}\} \text{ N} \cdot \text{m}
\]

\[
\alpha = 81.8^\circ
\]

\[
\beta = 75.7^\circ
\]

\[
\gamma = 163^\circ
\]
4–15.

Two men exert forces of $F = 80 \text{ lb}$ and $P = 50 \text{ lb}$ on the ropes. Determine the moment of each force about $A$. Which way will the pole rotate, clockwise or counterclockwise?

**SOLUTION**

$$\zeta + (M_A)_C = 80 \left( \frac{4}{5} \right)(12) = 768 \text{ lb} \cdot \text{ft}$$

$$\zeta + (M_A)_B = 50 \left( \cos 45^\circ \right)(18) = 636 \text{ lb} \cdot \text{ft}$$

Since $(M_A)_C > (M_A)_B$

Clockwise

**Ans:**

$(M_A)_C = 768 \text{ lb} \cdot \text{ft} \uparrow$

$(M_A)_B = 636 \text{ lb} \cdot \text{ft} \uparrow$

Clockwise
*4–16.

If the man at \( B \) exerts a force of \( P = 30 \text{ lb} \) on his rope, determine the magnitude of the force \( F \) the man at \( C \) must exert to prevent the pole from rotating, i.e., so the resultant moment about \( A \) of both forces is zero.

**SOLUTION**

\[
\zeta + 30 \cos 45^\circ)(18) = F \left( \frac{4}{5} \right)(12) = 0
\]

\[
F = 39.8 \text{ lb}
\]

Ans: \( F = 39.8 \text{ lb} \)
4–17.

The torque wrench $ABC$ is used to measure the moment or torque applied to a bolt when the bolt is located at $A$ and a force is applied to the handle at $C$. The mechanic reads the torque on the scale at $B$. If an extension $AO$ of length $d$ is used on the wrench, determine the required scale reading if the desired torque on the bolt at $O$ is to be $M$.

\[ \text{Ans:} \quad m = \frac{l}{d+l}M \]

**SOLUTION**

Moment at $A = m = Fl$

Moment at $O = M = (d+l)F$

\[
M = (d+l)\frac{m}{l}
\]

\[
m = \left( \frac{l}{d+l} \right) M \quad \text{Ans.}
\]
4–18.

The tongs are used to grip the ends of the drilling pipe $P$. Determine the torque (moment) $M_P$ that the applied force $F = 150 \text{ lb}$ exerts on the pipe about point $P$ as a function of $\theta$. Plot this moment $M_P$ versus $\theta$ for $0 \leq \theta \leq 90^\circ$.

**SOLUTION**

\[
M_P = 150 \cos \theta (43) + 150 \sin \theta (6)
\]
\[
= (6450 \cos \theta + 900 \sin \theta) \text{ lb} \cdot \text{in.}
\]
\[
= (537.5 \cos \theta + 75 \sin \theta) \text{ lb} \cdot \text{ft}
\]

\[
\frac{dM_P}{d\theta} = -537.5 \sin \theta + 75 \cos \theta = 0
\]
\[
\tan \theta = \frac{75}{537.5} \quad \theta = 7.943^\circ
\]

At $\theta = 7.943^\circ$, $M_P$ is maximum.

\[
(M_P)_{\text{max}} = 538 \cos 7.943^\circ + 75 \sin 7.943^\circ = 543 \text{ lb} \cdot \text{ft}
\]

Also $\quad (M_P)_{\text{max}} = 150 \text{ lb} \left( \left( \frac{43}{12} \right)^2 + \left( \frac{6}{12} \right)^2 \right) = 543 \text{ lb} \cdot \text{ft}$

\[
M_P = (537.5 \cos \theta + 75 \sin \theta) \text{ lb} \cdot \text{ft}
\]
4–19.

The tongs are used to grip the ends of the drilling pipe $P$. If a torque (moment) of $M_P = 800 \text{ lb}\cdot\text{ft}$ is needed at $P$ to turn the pipe, determine the cable force $F$ that must be applied to the tongs. Set $\theta = 30^\circ$.

**SOLUTION**

\[
M_P = F \cos 30^\circ(43) + F \sin 30^\circ(6)
\]
Set $M_P = 800(12) \text{ lb}\cdot\text{in.}$

\[
800(12) = F \cos 30^\circ(43) + F \sin 30^\circ(6)
\]

\[
F = 239 \text{ lb}
\]

Ans: $F = 239 \text{ lb}$
*4–20.

The handle of the hammer is subjected to the force of \( F = 20 \) lb. Determine the moment of this force about the point \( A \).

**SOLUTION**

Resolving the 20-lb force into components parallel and perpendicular to the hammer, Fig. \( a \), and applying the principle of moments,

\[
\zeta + M_A = -20 \cos 30^\circ (18) - 20 \sin 30^\circ (5)
\]

\[
= -361.77 \text{ lb} \cdot \text{in} = 362 \text{ lb} \cdot \text{in} \quad (\text{Clockwise}) \quad \text{Ans.}
\]
4–21.

In order to pull out the nail at B, the force $F$ exerted on the handle of the hammer must produce a clockwise moment of 500 lb · in. about point A. Determine the required magnitude of force $F$.

**SOLUTION**

Resolving force $F$ into components parallel and perpendicular to the hammer, Fig. a, and applying the principle of moments,

$$
\zeta + M_A = -500 = -F \cos 30^\circ(18) - F \sin 30^\circ(5)
$$

$$
F = 27.6 \text{ lb}
$$

AnS.

$F = 27.6 \text{ lb}$
4–22.

Old clocks were constructed using a fusee B to drive the gears and watch hands. The purpose of the fusee is to increase the leverage developed by the mainspring A as it uncoils and thereby loses some of its tension. The mainspring can develop a torque (moment) \( T_s = k\theta \), where \( k = 0.015 \text{ N} \cdot \text{m/rad} \) is the torsional stiffness and \( \theta \) is the angle of twist of the spring in radians. If the torque \( T_f \) developed by the fusee is to remain constant as the mainspring winds down, and \( x = 10 \text{ mm} \) when \( \theta = 4 \text{ rad} \), determine the required radius of the fusee when \( \theta = 3 \text{ rad} \).

**SOLUTION**

When \( \theta = 4 \text{ rad} \), \( r = 10 \text{ mm} \)

\[ T_s = 0.015(4) = 0.06 \text{ N} \cdot \text{m} \]

\[ F = \frac{0.06}{0.012} = 5 \text{ N} \]

\[ T_f = 5(0.010) = 0.05 \text{ N} \cdot \text{m} \] (constant)

When \( \theta = 3 \text{ rad} \),

\[ T_s = 0.015(3) = 0.045 \text{ N} \cdot \text{m} \]

\[ F = \frac{0.045}{0.012} = 3.75 \text{ N} \]

For the fusee require

\[ 0.05 = 3.75 \cdot r \]

\[ r = 0.0133 \text{ m} = 13.3 \text{ mm} \]

**Ans:**

\[ r = 13.3 \text{ mm} \]
4–23.

The tower crane is used to hoist the 2-Mg load upward at constant velocity. The 1.5-Mg jib \( BD \), 0.5-Mg jib \( BC \), and 6-Mg counterweight \( C \) have centers of mass at \( G_1 \), \( G_2 \), and \( G_3 \), respectively. Determine the resultant moment produced by the load and the weights of the tower crane jibs about point \( A \) and about point \( B \).

**SOLUTION**

Since the moment arms of the weights and the load measured to points \( A \) and \( B \) are the same, the resultant moments produced by the load and the weight about points \( A \) and \( B \) are the same.

\[
\zeta + (M_R)_A = (M_R)_B = \sum Fd; \quad (M_R)_A = (M_R)_B = 6000(9.81)(7.5) + 500(9.81)(4) - 1500(9.81)(9.5) - 2000(9.81)(12.5) = 76,027.5 \text{ N} \cdot \text{m} = 76.0 \text{ kN} \cdot \text{m (Counterclockwise)}
\]

**Ans:**

\[
(M_R)_A = (M_R)_B = 76.0 \text{ kN} \cdot \text{m}
\]
The tower crane is used to hoist a 2-Mg load upward at constant velocity. The 1.5-Mg jib $BD$ and 0.5-Mg jib $BC$ have centers of mass at $G_1$ and $G_2$, respectively. Determine the required mass of the counterweight $C$ so that the resultant moment produced by the load and the weight of the tower crane jibs about point $A$ is zero. The center of mass for the counterweight is located at $G_3$.

SOLUTION

\[ \zeta + (M_R)_A = \sum Fd; \quad 0 = M_C(9.81)(7.5) + 500(9.81)(4) - 1500(9.81)(9.5) - 2000(9.81)(12.5) \]

\[ M_C = 4966.67 \text{ kg} = 4.97 \text{ Mg} \]

**Ans:**

\[ M_C = 4.97 \text{ Mg} \]
4–25.

If the 1500-lb boom \( AB \), the 200-lb cage \( BCD \), and the 175-lb man have centers of gravity located at points \( G_1, G_2 \) and \( G_3 \), respectively, determine the resultant moment produced by each weight about point \( A \).

**SOLUTION**

**Moment of the weight of boom \( AB \) about point \( A \):**

\[
\zeta + (M_{AB})_A = -1500(10 \cos 75^\circ) = -3882.29 \text{ lb} \cdot \text{ft} \\
= 3.88 \text{ kip} \cdot \text{ft} \ (Clockwise) \quad \text{Ans.}
\]

**Moment of the weight of cage \( BCD \) about point \( A \):**

\[
\zeta + (M_{BCD})_A = -200(30 \cos 75^\circ + 2.5) = -2052.91 \text{ lb} \cdot \text{ft} \\
= 2.05 \text{ kip} \cdot \text{ft} \ (Clockwise) \quad \text{Ans.}
\]

**Moment of the weight of the man about point \( A \):**

\[
\zeta + (M_{man})_A = -175(30 \cos 75^\circ + 4.25) = -2102.55 \text{ lb} \cdot \text{ft} \\
= 2.10 \text{ kip} \cdot \text{ft} \ (Clockwise) \quad \text{Ans.}
\]

 Ans:

\[
(M_{AB})_A = 3.88 \text{ kip} \cdot \text{ft} \\
(M_{BCD})_A = 2.05 \text{ kip} \cdot \text{ft} \\
(M_{man})_A = 2.10 \text{ kip} \cdot \text{ft}
\]
4-26.

If the 1500-lb boom AB, the 200-lb cage BCD, and the 175-lb man have centers of gravity located at points $G_1$, $G_2$, and $G_3$, respectively, determine the resultant moment produced by all the weights about point $A$.

**SOLUTION**

Referring to Fig. $a$, the resultant moment of the weight about point $A$ is given by

$$
\zeta + (M_A)_A = \Sigma Fd; \quad (M_A)_A = -1500(10 \cos 75^\circ) - 200(30 \cos 75^\circ + 2.5) - 175(30 \cos 75^\circ + 4.25) \\
= -8037.75 \text{ lb} \cdot \text{ft} = 8.04 \text{ kip} \cdot \text{ft} \text{ (Clockwise)}
$$

**Ans:** 

$$(M_A)_A = 8.04 \text{ kip} \cdot \text{ft}$$
4–27.
Determine the moment of the force \( \mathbf{F} \) about point \( O \).
Express the result as a Cartesian vector.

**SOLUTION**

**Position Vector.** The coordinates of point \( A \) are \((1, -2, 6)\) m.
Thus,
\[
\mathbf{r}_{OA} = [i - 2j + 6k] \text{ m}
\]

**The moment of \( F \) About Point \( O \).**
\[
\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}
\]
\[
= \begin{vmatrix}
i & j & k \\
1 & -2 & 6 \\
-6 & 4 & 8
\end{vmatrix}
\]
\[
= [-40i - 44j - 8k] \text{ kN} \cdot \text{m}
\]

**Ans:**
\[
\mathbf{M}_O = [-40i - 44j - 8k] \text{ kN} \cdot \text{m}
\]
*4–28.

Determine the moment of the force \( \mathbf{F} \) about point \( P \). Express the result as a Cartesian vector.

**SOLUTION**

**Position Vector.** The coordinates of points \( A \) and \( P \) are \( A \) \((1, -2, 6) \) m and \( P \) \((0, 4, 3) \) m, respectively. Thus

\[
\mathbf{r}_{PA} = (1 - 0)i + (-2 - 4)j + (6 - 3)k = [i - 6j + 3k] \text{ m}
\]

**The moment of \( F \) About Point \( P \).**

\[
M_P = \mathbf{r}_{PA} \times \mathbf{F} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & -6 & 3 \\
-6 & 4 & 8
\end{vmatrix}
\]

\[
= [-60\mathbf{i} - 26\mathbf{j} - 32\mathbf{k}] \text{ kN} \cdot \text{m}
\]

**Ans.:**

\[
M_P = [-60\mathbf{i} - 26\mathbf{j} - 32\mathbf{k}] \text{ kN} \cdot \text{m}
\]
The force \( \mathbf{F} = [400\mathbf{i} - 100\mathbf{j} - 700\mathbf{k}] \text{ lb} \) acts at the end of the beam. Determine the moment of this force about point \( O \).

**SOLUTION**

**Position Vector.** The coordinates of point \( B \) are \( B(8, 0.25, 1.5) \text{ ft} \).

Thus,

\[
\mathbf{r}_{OB} = [8\mathbf{i} + 0.25\mathbf{j} + 1.5\mathbf{k}] \text{ ft}
\]

**Moments of \( \mathbf{F} \) About Point \( O \).**

\[
M_O = \mathbf{r}_{OB} \times \mathbf{F} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
8 & 0.25 & 1.5 \\
400 & -100 & -700
\end{vmatrix}
\]

\[
= [-25\mathbf{i} + 6200\mathbf{j} - 900\mathbf{k}] \text{ lb } \cdot \text{ ft}
\]

**Ans:**

\[
M_O = [-25\mathbf{i} + 6200\mathbf{j} - 900\mathbf{k}] \text{ lb } \cdot \text{ ft}
\]
4–30. The force \( \mathbf{F} = \{400\mathbf{i} - 100\mathbf{j} - 700\mathbf{k}\} \text{ lb} \) acts at the end of the beam. Determine the moment of this force about point \( A \).

**SOLUTION**

**Position Vector.** The coordinates of points \( A \) and \( B \) are \( A (0, 0, 1.5) \text{ ft} \) and \( B (8, 0.25, 1.5) \text{ ft} \), respectively. Thus,

\[
r_{AB} = (8 - 0)i + (0.25 - 0)j + (1.5 - 1.5)k
= 8i + 0.25j \text{ ft}
\]

**Moment of \( F \) About Point \( A \).**

\[
M_A = r_{AB} \times F
= \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
8 & 0.25 & 0 \\
400 & -100 & -700
\end{vmatrix}
\]

\[
= [-175\mathbf{i} + 5600\mathbf{j} - 900\mathbf{k}] \text{ lb} \cdot \text{ft}
\]

**Ans.**

\[
M_A = [-175\mathbf{i} + 5600\mathbf{j} - 900\mathbf{k}] \text{ lb} \cdot \text{ft}
\]
4-31.

Determine the moment of the force \( \mathbf{F} \) about point \( P \).
Express the result as a Cartesian vector.

**SOLUTION**

**Position Vector.** The coordinates of points \( A \) and \( P \) are \( A (3, 3, -1) \) m and \( P (-2, -3, 2) \) m respectively. Thus,

\[
\mathbf{r}_{PA} = [3 - (-2)] \mathbf{i} + [3 - (-3)] \mathbf{j} + (-1 - 2) \mathbf{k} \\
= [5 \mathbf{i} + 6 \mathbf{j} - 3 \mathbf{k}] \text{ m}
\]

**Moment of \( F \) About Point \( P \).**

\[
M_P = \mathbf{r}_{AP} \times \mathbf{F} \\
= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 6 & -3 \\ 2 & 4 & -6 \end{vmatrix} \\
= (-24 \mathbf{i} + 24 \mathbf{j} + 8 \mathbf{k}) \text{ kN \cdot m}
\]

Ans.

\[M_P = [-24 \mathbf{i} + 24 \mathbf{j} + 8 \mathbf{k}] \text{ kN \cdot m}\]
*4–32.

The pipe assembly is subjected to the force of \( \mathbf{F} = [600i + 800j - 500k] \) N. Determine the moment of this force about point \( A \).

**Solution**

**Position Vector.** The coordinates of point \( C \) are \( C (0.5, 0.7, -0.3) \) m. Thus

\[
\mathbf{r}_{AC} = [0.5i + 0.7j - 0.3k] \text{ m}
\]

**Moment of Force \( F \) About Point \( A \).**

\[
\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F}
\]

\[
= \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0.5 & 0.7 & -0.3 \\
600 & 800 & -500 \\
\end{vmatrix}
\]

\[
= [-110i + 70j - 20k] \text{ N} \cdot \text{m}
\]

Ans.

\[
\mathbf{M}_A = [-110i + 70j - 20k] \text{ N} \cdot \text{m}
\]
4–33.

The pipe assembly is subjected to the force of \( \mathbf{F} = (600 \mathbf{i} + 800 \mathbf{j} - 500 \mathbf{k}) \) N. Determine the moment of this force about point \( B \).

**SOLUTION**

**Position Vector.** The coordinates of points \( B \) and \( C \) are \( B (0.5, 0, 0) \) m and \( C (0.5, 0.7, -0.3) \) m, respectively. Thus,

\[
\mathbf{r}_{BC} = (0.5 - 0.5) \mathbf{i} + (0.7 - 0) \mathbf{j} + (-0.3 - 0) \mathbf{k} = \{0.7 \mathbf{j} - 0.3 \mathbf{k}\} \text{ m}
\]

**Moment of Force \( \mathbf{F} \) About Point \( B \).** Applying Eq. 4

\[
\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F}
\]

\[
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0 & 0.7 & -0.3 \\
600 & 800 & -500
\end{vmatrix}
\]

\[
= \{-110 \mathbf{i} - 180 \mathbf{j} - 420 \mathbf{k}\} \text{ N} \cdot \text{m}
\]

Ans.

\[\mathbf{M}_B = \{-110 \mathbf{i} - 180 \mathbf{j} - 420 \mathbf{k}\} \text{ N} \cdot \text{m}\]
4–34.
Determine the moment of the force of \( F = 600 \text{ N} \) about point \( A \).

**SOLUTION**

**Position Vectors And Force Vector.** The coordinates of points \( A, B \) and \( C \) are \( A (0, 0, 4) \text{ m}, B (4 \sin 45^\circ, 0, 4 \cos 45^\circ) \text{ m} \) and \( C (6, 6, 0) \text{ m} \), respectively. Thus

\[
\mathbf{r}_{AB} = (4 \sin 45^\circ - 0)i + (0 - 0)j + (4 \cos 45^\circ - 4)k = [2.8284i - 1.1716k] \text{ m}
\]

\[
\mathbf{r}_{AC} = (6 - 0)i + (6 - 0)j + (0 - 4)k = [6i + 6j - 4k] \text{ m}
\]

\[
\mathbf{r}_{BC} = (6 - 4 \sin 45^\circ)i + (6 - 0)j + (0 - 4 \cos 45^\circ)k = [3.1716i + 6j - 2.8284k] \text{ m}
\]

\[
\mathbf{F} = F \left( \frac{\mathbf{r}_{BC}}{\mathbf{r}_{BC}} \right) = 600 \left( \frac{3.1716i + 6j - 2.8284k}{\sqrt{3.1716^2 + 6^2 + (-2.8284)^2}} \right) = [258.82i + 489.63j - 230.81k] \text{ N}
\]

**The Moment of Force \( F \) About Point \( A \).**

\[
\mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} i & j & k \\ 2.8284 & 0 & -1.1716 \\ 258.82 & 489.63 & -230.81 \end{vmatrix}
\]

\[
= [573.64i + 349.62j + 1384.89k] \text{ N} \cdot \text{m}
\]

\[
= [574i + 350j + 1385k] \text{ N} \cdot \text{m}
\]

Ans.

**OR**

\[
\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F} = \begin{vmatrix} i & j & k \\ 6 & 6 & -4 \\ 258.82 & 489.63 & -230.81 \end{vmatrix}
\]

\[
= [573.64i + 349.62j + 1384.89k] \text{ N} \cdot \text{m}
\]

\[
= [574i + 350j + 1385k] \text{ N} \cdot \text{m}
\]

Ans.

\[
\mathbf{M}_A = [574i + 350j + 1385k] \text{ N} \cdot \text{m}
\]
**4–35.**

Determine the smallest force $F$ that must be applied along the rope in order to cause the curved rod, which has a radius of 4 m, to fail at the support $A$. This requires a moment of $M = 1500 \text{ N} \cdot \text{m}$ to be developed at $A$.

**SOLUTION**

**Position Vectors And Force Vector.** The coordinates of points $A$, $B$ and $C$ are $A (0, 0, 4)$ m, $B (4 \sin 45^\circ, 0, 4 \cos 45^\circ)$ m and $C (6, 6, 0)$ m, respectively.

Thus,

$$
\mathbf{r}_{AB} = (4 \sin 45^\circ - 0)\mathbf{i} + (0 - 0)\mathbf{j} + (4 \cos 45^\circ - 4)\mathbf{k} = [2.8284\mathbf{i} - 1.1716\mathbf{k}] \text{ m}
$$

$$
\mathbf{r}_{AC} = (6 - 0)\mathbf{i} + (6 - 0)\mathbf{j} + (0 - 4)\mathbf{k} = [6\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}] \text{ m}
$$

$$
\mathbf{r}_{BC} = (6 - 4 \sin 45^\circ)\mathbf{i} + (6 - 0)\mathbf{j} + (0 - 4 \cos 45^\circ)\mathbf{k} = [3.1716\mathbf{i} + 6\mathbf{j} - 2.8284\mathbf{k}] \text{ m}
$$

$$
\mathbf{F} = \mathbf{F} \left( \frac{\mathbf{r}_{BC}}{r_{BC}} \right) = \mathbf{F} \left( \frac{3.1716\mathbf{i} + 6\mathbf{j} - 2.8284\mathbf{k}}{\sqrt{3.1716^2 + 6^2 + (-2.8284)^2}} \right)
$$

$$
= 0.4314\mathbf{F}\mathbf{i} + 0.8161\mathbf{F}\mathbf{j} - 0.3847\mathbf{F}\mathbf{k}
$$

**The Moment of Force $F$ About Point $A$.**

$$
\mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{F}
$$

$$
= \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2.8284 & 0 & -1.1716 \\
0.4314\mathbf{F} & 0.8161\mathbf{F} & -0.3847\mathbf{F}
\end{vmatrix}
$$

$$
= 0.9561\mathbf{F}\mathbf{i} + 0.5827\mathbf{F}\mathbf{j} + 2.3081\mathbf{F}\mathbf{k}
$$

OR

$$
\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F}
$$

$$
= \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
6 & 6 & -4 \\
0.4314\mathbf{F} & 0.8161\mathbf{F} & -0.3847\mathbf{F}
\end{vmatrix}
$$

$$
= 0.9561\mathbf{F}\mathbf{i} + 0.5827\mathbf{F}\mathbf{j} + 2.3081\mathbf{F}\mathbf{k}
$$

The magnitude of $\mathbf{M}_A$ is

$$
M_A = \sqrt{(M_A)_x^2 + (M_A)_y^2 + (M_A)_z^2} = \sqrt{(0.9561\mathbf{F})^2 + (0.5827\mathbf{F})^2 + (2.3081\mathbf{F})^2}
$$

$$
= 2.5654\mathbf{F}
$$

It is required that $M_A = 1500 \text{ N} \cdot \text{m}$, then

$$
1500 = 2.5654\mathbf{F}
$$

$$
F = 584.71 \text{ N} = 585 \text{ N}
$$

**Ans:**

$$
F = 585 \text{ N}
$$
*4–36.

Determine the coordinate direction angles $\alpha, \beta, \gamma$ of force $\mathbf{F}$, so that the moment of $\mathbf{F}$ about $O$ is zero.

**SOLUTION**

**Position And Force Vectors.** The coordinates of point $A$ are $A(0.4, 0.5, -0.3)$ m. Thus,

$$
\mathbf{r}_{OA} = [0.4 \mathbf{i} + 0.5 \mathbf{j} - 0.3 \mathbf{k}] \text{ m}
$$

$$
\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{\mathbf{r}_{OA}} = \frac{0.4 \mathbf{i} + 0.5 \mathbf{j} - 0.3 \mathbf{k}}{\sqrt{0.4^2 + 0.5^2 + (-0.3)^2}} = \frac{4}{\sqrt{50}} \mathbf{i} + \frac{5}{\sqrt{50}} \mathbf{j} - \frac{3}{\sqrt{50}} \mathbf{k}
$$

$$
\mathbf{r}_{AO} = [-0.4 \mathbf{i} - 0.5 \mathbf{j} + 0.3 \mathbf{k}] \text{ m}
$$

$$
\mathbf{u}_{AO} = \frac{\mathbf{r}_{AO}}{\mathbf{r}_{AO}} = \frac{-0.4 \mathbf{i} - 0.5 \mathbf{j} + 0.3 \mathbf{k}}{\sqrt{(-0.4)^2 + (-0.5)^2 + 0.3^2}} = -\frac{4}{\sqrt{50}} \mathbf{i} - \frac{5}{\sqrt{50}} \mathbf{j} + \frac{3}{\sqrt{50}} \mathbf{k}
$$

**Moment of $\mathbf{F}$ About Point $O$.** To produce zero moment about point $O$, the line of action of $\mathbf{F}$ must pass through point $O$. Thus, $\mathbf{F}$ must directed from $O$ to $A$ (direction defined by $\mathbf{u}_{OA}$). Thus,

$$
\cos \alpha = \frac{-4}{\sqrt{50}}, \quad \alpha = 55.56^\circ = 55.6^\circ \text{ Ans.}
$$

$$
\cos \beta = \frac{-5}{\sqrt{50}}, \quad \beta = 45^\circ \text{ Ans.}
$$

$$
\cos \gamma = \frac{-3}{\sqrt{50}}, \quad \gamma = 115.1^\circ = 115^\circ \text{ Ans.}
$$

OR $\mathbf{F}$ must directed from $A$ to $O$ (direction defined by $\mathbf{u}_{AO}$). Thus

$$
\cos \alpha = \frac{-4}{\sqrt{50}}, \quad \alpha = 124.44^\circ = 124^\circ \text{ Ans.}
$$

$$
\cos \beta = \frac{-5}{\sqrt{50}}, \quad \beta = 135^\circ \text{ Ans.}
$$

$$
\cos \gamma = \frac{3}{\sqrt{50}}, \quad \gamma = 64.90^\circ = 64.9^\circ \text{ Ans.}
$$

**Ans:**

$\alpha = 55.6^\circ$

$\beta = 45^\circ$

$\gamma = 115^\circ$

OR

$\alpha = 124^\circ$

$\beta = 135^\circ$

$\gamma = 64.9^\circ$
4–37.
Determine the moment of force $F$ about point $O$. The force has a magnitude of 800 N and coordinate direction angles of $\alpha = 60^\circ$, $\beta = 120^\circ$, $\gamma = 45^\circ$. Express the result as a Cartesian vector.

**SOLUTION**

Position And Force Vectors. The coordinates of point $A$ are $A (0.4, 0.5, -0.3)$ m. Thus

$$r_{OA} = [0.4i + 0.5j - 0.3k] \text{ m}$$

$$F = F_i = 800 (\cos 60^\circ i + \cos 120^\circ j + \cos 45^\circ k)$$

$$= [400i - 400j + 565.69k] \text{ N}$$

Moment of $F$ About Point $O$.

$$M_O = r_{OA} \times F$$

$$= \begin{vmatrix}
i & j & k \\
0.4 & 0.5 & -0.3 \\
400 & -400 & 565.69 \\
\end{vmatrix}$$

$$= [162.84i - 346.27j - 360k] \text{ N \cdot m}$$

$$= [163i - 346j - 360k] \text{ N \cdot m} \quad \text{Ans.}$$

**Ans:**

$$M_O = [163i - 346j - 360k] \text{ N \cdot m}$$
4–38. Determine the moment of the force \( \mathbf{F} \) about the door hinge at \( A \). Express the result as a Cartesian vector.

**SOLUTION**

**Position Vectors And Force Vector.** The coordinates of points \( A, C \) and \( D \) are \( A (-6.5, -3, 0) \) ft, \( C [0, -(3 + 4 \cos 45^\circ), 4 \sin 45^\circ] \) ft and \( D (-5, 0, 0) \) ft, respectively. Thus,

\[
\mathbf{r}_{AC} = [0 - (-6.5)] \mathbf{i} + [-(3 + 4 \cos 45^\circ) - (-3)] \mathbf{j} + (4 \sin 45^\circ - 0) \mathbf{k} = [6.5 \mathbf{i} - 2.8284 \mathbf{j} + 2.8284 \mathbf{k}] \text{ ft}
\]

\[
\mathbf{r}_{AD} = [-5 - (-6.5)] \mathbf{i} + [0 - (-3)] \mathbf{j} + (0 - 0) \mathbf{k} = [1.5 \mathbf{i} + 3 \mathbf{j}] \text{ ft}
\]

\[
\mathbf{r}_{CD} = (-5 - 0) \mathbf{i} + [0 - [-3 + 4 \cos 45^\circ]] \mathbf{j} + (0 - 4 \sin 45^\circ) \mathbf{k} = [-5 \mathbf{i} + 5.8284 \mathbf{j} - 2.8284 \mathbf{k}] \text{ ft}
\]

\[
\mathbf{F} = 80 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5i + 5.8284j - 2.8284k \end{vmatrix} = 80 \left( \frac{-5i + 5.8284j - 2.8284k}{\sqrt{(-5)^2 + 5.8284^2 + (-2.8284)^2}} \right) = [-48.88i + 56.98j - 27.65k] \text{ lb}
\]

**Moment of \( \mathbf{F} \) About Point \( A \).**

\[
\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6.5 & -2.8284 & 2.8284 \\ -48.88 & 56.98 & -27.65 \end{vmatrix} = [-82.9496 \mathbf{i} + 41.47 \mathbf{j} + 232.10 \mathbf{k}] \text{ lb} \cdot \text{ft}
\]

\[
= [-82.91 \mathbf{i} + 41.5 \mathbf{j} + 232 \mathbf{k}] \text{ lb} \cdot \text{ft} \quad \text{Ans.}
\]

OR

\[
\mathbf{M}_A = \mathbf{r}_{AD} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 3 & 0 \\ -48.88 & 56.98 & -27.65 \end{vmatrix} = [-82.9496 \mathbf{i} + 41.47 \mathbf{j} + 232.10 \mathbf{k}] \text{ lb} \cdot \text{ft}
\]

\[
= [-82.91 \mathbf{i} + 41.5 \mathbf{j} + 232 \mathbf{k}] \text{ lb} \cdot \text{ft} \quad \text{Ans.}
\]

\[
\text{Ans:} \quad \mathbf{M}_A = [-82.91 \mathbf{i} + 41.5 \mathbf{j} + 232 \mathbf{k}] \text{ lb} \cdot \text{ft}
\]
Determine the moment of the force $F$ about the door hinge at $B$. Express the result as a Cartesian vector.

SOLUTION

**Position Vectors And Force Vector.** The coordinates of points $B$, $C$ and $D$ are $B (-1.5, -3, 0)$ ft, $C [0, -(3 + 4 \cos 45^\circ), 4 \sin 45^\circ]$ ft and $D (-5, 0, 0)$ ft, respectively. Thus,

$$
\mathbf{r}_{BC} = [0 - (-1.5)]\mathbf{i} + [-(3 + 4 \cos 45^\circ) - (-3)]\mathbf{j} + (4 \sin 45^\circ - 0)\mathbf{k} \\
= [1.5\mathbf{i} - 2.8284\mathbf{j} + 2.8284\mathbf{k}] \text{ ft}
$$

$$
\mathbf{r}_{BD} = [-5 - (-1.5)]\mathbf{i} + [0 - (3)]\mathbf{j} + (0 - 0)\mathbf{k} = [-3.5\mathbf{i} + 3\mathbf{j}] \text{ ft}
$$

$$
\mathbf{r}_{CD} = (-5-0)\mathbf{i} + [0 - [(3 + 4 \cos 45^\circ)]\mathbf{j} + (0 - 4 \sin 45^\circ)\mathbf{k} \\
= [-5\mathbf{i} + 5.8284\mathbf{j} - 2.8284\mathbf{k}] \text{ ft}
$$

$$
\mathbf{F} = F \left( \frac{\mathbf{r}_{CD}}{r_{CD}} \right) = 80 \left( \frac{-5\mathbf{i} + 5.8284\mathbf{j} - 2.8284\mathbf{k}}{\sqrt{(-5)^2 + (5.8284)^2 + (-2.8284)^2}} \right) \\
= \{-48.88\mathbf{i} + 56.98\mathbf{j} - 27.65\mathbf{k}\} \text{ lb}
$$

Moment of $\mathbf{F}$ About Point $B$.

$$
\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F} \\
= \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1.5 & -2.8284 & 2.8284 \\
-48.88 & 56.98 & -27.65
\end{vmatrix} \\
= \{-82.9496\mathbf{i} - 96.77\mathbf{j} - 52.78\mathbf{k}\} \text{ lb} \cdot \text{ft}
$$

Ans.

or

$$
\mathbf{M}_B = \mathbf{r}_{BD} \times \mathbf{F} \\
= \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-3.5 & 3 & 0 \\
-48.88 & 56.98 & -27.65
\end{vmatrix} \\
= \{-82.9496\mathbf{i} - 96.77\mathbf{j} - 52.78\mathbf{k}\} \text{ lb} \cdot \text{ft}
$$

Ans.

Ans:

$$
\mathbf{M}_B = \{-82.9\mathbf{i} - 96.8\mathbf{j} - 52.8\mathbf{k}\} \text{ lb} \cdot \text{ft}
$$
*4–40.

The curved rod has a radius of 5 ft. If a force of 60 lb acts at its end as shown, determine the moment of this force about point C.

**SOLUTION**

*Position Vector and Force Vector:*

\[
r_{CA} = \{(5 \sin 60^\circ - 0)\mathbf{j} + (5 \cos 60^\circ - 5)\mathbf{k}\} \text{ m}
= \{4.330\mathbf{j} - 2.50\mathbf{k}\} \text{ m}
\]

\[
F_{AB} = 60 \left( \frac{(6 - 0)\mathbf{i} + (7 - 5 \sin 60^\circ)\mathbf{j} + (0 - 5 \cos 60^\circ)\mathbf{k}}{\sqrt{(6 - 0)^2 + (7 - 5 \sin 60^\circ)^2 + (0 - 5 \cos 60^\circ)^2}} \right) \text{ lb}
= \{51.231\mathbf{i} + 22.797\mathbf{j} - 21.346\mathbf{k}\} \text{ lb}
\]

*Moment of Force \(F_{AB}\) About Point C:* Applying Eq. 4–7, we have

\[
M_C = r_{CA} \times F_{AB}
\]

\[
\begin{vmatrix}
i & j & k \\
0 & 4.330 & -2.50 \\
51.231 & 22.797 & -21.346 \\
\end{vmatrix}
\]

\[
= \{-35.4\mathbf{i} - 128\mathbf{j} - 222\mathbf{k}\} \text{ lb} \cdot \text{ft}
\]

Ans.

\[
M_C = \{-35.4\mathbf{i} - 128\mathbf{j} - 222\mathbf{k}\} \text{ lb} \cdot \text{ft}
\]
4-41.

Determine the smallest force $F$ that must be applied along the rope in order to cause the curved rod, which has a radius of 5 ft, to fail at the support $C$. This requires a moment of $M = 80 \text{ lb} \cdot \text{ft}$ to be developed at $C$.

**SOLUTION**

*Position Vector and Force Vector:*

$$\mathbf{r}_{CA} = [(5 \sin 60^\circ - 0) \mathbf{j} + (5 \cos 60^\circ - 5) \mathbf{k}] \text{ m}$$

$$= [4.330 \mathbf{j} - 2.50 \mathbf{k}] \text{ m}$$

$$\mathbf{F}_{AB} = F \left( \frac{(6 - 0) \mathbf{i} + (7 - 5 \sin 60^\circ) \mathbf{j} + (0 - 5 \cos 60^\circ) \mathbf{k}}{\sqrt{(6 - 0)^2 + (7 - 5 \sin 60^\circ)^2 + (0 - 5 \cos 60^\circ)^2}} \right) \text{ lb}$$

$$= 0.8539F\mathbf{i} + 0.3799F\mathbf{j} - 0.3558F\mathbf{k}$$

*Moment of Force $\mathbf{F}_{AB}$ About Point $C$:*

$$\mathbf{M}_C = \mathbf{r}_{CA} \times \mathbf{F}_{AB}$$

$$= \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0 & 4.330 & -2.50 \\
0.8539F & 0.3799F & -0.3558F \\
\end{vmatrix}$$

$$= -0.5909F\mathbf{i} - 2.135\mathbf{j} - 3.697\mathbf{k}$$

Require

$$80 = \sqrt{(0.5909)^2 + (-2.135)^2 + (-3.697)^2} F$$

$$F = 18.6 \text{ lb.} \quad \text{Ans.}$$
4–42.

A 20-N horizontal force is applied perpendicular to the handle of the socket wrench. Determine the magnitude and the coordinate direction angles of the moment created by this force about point \( O \).

**SOLUTION**

\[
\mathbf{r}_A = 0.2 \sin 15^\circ \mathbf{i} + 0.2 \cos 15^\circ \mathbf{j} + 0.075 \mathbf{k}
\]

\[
= 0.05176 \mathbf{i} + 0.1932 \mathbf{j} + 0.075 \mathbf{k}
\]

\[
\mathbf{F} = -20 \cos 15^\circ \mathbf{i} + 20 \sin 15^\circ \mathbf{j}
\]

\[
= -19.32 \mathbf{i} + 5.176 \mathbf{j}
\]

\[
M_O = \mathbf{r}_A \times \mathbf{F} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0.05176 & 0.1932 & 0.075 \\
-19.32 & 5.176 & 0
\end{vmatrix}
\]

\[
= [-0.3882 \mathbf{i} - 1.449 \mathbf{j} + 4.00 \mathbf{k}] \text{ N} \cdot \text{m}
\]

\[
M_O = 4.272 = 4.27 \text{ N} \cdot \text{m}
\]

\[
\alpha = \cos^{-1} \left( \frac{-0.3882}{4.272} \right) = 95.2^\circ
\]

\[
\beta = \cos^{-1} \left( \frac{-1.449}{4.272} \right) = 110^\circ
\]

\[
\gamma = \cos^{-1} \left( \frac{4}{4.272} \right) = 20.6^\circ
\]
4–43.

The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point A.

SOLUTION

Position Vector And Force Vector:

\[ \mathbf{r}_{AC} = [(0.55 - 0)i + (0.4 - 0)j + (-0.2 - 0)k] \text{ m} \]
\[ = [0.55i + 0.4j - 0.2k] \text{ m} \]

\[ \mathbf{F} = 80(\cos 30^\circ \sin 40^\circ \mathbf{i} + \cos 30^\circ \cos 40^\circ \mathbf{j} - \sin 30^\circ \mathbf{k}) \text{ N} \]
\[ = (44.53\mathbf{i} + 53.07\mathbf{j} - 40.0\mathbf{k}) \text{ N} \]

Moment of Force \( \mathbf{F} \) About Point A: Applying Eq. 4–7, we have

\[ \mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F} \]
\[ = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0.4 & -0.2 \\ 44.53 & 53.07 & -40.0 \end{vmatrix} \]
\[ = [-5.39\mathbf{i} + 13.1\mathbf{j} + 11.4\mathbf{k}] \text{ N} \cdot \text{m} \]

Ans.

\[ \mathbf{M}_A = [-5.39\mathbf{i} + 13.1\mathbf{j} + 11.4\mathbf{k}] \text{ N} \cdot \text{m} \]
4–44. The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point B.

**SOLUTION**

*Position Vector And Force Vector:*

\[ \mathbf{r}_{BC} = [(0.55 - 0)i + (0.4 - 0.4)j + (-0.2 - 0)k] \text{ m} \]
\[ = [0.55i - 0.2k] \text{ m} \]

\[ \mathbf{F} = 80 \left( \cos 30^\circ \sin 40^\circ \mathbf{i} + \cos 30^\circ \cos 40^\circ \mathbf{j} - \sin 30^\circ \mathbf{k} \right) \text{ N} \]
\[ = (44.53\mathbf{i} + 53.07\mathbf{j} - 40.0\mathbf{k}) \text{ N} \]

*Moment of Force \( \mathbf{F} \) About Point B:* Applying Eq. 4–7, we have

\[ \mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F} \]

\[ = \begin{vmatrix} i & j & k \\ 0.55 & 0 & -0.2 \\ 44.53 & 53.07 & -40.0 \end{vmatrix} \]

\[ = [10.6i + 13.1j + 29.2k] \text{ N} \cdot \text{m} \quad \text{Ans.} \]
4–45.

A force of \( \mathbf{F} = \{6\mathbf{i} - 2\mathbf{j} + 1\mathbf{k}\} \) kN produces a moment of \( \mathbf{M}_O = \{4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k}\} \) kN·m about the origin of coordinates, point \( O \). If the force acts at a point having an \( x \) coordinate of \( x = 1 \) m, determine the \( y \) and \( z \) coordinates. 

*Note:* The figure shows \( \mathbf{F} \) and \( \mathbf{M}_O \) in an arbitrary position.

**SOLUTION**

\[
\mathbf{M}_O = \mathbf{r} \times \mathbf{F}
\]

\[
\begin{vmatrix}
4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k} & \mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & y & z \\
6 & -2 & 1
\end{vmatrix}
\]

\[
4 = y + 2z
\]

\[
5 = -1 + 6z
\]

\[
-14 = -2 - 6y
\]

\[
y = 2 \text{ m} \quad \text{Ans.}
\]

\[
z = 1 \text{ m} \quad \text{Ans.}
\]
4–46.

The force \( \mathbf{F} = (6i + 8j + 10k) \) N creates a moment about point \( O \) of \( \mathbf{M}_O = (-14i + 8j + 2k) \) N \( \cdot \) m. If the force passes through a point having an \( x \) coordinate of 1 m, determine the \( y \) and \( z \) coordinates of the point. Also, realizing that \( \mathbf{M}_O = Fd \), determine the perpendicular distance \( d \) from point \( O \) to the line of action of \( \mathbf{F} \). Note: The figure shows \( \mathbf{F} \) and \( \mathbf{M}_O \) in an arbitrary position.

**SOLUTION**

\[
\begin{vmatrix}
  i & j & k \\
-14 & 8 & 2 \\
 1 & y & z
\end{vmatrix}
\]

\[-14 = 10y - 8z \]
\[8 = -10 + 6z \]
\[2 = 8 - 6y \]

\( y = 1 \) m \hspace{1cm} \text{Ans.}
\( z = 3 \) m \hspace{1cm} \text{Ans.}

\( \mathbf{M}_O = \sqrt{(-14)^2 + (8)^2 + (2)^2} = 16.25 \) N \( \cdot \) m

\( F = \sqrt{(6)^2 + (8)^2 + (10)^2} = 14.14 \) N

\( d = \frac{16.25}{14.14} = 1.15 \) m \hspace{1cm} \text{Ans.}

\( \text{Ans:} \)
\( y = 1 \) m
\( z = 3 \) m
\( d = 1.15 \) m
4-47.

A force $\mathbf{F}$ having a magnitude of $F = 100$ N acts along the diagonal of the parallelepiped. Determine the moment of $\mathbf{F}$ about point $A$, using $\mathbf{M}_A = \mathbf{r}_B \times \mathbf{F}$ and $\mathbf{M}_A = \mathbf{r}_C \times \mathbf{F}$.

**SOLUTION**

\[
\mathbf{F} = 100 \left( \frac{-0.4 \mathbf{i} + 0.6 \mathbf{j} + 0.2 \mathbf{k}}{0.7483} \right)
\]

\[
\mathbf{F} = \{-53.5 \mathbf{i} + 80.2 \mathbf{j} + 26.7 \mathbf{k}\} \text{ N}
\]

\[
\mathbf{M}_A = \mathbf{r}_B \times \mathbf{F} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0 & -0.6 & 0 \\
-53.5 & 80.2 & 26.7
\end{vmatrix} = \{-16.0 \mathbf{i} - 32.1 \mathbf{k}\} \text{ N} \cdot \text{m}
\]

Also,

\[
\mathbf{M}_A = \mathbf{r}_C \times \mathbf{F} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-0.4 & 0 & 0.2 \\
-53.5 & 80.2 & 26.7
\end{vmatrix} = \{-16.0 \mathbf{i} - 32.1 \mathbf{k}\} \text{ N} \cdot \text{m}
\]

Ans:

\[
\mathbf{M}_A = \{-16.0 \mathbf{i} - 32.1 \mathbf{k}\} \text{ N} \cdot \text{m}
\]
*4–48.

Force \( \mathbf{F} \) acts perpendicular to the inclined plane. Determine the moment produced by \( \mathbf{F} \) about point \( A \). Express the result as a Cartesian vector.

**SOLUTION**

**Force Vector:** Since force \( \mathbf{F} \) is perpendicular to the inclined plane, its unit vector \( \mathbf{u}_F \) is equal to the unit vector of the cross product, \( \mathbf{b} = \mathbf{r}_{AC} \times \mathbf{r}_{BC} \). Fig. 4a. Here

\[
\mathbf{r}_{AC} = (0 - 0)\mathbf{i} + (4 - 0)\mathbf{j} + (0 - 3)\mathbf{k} = [4\mathbf{j} - 3\mathbf{k}] \text{ m}
\]

\[
\mathbf{r}_{BC} = (0 - 3)\mathbf{i} + (4 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = [-3\mathbf{i} + 4\mathbf{j}] \text{ m}
\]

Thus,

\[
\mathbf{b} = \mathbf{r}_{CA} \times \mathbf{r}_{CB} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0 & 4 & -3 \\
-3 & 4 & 0
\end{vmatrix}
\]

\[
= [12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}] \text{ m}^2
\]

Then,

\[
\mathbf{u}_F = \frac{\mathbf{b}}{b} = \frac{12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}}{\sqrt{12^2 + 9^2 + 12^2}} = 0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k}
\]

And finally

\[
\mathbf{F} = F\mathbf{u}_F = 400(0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k})
\]

\[
= [249.88\mathbf{i} + 187.41\mathbf{j} + 249.88\mathbf{k}] \text{ N}
\]

**Vector Cross Product:** The moment of \( \mathbf{F} \) about point \( A \) is

\[
\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0 & 4 & -3 \\
249.88 & 187.41 & 249.88
\end{vmatrix}
\]

\[
= [1.56\mathbf{i} - 0.750\mathbf{j} - 1.00\mathbf{k}] \text{ kN} \cdot \text{m}
\]

**Ans:**

\[
\mathbf{M}_A = [1.56\mathbf{i} - 0.750\mathbf{j} - 1.00\mathbf{k}] \text{ kN} \cdot \text{m}
\]
4–49.

Force \( \mathbf{F} \) acts perpendicular to the inclined plane. Determine the moment produced by \( \mathbf{F} \) about point \( B \). Express the result as a Cartesian vector.

**SOLUTION**

**Force Vector:** Since force \( \mathbf{F} \) is perpendicular to the inclined plane, its unit vector \( \mathbf{u}_F \) is equal to the unit vector of the cross product, \( \mathbf{b} = \mathbf{r}_{AC} \times \mathbf{r}_{BC} \), Fig. a. Here

\[
\mathbf{r}_{AC} = (0 - 0)i + (4 - 0)j + (0 - 3)k = [4j - 3k] \text{ m}
\]

\[
\mathbf{r}_{BC} = (0 - 3)i + (4 - 0)j + (0 - 0)k = [-3k + 4j] \text{ m}
\]

Thus,

\[
\mathbf{b} = \mathbf{r}_{CA} \times \mathbf{r}_{CB} = \begin{vmatrix}
i & j & k \\
0 & 4 & -3 \\
-3 & 4 & 0 \\
\end{vmatrix} = [12i + 9j + 12k] \text{ m}^2
\]

Then,

\[
\mathbf{u}_F = \frac{\mathbf{b}}{b} = \frac{12i + 9j + 12k}{\sqrt{12^2 + 9^2 + 12^2}} = 0.6247i + 0.4685j + 0.6247k
\]

And finally

\[
\mathbf{F} = Fu_F = 400(0.6247i + 0.4685j + 0.6247k)
\]

\[
= [249.88i + 187.41j + 249.88k] \text{ N}
\]

**Vector Cross Product:** The moment of \( \mathbf{F} \) about point \( B \) is

\[
\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F} = \begin{vmatrix}
i & j & k \\
-3 & 4 & 0 \\
249.88 & 187.41 & 249.88 \\
\end{vmatrix}
\]

\[
= [1.00i + 0.750j - 1.56k] \text{ kN} \cdot \text{m}
\]

**Ans.:**

\[
\mathbf{M}_B = [1.00i + 0.750j - 1.56k] \text{ kN} \cdot \text{m}
\]
4–50.

Strut \( AB \) of the 1-m-diameter hatch door exerts a force of 450 N on point \( B \). Determine the moment of this force about point \( O \).

**SOLUTION**

*Position Vector And Force Vector:*

\[
\mathbf{r}_{OB} = (0 - 0)\mathbf{i} + (1 \cos 30° - 0)\mathbf{j} + (1 \sin 30° - 0)\mathbf{k} \text{ m}
\]

\[
= (0.8660\mathbf{j} + 0.5\mathbf{k}) \text{ m}
\]

\[
\mathbf{r}_{OA} = (0.5 \sin 30° - 0)\mathbf{i} + (0.5 + 0.5 \cos 30° - 0)\mathbf{j} + (0 - 0)\mathbf{k} \text{ m}
\]

\[
= (0.250\mathbf{i} + 0.9330\mathbf{j}) \text{ m}
\]

\[
\mathbf{F} = 450 \left( \frac{(0 - 0.5 \sin 30°)\mathbf{i} + [1 \cos 30° - (0.5 + 0.5 \cos 30°)]\mathbf{j} + (1 \sin 30° - 0)\mathbf{k}}{\sqrt{(0 - 0.5 \sin 30°)^2 + [1 \cos 30° - (0.5 + 0.5 \cos 30°)]^2 + (1 \sin 30° - 0)^2}} \right) \text{ N}
\]

\[
= (-199.82\mathbf{i} - 53.54\mathbf{j} + 399.63\mathbf{k}) \text{ N}
\]

**Moment of Force \( \mathbf{F} \) About Point \( O \):** Applying Eq. 4–7, we have

\[
\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{F}
\]

\[
= \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0 & 0.8660 & 0.5 \\
-199.82 & -53.54 & 399.63
\end{vmatrix}
\]

\[
= \{373\mathbf{i} - 99.9\mathbf{j} + 173\mathbf{k}\} \text{ N} \cdot \text{m} \quad \text{Ans.}
\]

Or

\[
\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}
\]

\[
= \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0.250 & 0.9330 & 0 \\
-199.82 & -53.54 & 399.63
\end{vmatrix}
\]

\[
= \{373\mathbf{i} - 99.9\mathbf{j} + 173\mathbf{k}\} \text{ N} \cdot \text{m} \quad \text{Ans.}
\]

Ans:

\[
\mathbf{M}_O = \{373\mathbf{i} - 99.9\mathbf{j} + 173\mathbf{k}\} \text{ N} \cdot \text{m}
\]
4-51.

Using a ring collar the 75-N force can act in the vertical plane at various angles $\theta$. Determine the magnitude of the moment it produces about point $A$, plot the result of $M$ (ordinate) versus $\theta$ (abscissa) for $0^\circ \leq \theta \leq 180^\circ$, and specify the angles that give the maximum and minimum moment.

**SOLUTION**

\[
\mathbf{M}_A = \begin{bmatrix} i & j & k \\ 2 & 1.5 & 0 \\ 0 & 75 \cos \theta & 75 \sin \theta \end{bmatrix}
\]

\[
= 112.5 \sin \theta \mathbf{i} - 150 \sin \theta \mathbf{j} + 150 \cos \theta \mathbf{k}
\]

\[
M_A = \sqrt{(112.5 \sin \theta)^2 + (-150 \sin \theta)^2 + (150 \cos \theta)^2} = \sqrt{12656.25 \sin^2 \theta + 22500}
\]

\[
\frac{dM_A}{d\theta} = \frac{1}{2} (12656.25 \sin^2 \theta + 22500)^{-\frac{1}{2}} (12656.25) (2 \sin \theta \cos \theta) = 0
\]

\[
\sin \theta \cos \theta = 0; \quad \theta = 0^\circ, 90^\circ, 180^\circ
\]

\[
\theta_{\text{max}} = 187.5 \text{ N} \cdot \text{m} \text{ at } \theta = 90^\circ
\]

\[
\theta_{\text{min}} = 150 \text{ N} \cdot \text{m} \text{ at } \theta = 0^\circ, 180^\circ
\]
**4–52.**

The lug nut on the wheel of the automobile is to be removed using the wrench and applying the vertical force of \( F = 30 \text{ N} \) at \( A \). Determine if this force is adequate, provided 14 N \( \cdot \) m of torque about the \( x \) axis is initially required to turn the nut. If the 30-N force can be applied at \( A \) in any other direction, will it be possible to turn the nut?

**SOLUTION**

\[
M_x = 30 \left( \sqrt{(0.5)^2 - (0.3)^2} \right) = 12 \text{ N} \cdot \text{m} < 14 \text{ N} \cdot \text{m}, \quad \text{No}
\]

For \( (M_x)_{\text{max}} \), apply force perpendicular to the handle and the \( x \)-axis.

\[
(M_x)_{\text{max}} = 30 \cdot (0.5) = 15 \text{ N} \cdot \text{m} > 14 \text{ N} \cdot \text{m}, \quad \text{Yes}
\]
4–53.

Solve Prob. 4–52 if the cheater pipe $AB$ is slipped over the handle of the wrench and the 30-N force can be applied at any point and in any direction on the assembly.

SOLUTION

\[ M_x = 30 \times 0.75 \times \left(\frac{4}{5}\right) = 18 \, \text{N} \cdot \text{m} > 14 \, \text{N} \cdot \text{m}, \quad \text{Yes} \]

\( (M_x)_{\text{max}} \) occurs when force is applied perpendicular to both the handle and the $x$-axis.

\[ (M_x)_{\text{max}} = 30 \times 0.75 \times 22.5 \, \text{N} \cdot \text{m} > 14 \, \text{N} \cdot \text{m}, \quad \text{Yes} \]
4–54. The A-frame is being hoisted into an upright position by the vertical force of \( F = 80 \) lb. Determine the moment of this force about the \( y' \) axis passing through points \( A \) and \( B \) when the frame is in the position shown.

**SOLUTION**

Scalar analysis:
\[
M_{y'} = 80 \left( 6 \cos 15^\circ \right) = 464 \text{ lb} \cdot \text{ft}
\]

Vector analysis:
\[
u_{AB} = \cos 60^\circ \mathbf{i} + \cos 30^\circ \mathbf{j}
\]

Coordinates of point \( C \):
\[
x = 3 \sin 30^\circ - 6 \cos 15^\circ \cos 30^\circ = -3.52 \text{ ft}
\]
\[
y = 3 \cos 30^\circ + 6 \cos 15^\circ \sin 30^\circ = 5.50 \text{ ft}
\]
\[
z = 6 \sin 15^\circ = 1.55 \text{ ft}
\]
\[
r_{AC} = -3.52 \mathbf{i} + 5.50 \mathbf{j} + 1.55 \mathbf{k}
\]
\[
F = 80 \mathbf{k}
\]

\[
M_{y'} = \begin{vmatrix}
\sin 30^\circ & \cos 30^\circ & 0 \\
-3.52 & 5.50 & 1.55 \\
0 & 0 & 80
\end{vmatrix}
\]

\[
M_{y'} = 464 \text{ lb} \cdot \text{ft}
\]

**Ans.**

\[
M_{y'} = 464 \text{ lb} \cdot \text{ft}
\]
4–55.

The A-frame is being hoisted into an upright position by the vertical force of \( F = 80 \text{ lb} \). Determine the moment of this force about the \( x \) axis when the frame is in the position shown.

**SOLUTION**

Using \( x', y', z' \):

\[
\mathbf{u}_x = \cos 30^\circ \mathbf{i}' + \sin 30^\circ \mathbf{j}'
\]

\[
\mathbf{r}_{AC} = -6 \cos 15^\circ \mathbf{i}' + 3 \mathbf{j}' + 6 \sin 15^\circ \mathbf{k}
\]

\( \mathbf{F} = 80 \mathbf{k} \)

\[
M_x = \begin{bmatrix}
\cos 30^\circ & \sin 30^\circ & 0 \\
-6 \cos 15^\circ & 3 & 6 \sin 15^\circ \\
0 & 0 & 80
\end{bmatrix} = 207.85 + 231.82 + 0
\]

\( M_x = 440 \text{ lb} \cdot \text{ft} \)

Also, using \( x, y, z \),

Coordinates of point \( C \):

\[
x = 3 \sin 30^\circ - 6 \cos 15^\circ \cos 30^\circ = -3.52 \text{ ft}
\]

\[
y = 3 \cos 30^\circ + 6 \cos 15^\circ \sin 30^\circ = 5.50 \text{ ft}
\]

\[
z = 6 \sin 15^\circ = 1.55 \text{ ft}
\]

\[
\mathbf{r}_{AC} = -3.52 \mathbf{i} + 5.50 \mathbf{j} + 1.55 \mathbf{k}
\]

\( \mathbf{F} = 80 \mathbf{k} \)

\[
M_x = \begin{bmatrix}
1 & 0 & 0 \\
-3.52 & 5.50 & 1.55 \\
0 & 0 & 80
\end{bmatrix} = 440 \text{ lb} \cdot \text{ft} \quad \text{Ans.}
\]
*4–56.

Determine the magnitude of the moments of the force \( \mathbf{F} \) about the \( x, y, \) and \( z \) axes. Solve the problem (a) using a Cartesian vector approach and (b) using a scalar approach.

**SOLUTION**

a) **Vector Analysis**

**Position Vector:**

\[
\mathbf{r}_{AB} = [(4 - 0) \mathbf{i} + (3 - 0) \mathbf{j} + (-2 - 0) \mathbf{k}] \text{ ft} = [4 \mathbf{i} + 3 \mathbf{j} - 2 \mathbf{k}] \text{ ft}
\]

**Moment of Force \( \mathbf{F} \) About \( x, y, \) and \( z \) Axes:** The unit vectors along \( x, y, \) and \( z \) axes are \( \mathbf{i}, \mathbf{j}, \) and \( \mathbf{k} \) respectively. Applying Eq. 4–11, we have

\[
M_x = \mathbf{i} \cdot (\mathbf{r}_{AB} \times \mathbf{F})
\]

\[
= \begin{vmatrix}
1 & 0 & 0 \\
4 & 3 & -2 \\
4 & 12 & -3
\end{vmatrix}
\]

\[
= 1[3(-3) - (12)(-2)] - 0 + 0 = 15.0 \text{ lb} \cdot \text{ft} \quad \text{Ans.}
\]

\[
M_y = \mathbf{j} \cdot (\mathbf{r}_{AB} \times \mathbf{F})
\]

\[
= \begin{vmatrix}
0 & 1 & 0 \\
4 & 3 & -2 \\
4 & 12 & -3
\end{vmatrix}
\]

\[
= 0 - 1[4(-3) - (4)(-2)] + 0 = 4.00 \text{ lb} \cdot \text{ft} \quad \text{Ans.}
\]

\[
M_z = \mathbf{k} \cdot (\mathbf{r}_{AB} \times \mathbf{F})
\]

\[
= \begin{vmatrix}
0 & 0 & 1 \\
4 & 3 & -2 \\
4 & 12 & -3
\end{vmatrix}
\]

\[
= 0 - 0 + 1[4(12) - (4)(3)] = 36.0 \text{ lb} \cdot \text{ft} \quad \text{Ans.}
\]

b) **Scalar Analysis**

\[
M_x = \sum M_x; \quad M_x = 12(2) - 3(3) = 15.0 \text{ lb} \cdot \text{ft} \quad \text{Ans.}
\]

\[
M_y = \sum M_y; \quad M_y = -4(2) + 3(4) = 4.00 \text{ lb} \cdot \text{ft} \quad \text{Ans.}
\]

\[
M_z = \sum M_z; \quad M_z = -4(3) + 12(4) = 36.0 \text{ lb} \cdot \text{ft} \quad \text{Ans.}
\]

\[
\text{Ans:} \quad M_x = 15.0 \text{ lb} \cdot \text{ft} \quad M_y = 4.00 \text{ lb} \cdot \text{ft} \quad M_z = 36.0 \text{ lb} \cdot \text{ft}
\]
4-57.

Determine the moment of the force \( F \) about an axis extending between \( A \) and \( C \). Express the result as a Cartesian vector.

**SOLUTION**

*Position Vector:*

\[
r_{CB} = [-2k] \text{ ft} \\
r_{AB} = [(4 - 0)i + (3 - 0)j + (-2 - 0)k] \text{ ft} = [4i + 3j - 2k] \text{ ft}
\]

*Unit Vector Along AC Axis:*

\[
u_{AC} = \frac{(4 - 0)i + (3 - 0)j}{\sqrt{(4 - 0)^2 + (3 - 0)^2}} = 0.8i + 0.6j
\]

*Moment of Force \( F \) About AC Axis:*

With \( F = [4i + 12j - 3k] \text{ lb} \), applying Eq. 4-7, we have

\[
M_{AC} = u_{AC} \cdot (r_{CB} \times F)
\]

\[
= \begin{bmatrix}
0.8 & 0.6 & 0 \\
0 & 0 & -2 \\
4 & 12 & -3
\end{bmatrix}
\]

\[
= 0.8[(0)(-3) - 12(-2)] - 0.6[0(-3) - 4(-2)] + 0
\]

\[
= 14.4 \text{ lb} \cdot \text{ft}
\]

Or

\[
M_{AC} = u_{AC} \cdot (r_{AB} \times F)
\]

\[
= \begin{bmatrix}
0.8 & 0.6 & 0 \\
4 & 3 & -2 \\
4 & 12 & -3
\end{bmatrix}
\]

\[
= 0.8[(3)(-3) - 12(-2)] - 0.6[(-3) - 4(-2)] + 0
\]

\[
= 14.4 \text{ lb} \cdot \text{ft}
\]

Expressing \( M_{AC} \) as a Cartesian vector yields

\[
M_{AC} = M_{AC}u_{AC}
\]

\[
= 14.4(0.8i + 0.6j)
\]

\[
= [11.5i + 8.64j] \text{ lb} \cdot \text{ft}
\]

Ans: \( M_{AC} = [11.5i + 8.64j] \text{ lb} \cdot \text{ft} \)

Ans:
The board is used to hold the end of a four-way lug wrench in the position shown when the man applies a force of \( F = 100 \text{ N} \). Determine the magnitude of the moment produced by this force about the \( x \) axis. Force \( \mathbf{F} \) lies in a vertical plane.

**Solution**

**Vector Analysis**

*Moment About the \( x \) Axis:* The position vector \( \mathbf{r}_{AB} \), Fig. a, will be used to determine the moment of \( \mathbf{F} \) about the \( x \) axis.

\[
\mathbf{r}_{AB} = (0.25 - 0.25)i + (0.25 - 0)j + (0 - 0)k = [0.25] \text{ m}
\]

The force vector \( \mathbf{F} \), Fig. a, can be written as

\[
\mathbf{F} = 100(\cos 60^\circ \mathbf{j} - \sin 60^\circ \mathbf{k}) = [50 \mathbf{j} - 86.60 \mathbf{k}] \text{ N}
\]

Knowing that the unit vector of the \( x \) axis is \( \mathbf{i} \), the magnitude of the moment of \( \mathbf{F} \) about the \( x \) axis is given by

\[
M_x = \mathbf{i} \cdot \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix}
1 & 0 & 0 \\
0 & 0.25 & 0 \\
0 & 50 & -86.60
\end{vmatrix}
= 1[0.25(-86.60) - 50(0)] + 0 + 0
= -21.7 \text{ N} \cdot \text{m}
\]

The negative sign indicates that \( M_x \) is directed towards the negative \( x \) axis.

**Scalar Analysis**

This problem can be solved by summing the moment about the \( x \) axis

\[
M_x = \sum M_x; \quad M_x = -100 \ \sin 60^\circ (0.25) + 100 \ \cos 60^\circ (0)
= -21.7 \text{ N} \cdot \text{m} \quad \text{Ans.}
\]
4–59.

The board is used to hold the end of a four-way lug wrench in position. If a torque of 30 N·m about the x axis is required to tighten the nut, determine the required magnitude of the force $F$ that the man’s foot must apply on the end of the wrench in order to turn it. Force $F$ lies in a vertical plane.

SOLUTION

**Vector Analysis**

**Moment About the x Axis:** The position vector $\mathbf{r}_{AB}$, Fig. a, will be used to determine the moment of $\mathbf{F}$ about the $x$ axis.

$$\mathbf{r}_{AB} = (0.25 - 0.25)\mathbf{i} + (0.25 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = [0.25\mathbf{j}] \text{ m}$$

The force vector $\mathbf{F}$, Fig. a, can be written as

$$\mathbf{F} = F(\cos 60^\circ \mathbf{j} - \sin 60^\circ \mathbf{k}) = 0.5F\mathbf{j} - 0.8660F\mathbf{k}$$

Knowing that the unit vector of the $x$ axis is $\mathbf{i}$, the magnitude of the moment of $\mathbf{F}$ about the $x$ axis is given by

$$M_x = \mathbf{i} \cdot \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0.5F & -0.8660F & 3 \end{vmatrix}$$

$$= 1[0.25(-0.8660F) - 0.5F(0)] + 0 + 0$$

$$= -0.2165F$$

Ans.

The negative sign indicates that $M_x$ is directed towards the negative $x$ axis. The magnitude of $\mathbf{F}$ required to produce $M_x = 30 \text{ N·m}$ can be determined from

$$30 = 0.2165F$$

$$F = 139 \text{ N}$$

Ans.

**Scalar Analysis**

This problem can be solved by summing the moment about the $x$ axis

$$M_x = \sum M_x; \quad -30 = -F \sin 60^\circ (0.25) + F \cos 60^\circ (0)$$

$$F = 139 \text{ N}$$

Ans.
The A-frame is being hoisted into an upright position by the vertical force of $F = 80$ lb. Determine the moment of this force about the $y$ axis when the frame is in the position shown.

**SOLUTION**

Using $x', y', z$: 

$u_y = -\sin 30^\circ i' + \cos 30^\circ j'$

$r_{AC} = -6 \cos 15^\circ i' + 3 j' + 6 \sin 15^\circ k$

$F = 80 k$

$$
M_y = \begin{bmatrix}
-\sin 30^\circ & \cos 30^\circ & 0 \\
-6 \cos 15^\circ & 3 & 6 \sin 15^\circ \\
0 & 0 & 80
\end{bmatrix}
$$

$M_y = 282 \text{ lb} \cdot \text{ft}$

Also, using $x, y, z$:

Coordinates of point $C$:

$$
x = 3 \sin 30^\circ - 6 \cos 15^\circ \cos 30^\circ = -3.52 \text{ ft}
$$

$$
y = 3 \cos 30^\circ + 6 \cos 15^\circ \sin 30^\circ = 5.50 \text{ ft}
$$

$$
z = 6 \sin 15^\circ = 1.55 \text{ ft}
$$

$r_{AC} = -3.52 i + 5.50 j + 1.55 k$

$F = 80 k$

$$
M_y = \begin{bmatrix}
-3.52 & 5.50 & 1.55 \\
0 & 0 & 80
\end{bmatrix}
$$

$M_y = 282 \text{ lb} \cdot \text{ft}$

**Ans:**

$M_y = 282 \text{ lb} \cdot \text{ft}$
4-61.

Determine the magnitude of the moment of the force \( \mathbf{F} = [50 \mathbf{i} - 20 \mathbf{j} - 80 \mathbf{k}] \) N about the base line \( AB \) of the tripod.

**SOLUTION**

\[
\mathbf{u}_{AB} = \frac{[3.5 \mathbf{i} + 0.5 \mathbf{j}]}{\sqrt{(3.5)^2 + (0.5)^2}}
\]

\[
\mathbf{u}_{AB} = [0.9899 \mathbf{i} + 0.1414 \mathbf{j}]
\]

\[
M_{AB} = \mathbf{u}_{AB} \cdot (\mathbf{r}_{AD} \times \mathbf{F}) = \begin{vmatrix} 0.9899 & 0.1414 & 0 \\ 2.5 & 0 & 4 \\ 50 & -20 & -80 \end{vmatrix}
\]

\[
M_{AB} = 136 \text{ N} \cdot \text{m}
\]

\text{Ans.}

\[
M_{AB} = 136 \text{ N} \cdot \text{m}
\]
4–62.

Determine the magnitude of the moment of the force $\mathbf{F} = [50 \mathbf{i} - 20 \mathbf{j} - 80 \mathbf{k}]$ N about the base line $BC$ of the tripod.

**SOLUTION**

$$\mathbf{u}_{BC} = \frac{[-1.5i - 2.5j]}{\sqrt{(-1.5)^2 + (-2.5)^2}}$$

$$\mathbf{u}_{BC} = [-0.5145i - 0.8575j]$$

$$M_{RC} = \mathbf{u}_{BC} \cdot (\mathbf{r}_{CD} \times \mathbf{F}) = \begin{vmatrix}
-0.5145 & -0.8575 & 0 \\
0.5 & 2 & 4 \\
50 & -20 & -80
\end{vmatrix}$$

$$M_{RC} = 165 \text{ N} \cdot \text{m}$$

Ans.

Ans: $M_{RC} = 165 \text{ N} \cdot \text{m}$
4-63.

Determine the magnitude of the moment of the force \( \mathbf{F} = [50 \mathbf{i} - 20 \mathbf{j} - 80 \mathbf{k}] \) N about the base line \( CA \) of the tripod.

\[ \mathbf{u}_{CA} = \frac{[-2 \mathbf{i} + 2 \mathbf{j}]}{\sqrt{(-2)^2 + (2)^2}} \]

\[ \mathbf{u}_{CA} = \{ -0.707 \mathbf{i} + 0.707 \mathbf{j} \} \]

\[ M_{CA} = \mathbf{u}_{CA} \cdot (\mathbf{r}_{AD} \times \mathbf{F}) = \begin{vmatrix} -0.707 & 0.707 & 0 \\ 2.5 & 0 & 4 \\ 50 & -20 & -80 \end{vmatrix} \]

\[ M_{CA} = 226 \text{ N} \cdot \text{m} \]

\[ \text{Ans.} \]
*4–64.

A horizontal force of \( \mathbf{F} = [-50\mathbf{i}] \) N is applied perpendicular to the handle of the pipe wrench. Determine the moment that this force exerts along the axis \( OA \) (\( z \) axis) of the pipe assembly. Both the wrench and pipe assembly, \( OABC \), lie in the \( y-z \) plane. *Suggestion:* Use a scalar analysis.

**SOLUTION**

\[
M_z = 50(0.8 + 0.2) \cos 45^\circ = 35.36 \text{ N} \cdot \text{m}
\]

\[
M_z = [35.4 \mathbf{k}] \text{ N} \cdot \text{m}
\]

Ans.

\[
M_z = (35.4 \mathbf{k}) \text{ N} \cdot \text{m}
\]

Ans:
4-65.

Determine the magnitude of the horizontal force \( F = -F_i \) acting on the handle of the wrench so that this force produces a component of the moment along the \( OA \) axis (\( z \) axis) of the pipe assembly of \( M_z = [4k] \) N\( \cdot \)m. Both the wrench and the pipe assembly, \( OABC \), lie in the \( y-z \) plane. 

_Suggestion:_ Use a scalar analysis.

**SOLUTION**

\[
M_z = F(0.8 + 0.2) \cos 45^\circ = 4
\]

\( F = 5.66 \) N

**Ans:**

\( F = 5.66 \) N
The force of $F = 30$ N acts on the bracket as shown. Determine the moment of the force about the $a-a$ axis of the pipe if $\alpha = 60^\circ$, $\beta = 60^\circ$, and $\gamma = 45^\circ$. Also, determine the coordinate direction angles of $F$ in order to produce the maximum moment about the $a-a$ axis. What is this moment?

**SOLUTION**

$\mathbf{F} = 30 \left( \cos 60^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 45^\circ \mathbf{k} \right)$

$= \{15 \mathbf{i} + 15 \mathbf{j} + 21.21 \mathbf{k} \} \text{ N}$

$\mathbf{r} = \{-0.1 \mathbf{i} + 0.15 \mathbf{k} \} \text{ m}$

$\mathbf{u} = \mathbf{j}$

$M_a = \begin{vmatrix} 0 & 1 & 0 \\ -0.1 & 0 & 0.15 \\ 15 & 15 & 21.21 \end{vmatrix} = 4.37 \text{ N} \cdot \text{m}$

$\mathbf{F}$ must be perpendicular to $\mathbf{u}$ and $\mathbf{r}$.

$u_F = \frac{0.15}{0.1803} \mathbf{i} + \frac{0.1}{0.1803} \mathbf{k}$

$= 0.8321 \mathbf{i} + 0.5547 \mathbf{k}$

$\alpha = \cos^{-1} 0.8321 = 33.7^\circ$

$\beta = \cos^{-1} 0 = 90^\circ$

$\gamma = \cos^{-1} 0.5547 = 56.3^\circ$

$M = 30 (0.1803) = 5.41 \text{ N} \cdot \text{m}$

Ans: $M_a = 4.37 \text{ N} \cdot \text{m}$

$\alpha = 33.7^\circ$

$\beta = 90^\circ$

$\gamma = 56.3^\circ$

$M = 5.41 \text{ N} \cdot \text{m}$
4–67.

A clockwise couple \( M = 5 \text{ N} \cdot \text{m} \) is resisted by the shaft of the electric motor. Determine the magnitude of the reactive forces \(-R\) and \(R\) which act at supports \(A\) and \(B\) so that the resultant of the two couples is zero.

**SOLUTION**

\[
\zeta + M_C = -5 + R \left( 2(0.15)/\tan 60^\circ \right) = 0
\]

\[R = 28.9 \text{ N}\]  \(\text{Ans.}\)
A twist of 4 N·m is applied to the handle of the screwdriver. Resolve this couple moment into a pair of couple forces $F$ exerted on the handle and $P$ exerted on the blade.

**SOLUTION**

For the handle

\[ M_C = \sum M_x; \quad F(0.03) = 4 \]

\[ F = 133 \text{ N} \]  
**Ans.**

For the blade,

\[ M_C = \sum M_x; \quad P(0.005) = 4 \]

\[ P = 800 \text{ N} \]  
**Ans.**

---

**Ans:**

\[ F = 133 \text{ N} \]

\[ P = 800 \text{ N} \]
4–69.

If the resultant couple of the three couples acting on the triangular block is to be zero, determine the magnitude of forces \( F \) and \( P \).

**SOLUTION**

\( BA = 0.5 \) m

The couple created by the 150-N forces is

\[ M_{C1} = 150 \times 0.5 = 75 \text{ N} \cdot \text{m} \]

Then

\[ M_{C1} = 75 \left( \frac{3}{5} \right) j + 75 \left( \frac{4}{5} \right) k \]

\[ = 45j + 60k \]

\[ M_{C2} = -P \times 0.6 \times k \]

\[ M_{C3} = -F \times 0.6 \times j \]

Require

\[ M_{C1} + M_{C2} + M_{C3} = 0 \]

\[ 45j + 60k - P \times 0.6 \times k - F \times 0.6 \times j = 0 \]

Equate the j and k components

\[ 45 - F \times 0.6 = 0 \]

\[ F = 75 \text{ N} \quad \text{Ans.} \]

\[ 60 - P \times 0.6 = 0 \]

\[ P = 100 \text{ N} \quad \text{Ans.} \]
4–70.

Two couples act on the beam. If \( F = 125 \text{ lb} \), determine the resultant couple moment.

**SOLUTION**

125 lb couple is resolved in to their horizontal and vertical components as shown in Fig. \( a \).

\[
\zeta + (M_R)_C = 200(1.5) + 125 \cos 30^\circ (1.25)
\]

\[
= 435.32 \text{ lb} \cdot \text{ft} = 435 \text{ lb} \cdot \text{ft}
\]

\[
(M_R)_C = 200(1.5) + 125 \cos 30^\circ (1.25)
\]

\[
= 435 \text{ lb} \cdot \text{ft}
\]

**Ans:**

\((M_R)_C = 435 \text{ lb} \cdot \text{ft}\)
4-71.

Two couples act on the beam. Determine the magnitude of \( F \) so that the resultant couple moment is 450 lb-ft, counterclockwise. Where on the beam does the resultant couple moment act?

**SOLUTION**

\[
\sum F + M_R = \Sigma M; \quad 450 = 200(1.5) + F \cos 30^\circ(1.25)
\]

\[ F = 139 \text{ lb} \quad \text{Ans.} \]

The resultant couple moment is a free vector. It can act at any point on the beam.
Determine the magnitude of the couple force $F$ so that the resultant couple moment on the crank is zero.

**SOLUTION**

By resolving $F$ and the 150-lb couple into components parallel and perpendicular to the lever arm of the crank, Fig. a, and summing the moment of these two force components about point $A$, we have

$$
\Sigma \tau = \Sigma M_A:\quad 0 = 150 \cos 15^\circ(10) - F \cos 15^\circ(5) - F \sin 15^\circ(4) - 150 \sin 15^\circ(8)
$$

Ansol. $F = 194$ lb

Note: Since the line of action of the force component parallel to the lever arm of the crank passes through point $A$, no moment is produced about this point.
4-73.

The ends of the triangular plate are subjected to three couples. Determine the magnitude of the force $F$ so that the resultant couple moment is $400 \text{ N} \cdot \text{m}$ clockwise.

**SOLUTION**

\[ \zeta + M_R = \Sigma M; \quad -400 = 600 \left( \frac{0.5}{\cos 40^\circ} \right) - F \left( \frac{0.5}{\cos 40^\circ} \right) - 250(1) \]

\[ F = 830 \text{ N} \]
4–74.

The man tries to open the valve by applying the couple forces of $F = 75$ N to the wheel. Determine the couple moment produced.

**SOLUTION**

\[ \sum F = \Sigma M; \quad M_c = -75(0.15 + 0.15) \]

\[ = -22.5 \text{ N} \cdot \text{m} = 22.5 \text{ N} \cdot \text{m} \]

**Ans:**

\[ M_c = 22.5 \text{ N} \cdot \text{m} \]
4-75.

If the valve can be opened with a couple moment of 25 N \cdot m, determine the required magnitude of each couple force which must be applied to the wheel.

\[ \zeta + M_c = \Sigma M; \quad -25 = -F (0.15 + 0.15) \]
\[ F = 83.3 \text{ N} \]

Ans: 
\[ F = 83.3 \text{ N} \]
*4–76.

Determine the magnitude of \( F \) so that the resultant couple moment is 12 kN \( \cdot \) m, counterclockwise. Where on the beam does the resultant couple moment act?

**SOLUTION**

\[ \zeta + M_R = \Sigma M_C; \quad 12 = (F \cos 30^\circ)(0.3) + 8(1.2) \]

\[ F = 9.238 \text{ kN} = 9.24 \text{ kN} \quad \text{Ans.} \]

Since the couple moment is a free vector, the resultant couple moment can act at any point on or off the beam.

**Ans:**

\[ F = 9.24 \text{ kN} \]
4-77.

Two couples act on the beam as shown. If $F = 150$ lb, determine the resultant couple moment.

**SOLUTION**

150 lb couple is resolved into their horizontal and vertical components as shown in Fig. a

$$\zeta + (M_R)_c = 150 \left(\frac{4}{5}\right)(1.5) + 150 \left(\frac{3}{5}\right)(4) - 200(1.5)$$

$$= 240 \text{ lb} \cdot \text{ft}$$

**Ans:**

$$(M_R)_C = 240 \text{ lb} \cdot \text{ft}$$
Two couples act on the beam as shown. Determine the magnitude of $F$ so that the resultant couple moment is 300 lb·ft counterclockwise. Where on the beam does the resultant couple act?

**SOLUTION**

\[ \zeta + (M_C)_R = \frac{3}{5} F(4) + \frac{4}{5} F(1.5) - 200(1.5) = 300 \]

\[ F = 167 \text{ lb} \]

Resultant couple can act anywhere.

**Ans:**

$F = 167$ lb

Resultant couple can act anywhere.
4–79.

Two couples act on the frame. If the resultant couple moment is to be zero, determine the distance \( d \) between the 80-lb couple forces.

**SOLUTION**

\[
\zeta + M_c = -50 \cos 30^\circ (3) + \frac{4}{5} (80)(d) = 0
\]

\[
d = 2.03 \text{ ft}
\]

\text{Ans.}
Two couples act on the frame. If $d = 4\text{ ft}$, determine the resultant couple moment. Compute the result by resolving each force into $x$ and $y$ components and (a) finding the moment of each couple (Eq. 4–13) and (b) summing the moments of all the force components about point $A$.

**SOLUTION**

(a) $\mathbf{M}_C = \sum (\mathbf{r} \times \mathbf{F})$

$$\begin{align*}
\mathbf{M}_C &= \begin{vmatrix}
i & j & k \\
3 & 0 & 0 \\
-50 \sin 30^\circ & -50 \cos 30^\circ & 0 \\
-\frac{4}{5}(80) & -\frac{4}{5}(80) & 0
\end{vmatrix} + \begin{vmatrix}
i & j & k \\
0 & 4 & 0 \\
-\frac{3}{5}(80) & -\frac{3}{5}(80) & 0
\end{vmatrix} \\
&= \{126k\} \text{ lb} \cdot \text{ft}
\end{align*}$$

Ans.

(b) $\mathbf{C} + \mathbf{M}_C = -\frac{4}{5}(80)(3) + \frac{4}{5}(80)(7) + 50 \cos 30^\circ(2) - 50 \cos 30^\circ(5)$

$$\begin{align*}
\mathbf{M}_C &= 126 \text{ lb} \cdot \text{ft}
\end{align*}$$

Ans.
4–81.

Two couples act on the frame. If \( d = 4 \) ft, determine the resultant couple moment. Compute the result by resolving each force into \( x \) and \( y \) components and (a) finding the moment of each couple (Eq. 4–13) and (b) summing the moments of all the force components about point \( B \).

**SOLUTION**

(a) \( \mathbf{M}_C = \sum (\mathbf{r} \times \mathbf{F}) \)

\[
\begin{vmatrix}
1 & j & k \\
3 & 0 & 0 \\
\end{vmatrix} + \begin{vmatrix}
1 & j & k \\
-50 \sin 30^\circ & -50 \cos 30^\circ & 0 \\
\frac{4}{5}(80) & \frac{3}{5}(80) & 0 \\
\end{vmatrix}
\]

\[ \mathbf{M}_C = [126k] \text{ lb} \cdot \text{ft} \quad \text{Ans.} \]

(b) \( \zeta + \mathbf{M}_C = 50 \cos 30^\circ(2) - 50 \cos 30^\circ(5) - \frac{4}{5}(80)(1) + \frac{4}{5}(80)(5) \)

\[ \mathbf{M}_C = 126 \text{ lb} \cdot \text{ft} \quad \text{Ans.} \]
4–82. Express the moment of the couple acting on the pipe assembly in Cartesian vector form. What is the magnitude of the couple moment?

**SOLUTION**

\[ \mathbf{r}_{CB} = (-3 \mathbf{i} - 2.5 \mathbf{j}) \text{ ft} \]

\[ \mathbf{M}_C = \mathbf{r}_{CB} \times \mathbf{F} \]

\[ = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2.5 & 0 \\ 0 & 0 & 20 \end{vmatrix} \]

\[ \mathbf{M}_C = (-50 \mathbf{i} + 60 \mathbf{j}) \text{ lb} \cdot \text{ft} \]

\[ M_C = \sqrt{(-50)^2 + (60)^2} = 78.1 \text{ lb} \cdot \text{ft} \]

Ans: \[ \mathbf{M}_C = 78.1 \text{ lb} \cdot \text{ft} \]
4–83.

If $M_1 = 180 \text{ lb-ft}, \ M_2 = 90 \text{ lb-ft}, \text{ and } M_3 = 120 \text{ lb-ft}$, determine the magnitude and coordinate direction angles of the resultant couple moment.

**SOLUTION**

Since the couple moment is a free vector, it can act at any point without altering its effect. Thus, the couple moments $M_1, M_2, M_3$, and $M_4$ acting on the gear deducer can be simplified, as shown in Fig. a. Expressing each couple moment in Cartesian vector form,

\[ M_1 = [180] \text{ lb-ft} \]
\[ M_2 = [-90i] \text{ lb-ft} \]
\[ M_3 = M_3 \mathbf{u} = 120 \left[ \frac{(2 - 0)i + (-2 - 0)j + (1 + 0)k}{\sqrt{(2 - 0)^2 + (-2 - 0)^2 + (1 - 0)^2}} \right] = [80i - 80j + 40k] \text{ lb-ft} \]
\[ M_4 = 150[\cos 45^\circ \sin 45^\circ i - \cos 45^\circ \cos 45^\circ j - \sin 45^\circ k] = [75i - 75j - 106.07k] \text{ lb-ft} \]

The resultant couple moment is given by

\[ (\mathbf{M}_c)_R = \sum \mathbf{M}_i; \quad (\mathbf{M}_c)_R = M_1 + M_2 + M_3 + M_4 \]
\[ = 180j - 90i + (80i - 80j + 40k) + (75i - 75j - 106.07k) \]
\[ = [65i + 25j - 66.07k] \text{ lb-ft} \]

The magnitude of $(\mathbf{M}_c)_R$ is

\[ (\mathbf{M}_c)_R = \sqrt{[(\mathbf{M}_c)_R]_x^2 + [(\mathbf{M}_c)_R]_y^2 + [(\mathbf{M}_c)_R]_z^2} \]
\[ = \sqrt{(65)^2 + (25)^2 + (-66.07)^2} \]
\[ = 95.99 \text{ lb-ft} = 96.0 \text{ lb-ft} \quad \text{Ans.} \]

The coordinate angles of $(\mathbf{M}_c)_R$ are

\[ \alpha = \cos^{-1} \left( \frac{[(\mathbf{M}_c)_R]_x}{(\mathbf{M}_c)_R} \right) = \cos \left( \frac{65}{95.99} \right) = 47.4^\circ \quad \text{Ans.} \]
\[ \beta = \cos^{-1} \left( \frac{[(\mathbf{M}_c)_R]_y}{(\mathbf{M}_c)_R} \right) = \cos \left( \frac{25}{95.99} \right) = 74.9^\circ \quad \text{Ans.} \]
\[ \gamma = \cos^{-1} \left( \frac{[(\mathbf{M}_c)_R]_z}{(\mathbf{M}_c)_R} \right) = \cos \left( \frac{-66.07}{95.99} \right) = 133^\circ \quad \text{Ans.} \]

Ans:

$M_R = 96.0 \text{ lb-ft}, \alpha = 47.4^\circ, \beta = 74.9^\circ, \gamma = 133^\circ$
Determine the magnitudes of couple moments $M_1$, $M_2$, and $M_3$ so that the resultant couple moment is zero.

**SOLUTION**

Since the couple moment is a free vector, it can act at any point without altering its effect. Thus, the couple moments $M_1$, $M_2$, $M_3$, and $M_4$ acting on the gear deducer can be simplified, as shown in Fig. a. Expressing each couple moment in Cartesian vector form,

$M_1 = M_2 \hat{j}$

$M_2 = -M_3 \hat{i}$

$M_3 = M_3 \hat{u} = M_3 \left[ \frac{(2 - 0)i + (-2 - 0)j + (1 + 0)k}{\sqrt{(2 - 0)^2 + (-2 - 0)^2 + (1 - 0)^2}} \right] = \frac{2}{3}M_3 \hat{i} - \frac{2}{3}M_3 \hat{j} + \frac{1}{3}M_3 \hat{k}$

$M_4 = 150[\cos 45^\circ \sin 45^\circ \hat{i} - \cos 45^\circ \cos 45^\circ \hat{j} - \sin 45^\circ \hat{k}] = [75\hat{i} - 75\hat{j} - 106.07\hat{k}] \text{lb} \cdot \text{ft}$

The resultant couple moment is required to be zero. Thus,

$$(M_c)_R = \sum M_c = 0 = M_1 + M_2 + M_3 + M_4$$

$$0 = M_1 \hat{j} + (-M_3 \hat{i}) + \left( \frac{2}{3}M_3 \hat{i} - \frac{2}{3}M_3 \hat{j} + \frac{1}{3}M_3 \hat{k} \right) + (75\hat{i} - 75\hat{j} - 106.07\hat{k})$$

$$0 = \left( -M_2 + \frac{2}{3}M_3 + 75 \right) \hat{i} + \left( M_1 - \frac{2}{3}M_3 - 75 \right) \hat{j} + \left( \frac{1}{3}M_3 - 106.07 \right) \hat{k}$$

Equating the $\hat{i}$, $\hat{j}$, and $\hat{k}$ components,

$$0 = -M_2 + \frac{2}{3}M_3 + 75 \quad \text{(1)}$$

$$0 = M_1 - \frac{2}{3}M_3 - 75 \quad \text{(2)}$$

$$0 = \frac{1}{3}M_3 - 106.07 \quad \text{(3)}$$

Solving Eqs. (1), (2), and (3) yields

$M_3 = 318 \text{ lb} \cdot \text{ft}$

$M_1 = M_2 = 287 \text{ lb} \cdot \text{ft}$
4–85.
The gears are subjected to the couple moments shown. Determine the magnitude and coordinate direction angles of the resultant couple moment.

**SOLUTION**

\[
M_1 = 40 \cos 20^\circ \sin 15^\circ \mathbf{i} + 40 \cos 20^\circ \cos 15^\circ \mathbf{j} - 40 \sin 20^\circ \mathbf{k} \\
= 9.728 \mathbf{i} + 36.307 \mathbf{j} - 13.681 \mathbf{k}
\]

\[
M_2 = -30 \sin 30^\circ \mathbf{i} + 30 \cos 30^\circ \mathbf{j} \\
= -15 \mathbf{i} + 25.981 \mathbf{j}
\]

\[
M_R = M_1 + M_2 = -5.272 \mathbf{i} + 62.288 \mathbf{j} - 13.681 \mathbf{k}
\]

\[
M_R = \sqrt{(-5.272)^2 + (62.288)^2 + (-13.681)^2} = 63.990 = 64.0 \text{ lb} \cdot \text{ft}
\]

\[
\alpha = \cos^{-1} \left( \frac{-5.272}{63.990} \right) = 94.7^\circ
\]

\[
\beta = \cos^{-1} \left( \frac{62.288}{63.990} \right) = 13.2^\circ
\]

\[
\gamma = \cos^{-1} \left( \frac{-13.681}{63.990} \right) = 102^\circ
\]

**Ans:**

\[
M_R = 64.0 \text{ lb} \cdot \text{ft} \\
\alpha = 94.7^\circ \\
\beta = 13.2^\circ \\
\gamma = 102^\circ
\]
4–86.

Determine the required magnitude of the couple moments \( M_2 \) and \( M_3 \) so that the resultant couple moment is zero.

**SOLUTION**

Since the couple moment is the free vector, it can act at any point without altering its effect. Thus, the couple moments \( M_1, M_2, \) and \( M_3 \) can be simplified as shown in Fig. a. Since the resultant of \( M_1, M_2, \) and \( M_3 \) is required to be zero,

\[
\begin{align*}
(M_R)_y &= \sum M_y ; \quad 0 = M_2 \sin 45^\circ - 300 \\
(M_R)_x &= \sum M_x ; \quad 0 = 424.26 \cos 45^\circ - M_3 \\
\end{align*}
\]

Ans. \( M_2 = 424.26 \text{ N} \cdot \text{m} = 424 \text{ N} \cdot \text{m} \)

Ans. \( M_3 = 300 \text{ N} \cdot \text{m} \)
4–87.

Determine the resultant couple moment of the two couples that act on the assembly. Specify its magnitude and coordinate direction angles.

**SOLUTION**

\[
M_R = \begin{vmatrix}
i & j & k \\
4 \cos 30^\circ & 5 & -4 \sin 30^\circ \\
0 & 0 & 60 \\
4 \cos 30^\circ & 0 & -4 \sin 30^\circ \\
0 & 80 & 0 \\
\end{vmatrix} + \begin{vmatrix}
i & j & k \\
4 \cos 30^\circ & 0 & -4 \sin 30^\circ \\
0 & 80 & 0 \\
4 \cos 30^\circ & 5 & -4 \sin 30^\circ \\
0 & 0 & 60 \\
\end{vmatrix}
\]

\[
= 300 \mathbf{i} - 207.85 \mathbf{j} + 160 \mathbf{i} + 277.13 \mathbf{k}
\]

\[
= [460 \mathbf{i} - 207.85 \mathbf{j} + 277.13 \mathbf{k}] \text{ lb} \cdot \text{in.}
\]

\[
M_R = \sqrt{(460)^2 + (-207.85)^2 + (277.13)^2} = 575.85 = 576 \text{ lb} \cdot \text{in.}
\]

\[
\alpha = \cos^{-1}\left(\frac{460}{575.85}\right) = 37.0^\circ
\]

\[
\beta = \cos^{-1}\left(\frac{-207.85}{575.85}\right) = 111^\circ
\]

\[
\gamma = \cos^{-1}\left(\frac{277.13}{575.85}\right) = 61.2^\circ
\]
Express the moment of the couple acting on the frame in Cartesian vector form. The forces are applied perpendicular to the frame. What is the magnitude of the couple moment? Take $F = 50 \text{ N}$. 

**SOLUTION**

$$M_C = 80(1.5) = 75 \text{ N} \cdot \text{m}$$

$$M_C = -75(\cos 30^\circ \hat{i} + \cos 60^\circ \hat{k})$$

$$= \{-65.0\hat{i} - 37.5\hat{k}\} \text{ N} \cdot \text{m}$$
In order to turn over the frame, a couple moment is applied as shown. If the component of this couple moment along the x axis is \( M_x = \{-20i\} \text{ N} \cdot \text{m} \), determine the magnitude \( F \) of the couple forces.

**SOLUTION**

\[ M_C = F (1.5) \]

Thus

\[ 20 = F (1.5) \cos 30° \]

\[ F = 15.4 \text{ N} \]
4–90.

Express the moment of the couple acting on the pipe in Cartesian vector form. What is the magnitude of the couple moment? Take $F = 125$ N.

**SOLUTION**

$$M_C = r_{AB} \times (125 \mathbf{k})$$

$$M_C = (0.2i + 0.3j) \times (125 \mathbf{k})$$

$$M_C = [37.5i - 25j] \text{ N} \cdot \text{m}$$

$$M_C = \sqrt{(37.5)^2 + (-25)^2} = 45.1 \text{ N} \cdot \text{m}$$

**Ans:**

$$M_C = 45.1 \text{ N} \cdot \text{m}$$
4–91.

If the couple moment acting on the pipe has a magnitude of 300 N \cdot m, determine the magnitude \( F \) of the forces applied to the wrenches.

**SOLUTION**

\[
M_C = \mathbf{r}_{AB} \times (F \mathbf{k})
\]

\[
= (0.2 \mathbf{i} + 0.3 \mathbf{j}) \times (F \mathbf{k})
\]

\[
= [0.2Fi - 0.3Fj] \text{ N} \cdot \text{m}
\]

\[
M_C = F \sqrt{(0.2F)^2 + (-0.3F)^2} = 0.3606 \quad F
\]

\[300 = 0.3606 \quad F\]

\[F = 832 \quad \text{N}\]

**Ans:**

\[F = 832 \quad \text{N}\]
If $F = 80$ N, determine the magnitude and coordinate direction angles of the couple moment. The pipe assembly lies in the $x$–$y$ plane.

**SOLUTION**

It is easiest to find the couple moment of $F$ by taking the moment of $F$ or $-F$ about point $A$ or $B$, respectively, Fig. (a). Here the position vectors $r_{BA}$ and $r_{AB}$ must be determined first.

\[
\begin{align*}
 r_{AB} & = (0.3 - 0.2)i + (0.8 - 0.3)j + (0 - 0)k = [0.1i + 0.5j] \text{ m} \\
 r_{BA} & = (0.2 - 0.3)i + (0.3 - 0.8)j + (0 - 0)k = [-0.1i - 0.5j] \text{ m}
\end{align*}
\]

The force vectors $F$ and $-F$ can be written as

\[
\begin{align*}
 F & = [80 \mathbf{k}] \text{ N} \quad \text{and} \quad -F = [-80 \mathbf{k}] \text{ N}
\end{align*}
\]

Thus, the couple moment of $F$ can be determined from

\[
\begin{align*}
 M_c & = r_{AB} \times F = \begin{vmatrix} i & j & k \\ 0.1 & 0.5 & 0 \\ 0 & 0 & 80 \end{vmatrix} = [40i - 8j] \text{ N} \cdot \text{m} \\
\text{or} \\
 M_c & = r_{BA} \times -F = \begin{vmatrix} i & j & k \\ -0.1 & -0.5 & 0 \\ 0 & 0 & -80 \end{vmatrix} = [40i - 8j] \text{ N} \cdot \text{m}
\end{align*}
\]

The magnitude of $M_c$ is given by

\[
M_c = \sqrt{M_x^2 + M_y^2 + M_z^2} = \sqrt{40^2 + (-8)^2 + 0^2} = 40.79 \text{ N} \cdot \text{m} = 40.8 \text{ N} \cdot \text{m}
\]

The coordinate angles of $M_c$ are

\[
\begin{align*}
 \alpha & = \cos^{-1} \left( \frac{M_x}{M} \right) = \cos \left( \frac{40}{40.79} \right) = 11.3^\circ \\
 \beta & = \cos^{-1} \left( \frac{M_y}{M} \right) = \cos \left( \frac{-8}{40.79} \right) = 101^\circ \\
 \gamma & = \cos^{-1} \left( \frac{M_z}{M} \right) = \cos \left( \frac{0}{40.79} \right) = 90^\circ
\end{align*}
\]

**Ans:**

$M_c = 40.8 \text{ N} \cdot \text{m}$

$\alpha = 11.3^\circ$

$\beta = 101^\circ$

$\gamma = 90^\circ$
4–93.

If the magnitude of the couple moment acting on the pipe assembly is 50 N ⋅ m, determine the magnitude of the couple forces applied to each wrench. The pipe assembly lies in the x–y plane.

**SOLUTION**

It is easiest to find the couple moment of \( \mathbf{F} \) by taking the moment of either \( \mathbf{F} \) or \(-\mathbf{F}\) about point \( A \) or \( B \), respectively, Fig. a. Here the position vectors \( \mathbf{r}_{AB} \) and \( \mathbf{r}_{BA} \) must be determined first.

\[
\mathbf{r}_{AB} = (0.3 - 0.2)i + (0.8 - 0.3)j + (0 - 0)k = [0.1i + 0.5j] \text{ m}
\]

\[
\mathbf{r}_{BA} = (0.2 - 0.3)i + (0.3 - 0.8)j + (0 - 0)k = [-0.1i - 0.5j] \text{ m}
\]

The force vectors \( \mathbf{F} \) and \(-\mathbf{F}\) can be written as

\( \mathbf{F} = [F_k] \text{ N} \) and \(-\mathbf{F} = [-F_k] \text{ N} \)

Thus, the couple moment of \( \mathbf{F} \) can be determined from

\[
\mathbf{M}_c = \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} i & j & k \\ 0.1 & 0.5 & 0 \\ 0 & 0 & F \end{vmatrix} = 0.5Fi - 0.1Fj
\]

The magnitude of \( M_c \) is given by

\[
M_c = \sqrt{M_x^2 + M_y^2 + M_z^2} = \sqrt{(0.5F)^2 + (0.1F)^2 + 0^2} = 0.5099F
\]

Since \( M_c \) is required to equal 50 N ⋅ m,

\[
50 = 0.5099F
\]

\[
F = 98.1 \text{ N}
\]

Ans: \( F = 98.1 \text{ N} \)
4–94.

Express the moment of the couple acting on the rod in Cartesian vector form. What is the magnitude of the couple moment?

**SOLUTION**

**Position Vector.** The coordinates of points $A$ and $B$ are $A (0, 0, 1)$ m and $B (3, 2, -1)$ m, respectively. Thus,

$$\mathbf{r}_{AB} = (3 - 0)\mathbf{i} + (2 - 0)\mathbf{j} + (-1 - 1)\mathbf{k} = [3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}] \text{ m}$$

**Couple Moment.**

$$\mathbf{M}_C = \mathbf{r}_{AB} \times \mathbf{F}$$

$$\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
3 & 2 & -2 \\
-4 & 3 & -4
\end{vmatrix}$$

$$= [-2\mathbf{i} + 20\mathbf{j} + 17\mathbf{k}] \text{ kN} \cdot \text{m}$$

**Ans.**

The magnitude of $\mathbf{M}_C$ is

$$M_C = \sqrt{(M_C)_x^2 + (M_C)_y^2 + (M_C)_z^2}$$

$$= \sqrt{(-2)^2 + 20^2 + 17^2}$$

$$= 26.32 \text{ kN} \cdot \text{m} = 26.3 \text{ kN} \cdot \text{m}$$

**Ans.**
If \( F_1 = 100 \text{ N} \), \( F_2 = 120 \text{ N} \) and \( F_3 = 80 \text{ N} \), determine the magnitude and coordinate direction angles of the resultant couple moment.

**SOLUTION**

**Couple Moment:** The position vectors \( r_1, r_2, r_3, \) and \( r_4 \), Fig. a, must be determined first.

\[
r_1 = [0.2i] \text{ m} \\
r_2 = [0.2j] \text{ m} \\
r_3 = [0.2j] \text{ m}
\]

From the geometry of Figs. b and c, we obtain

\[
r_4 = 0.3 \cos 30^\circ \cos 45^\circ i + 0.3 \cos 30^\circ \sin 45^\circ j - 0.3 \sin 30^\circ k
\]

\[
= [0.1837i + 0.1837j - 0.15k] \text{ m}
\]

The force vectors \( F_1, F_2, \) and \( F_3 \) are given by

\[
F_1 = [100k] \text{ N} \\
F_2 = [120k] \text{ N} \\
F_3 = [80i] \text{ N}
\]

Thus,

\[
M_1 = r_1 \times F_1 = (0.2i) \times (100k) = [-20j] \text{ N} \cdot \text{m}
\]

\[
M_2 = r_2 \times F_2 = (0.2j) \times (120k) = [24i] \text{ N} \cdot \text{m}
\]

\[
M_3 = r_3 \times F_3 = (0.2j) \times (80i) = [-16k] \text{ N} \cdot \text{m}
\]

\[
M_4 = r_4 \times F_4 = (0.1837i + 0.1837j - 0.15k) \times (150k) = [27.56i - 27.56j] \text{ N} \cdot \text{m}
\]

**Resultant Moment:** The resultant couple moment is given by

\[
(M_c)_R = \sum M_i = M_1 + M_2 + M_3 + M_4
\]

\[
= (-20j) + (24i) + (-16k) + (27.56i - 27.56j)
\]

\[
= [51.56i - 47.56j - 16k] \text{ N} \cdot \text{m}
\]

The magnitude of the couple moment is

\[
(M_c)_R = \sqrt{[(M_c)_R]_x^2 + [(M_c)_R]_y^2 + [(M_c)_R]_z^2}
\]

\[
= \sqrt{(51.56)^2 + (-47.56)^2 + (-16)^2}
\]

\[
= 71.94 \text{ N} \cdot \text{m}
\]

The coordinate angles of \( (M_c)_R \) are

\[
\alpha = \cos^{-1}\left(\frac{[(M_c)_R]_x}{(M_c)_R}\right) = \cos\left(\frac{51.56}{71.94}\right) = 44.2^\circ
\]

\[
\beta = \cos^{-1}\left(\frac{[(M_c)_R]_y}{(M_c)_R}\right) = \cos\left(\frac{-47.56}{71.94}\right) = 131^\circ
\]

\[
\gamma = \cos^{-1}\left(\frac{[(M_c)_R]_z}{(M_c)_R}\right) = \cos\left(\frac{-16}{71.94}\right) = 103^\circ
\]
Determine the required magnitude of \( F_1, F_2, \) and \( F_3 \) so that the resultant couple moment is \((M_c)_R = [50\,\text{i} - 45\,\text{j} - 20\,\text{k}]\,\text{N}\cdot\text{m}\).

**SOLUTION**

**Couple Moment:** The position vectors \( r_1, r_2, r_3, \) and \( r_4 \), Fig. a, must be determined first.

\[
\begin{align*}
 r_1 &= [0.2\,\text{i}]\,\text{m} & r_2 &= [0.2\,\text{j}]\,\text{m} & r_3 &= [0.2\,\text{j}]\,\text{m}
\end{align*}
\]

From the geometry of Figs. b and c, we obtain

\[
\begin{align*}
 r_4 &= 0.3\,\cos 30^\circ\,\cos 45^\circ\,\text{i} + 0.3\,\cos 30^\circ\,\sin 45^\circ\,\text{j} - 0.3\,\sin 30^\circ\,\text{k} \\
 &= [0.1837\,\text{i} + 0.1837\,\text{j} - 0.15\,\text{k}]\,\text{m}
\end{align*}
\]

The force vectors \( F_1, F_2, \) and \( F_3 \) are given by

\[
\begin{align*}
 F_1 &= F_3\,\text{k} & F_2 &= F_3\,\text{k} & F_3 &= F_3\,\text{k}
\end{align*}
\]

Thus,

\[
\begin{align*}
 M_1 &= r_1 \times F_1 = (0.2\,\text{i}) \times (F_3\,\text{k}) = -0.2\,F_3\,\text{j} \\
 M_2 &= r_2 \times F_2 = (0.2\,\text{j}) \times (F_3\,\text{k}) = 0.2\,F_3\,\text{i} \\
 M_3 &= r_3 \times F_3 = (0.2\,\text{j}) \times (F_3\,\text{k}) = -0.2\,F_3\,\text{k} \\
 M_4 &= r_4 \times F_4 = (0.1837\,\text{i} + 0.1837\,\text{j} - 0.15\,\text{k}) \times (150\,\text{k}) = [27.56\,\text{i} - 27.56\,\text{j}]\,\text{N}\cdot\text{m}
\end{align*}
\]

**Resultant Moment:** The resultant couple moment required to equal \((M_c)_R = [50\,\text{i} - 45\,\text{j} - 20\,\text{k}]\,\text{N}\cdot\text{m}\). Thus,

\[
\begin{align*}
 (M_c)_R &= \sum M_i \\
 (M_c)_R &= M_1 + M_2 + M_3 + M_4 \\
 &= 50\,\text{i} - 45\,\text{j} - 20\,\text{k} = (-0.2F_1\,\text{j}) + (0.2F_2\,\text{j}) + (-0.2F_3\,\text{k}) + (27.56\,\text{i} - 27.56\,\text{j}) \\
 &= 50\,\text{i} - 45\,\text{j} - 20\,\text{k} = (0.2F_2 + 27.56)\,\text{i} + (-0.2F_1 - 27.56)\,\text{j} - 0.2F_3\,\text{k}
\end{align*}
\]

Equating the \( \text{i}, \text{j}, \) and \( \text{k} \) components yields

\[
\begin{align*}
 50 &= 0.2F_2 + 27.56 & F_2 &= 112\,\text{N} & \text{Ans.} \\
 -45 &= -0.2F_1 - 27.56 & F_1 &= 87.2\,\text{N} & \text{Ans.} \\
 -20 &= -0.2F_3 & F_3 &= 100\,\text{N} & \text{Ans.}
\end{align*}
\]
4–97.
Replace the force system by an equivalent resultant force and couple moment at point $O$.

**SOLUTION**

Equivalent Resultant Force And Couple Moment At $O$.

\[
\begin{align*}
\sum (F_R)_x &= \sum F_i; \quad (F_R)_x = 600 \cos 60^\circ - 455\left(\frac{12}{13}\right) = -120 \text{ N} = 120 \text{ N} \leftarrow \\
\sum (F_R)_y &= \sum F_i; \quad (F_R)_y = 455\left(\frac{5}{13}\right) - 600 \sin 60^\circ = -344.62 \text{ N} = 344.62 \text{ N} \downarrow \\
\end{align*}
\]

As indicated in Fig. $a$

\[F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{120^2 + 344.62^2} = 364.91 \text{ N} = 365 \text{ N} \quad \text{Ans.}\]

And

\[\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{344.62}{120}\right) = 70.8^\circ = 70.8^\circ \uparrow \quad \text{Ans.}\]

Also,

\[\zeta + (M_R)_O = \sum M_O; \quad (M_R)_O = 455\left(\frac{12}{13}\right)(2) + 600 \cos 60^\circ (0.75) + 600 \sin 60^\circ (2.5)\]
\[= 2364.04 \text{ N} \cdot \text{m}\]
\[= 2364 \text{ N} \cdot \text{m (counterclockwise)} \quad \text{Ans.}\]
Replace the force system by an equivalent resultant force and couple moment at point $P$.

**SOLUTION**

Equivalent Resultant Force And Couple Moment At $P$.

$$
\sum (F_R)_x = \Sigma F_i; \quad (F_R)_x = 600 \cos 60^\circ - 455 \left( \frac{12}{13} \right) = -120 \text{ N} = 120 \text{ N}
$$

$$
\sum (F_R)_y = \Sigma F_i; \quad (F_R)_y = 455 \left( \frac{5}{13} \right) - 600 \sin 60^\circ = -344.62 \text{ N} = 344.62 \text{ N}
$$

As indicated in Fig. a,

$$
F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{120^2 + 344.62^2} = 364.91 \text{ N} = 365 \text{ N}
$$

Ans.

And

$$
\theta = \tan^{-1} \left[ \frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left( \frac{344.62}{120} \right) = 70.8^\circ = 70.8^\circ \text{ v}\text{p}
$$

Ans.

Also,

$$
\zeta + (M_R)_P = \Sigma M_i; \quad (M_R)_P = 455 \left( \frac{12}{13} \right) (2.75) - 455 \left( \frac{5}{13} \right) (1) + 600 \sin 60^\circ (3.5)
$$

$$
= 2798.65 \text{ N} \cdot \text{m}
$$

$$
= 2799 \text{ N} \cdot \text{m (counterclockwise)}
$$

Ans.
4-99.

Replace the force system acting on the beam by an equivalent force and couple moment at point A.

**SOLUTION**

\[ \sum F_x = \sum F_y; \quad F_{R_x} = 1.5 \sin 30^\circ - 2.5 \left( \frac{4}{5} \right) \]

\[ = -1.25 \text{ kN} = -1.25 \text{ kN} \]

\[ \sum F_y = \sum F_x; \quad F_{R_y} = -1.5 \cos 30^\circ - 2.5 \left( \frac{3}{5} \right) - 3 \]

\[ = -5.799 \text{ kN} = -5.799 \text{ kN} \]

Thus,

\[ F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{(-1.25)^2 + (-5.799)^2} = 5.93 \text{ kN} \]

and

\[ \theta = \tan^{-1} \left( \frac{F_{R_y}}{F_{R_x}} \right) = \tan^{-1} \left( \frac{-5.799}{1.25} \right) = 77.8^\circ \]

\[ \zeta + M_{RA} = \Sigma M_A; \quad M_{RA} = -2.5 \left( \frac{3}{5} \right)(2) - 1.5 \cos 30^\circ(6) - 3(8) \]

\[ = -34.8 \text{ kN} \cdot \text{m} = -34.8 \text{ kN} \cdot \text{m} \text{ (Clockwise)} \]

**Ans:**

\[ F_R = 5.93 \text{ kN} \]

\[ \theta = 77.8^\circ \]

\[ M_{RA} = -34.8 \text{ kN} \cdot \text{m} \]

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Replace the force system acting on the beam by an equivalent force and couple moment at point B.

**SOLUTION**

\[ \sum F_x = \Sigma F_y; \quad F_{R_x} = 1.5 \sin 30^\circ - 2.5\left(\frac{4}{5}\right) \]
\[ = -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow \]

\[ + \uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = -1.5 \cos 30^\circ - 2.5\left(\frac{3}{5}\right) - 3 \]
\[ = -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow \]

Thus,

\[ F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN} \]

and

\[ \theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8^\circ \uparrow \]

\[ \zeta + M_{R_y} = \Sigma M_{R_y}; \quad M_B = 1.5 \cos 30^\circ (2) + 2.5\left(\frac{3}{5}\right)(6) \]
\[ = 11.6 \text{ kN} \cdot \text{m (Counterclockwise)} \]

**Ans:**

\[ F_R = 5.93 \text{ kN} \]
\[ \theta = 77.8^\circ \uparrow \]
\[ M_B = 11.6 \text{ kN} \cdot \text{m (Counterclockwise)} \]
4–101.
Replace the loading system acting on the beam by an equivalent resultant force and couple moment at point $O$.

**SOLUTION**

\[ F_{Rx} = \Sigma F_x; \quad F_{Rx} = 450 \sin 30^\circ = 225.0 \]

\[ F_{Ry} = \Sigma F_y; \quad F_{Ry} = 450 \cos 30^\circ - 200 = 189.7 \]

\[ F_R = \sqrt{(225)^2 + (189.7)^2} = 294 \text{ N} \]

\[ \theta = \tan^{-1} \left( \frac{189.7}{225} \right) = 40.1^\circ \]

\[ M_{RO} = \Sigma M_O; \quad M_{RO} = 450 \cos 30^\circ (1.5) - 450 \sin 30^\circ (0.2) - 200 (3.5) + 200 \]

\[ M_{RO} = 39.6 \text{ N} \cdot \text{m} \]

Ans:

\[ F_R = 294 \text{ N} \]

\[ \theta = 40.1^\circ \]

\[ M_{RO} = 39.6 \text{ N} \cdot \text{m} \]
4–102.

Replace the loading system acting on the post by an equivalent resultant force and couple moment at point A.

**SOLUTION**

**Equivalent Resultant Force And Couple Moment at Point A.**

\[
\begin{align*}
\sum (F_R)_x &= \Sigma F_x; \quad (F_R)_x = 650 \sin 30^\circ - 500 \cos 60^\circ = 75 \text{ N} \\
\sum (F_R)_y &= \Sigma F_y; \quad (F_R)_y = -650 \cos 30^\circ - 300 - 500 \sin 60^\circ = -1295.93 \text{ N} = 1295.93 \text{ N}
\end{align*}
\]

As indicated in Fig. 4–102,

\[F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{75^2 + 1295.93^2} = 1298.10 \text{ N} = 1.30 \text{ kN} \quad \text{Ans.}\]

And

\[\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{1295.93}{75}\right) = 86.69^\circ = 86.7^\circ \quad \text{Ans.}\]

Also,

\[\sum (M_R)_A = \Sigma M_A; \quad (M_R)_A = 650 \cos 30^\circ (3) + 1500 - 500 \sin 60^\circ (5) = 1023.69 \text{ N} \cdot \text{m} = 1.02 \text{ kN} \cdot \text{m (counter clockwise)} \quad \text{Ans.}\]
4–103.
Replace the loading system acting on the post by an equivalent resultant force and couple moment at point B.

**SOLUTION**

**Equivalent Resultant Force And Couple Moment At Point B.**

\[ (F_R)_x = \Sigma F_x; \quad (F_R)_x = 650 \sin 30^\circ - 500 \cos 60^\circ = 75 \text{ N} \rightarrow \]
\[ + \uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = -650 \cos 30^\circ - 300 - 500 \sin 60^\circ \]
\[ = -1295.93 \text{ N} = 1295.93 \text{ N} \downarrow \]

As indicated in Fig. a,

\[ F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{75^2 + 1295.93^2} = 1298.10 \text{ N} = 1.30 \text{ kN} \text{ Ans.} \]

And

\[ \theta = \tan^{-1}\left( \frac{(F_R)_y}{(F_R)_x} \right) = \tan^{-1}\left( \frac{1295.93}{75} \right) = 86.69^\circ = 86.7^\circ \approx \]

\[ \text{Ans.} \]

Also,

\[ \zeta + (M_R)_B = \Sigma M_B; \quad (M_R)_B = 650 \cos 30^\circ (10) + 300(7) + 500 \sin 60^\circ (2) + 1500 \]
\[ = 10,095.19 \text{ N} \cdot \text{m} \]
\[ = 10.1 \text{ kN} \cdot \text{m} \text{ (counterclockwise)} \text{ Ans.} \]

**Ans:**

\[ F_R = 1.30 \text{ kN} \]
\[ \theta = 86.7^\circ \approx \]
\[ (M_R)_B = 1.01 \text{ kN} \cdot \text{m} \text{ (counterclockwise)} \]**
Replace the force system acting on the post by a resultant force and couple moment at point $O$.

**SOLUTION**

*Equivalent Resultant Force:* Forces $F_1$ and $F_2$ are resolved into their $x$ and $y$ components, Fig. $a$. Summing these force components algebraically along the $x$ and $y$ axes, we have

$$ \sum (F_R)_x = \sum F_x; \quad (F_R)_x = 300 \cos 30^\circ - 150\left(\frac{4}{5}\right) + 200 = 339.81 \text{ lb} \to $$

$$ + \sum (F_R)_y = \sum F_y; \quad (F_R)_y = 300 \sin 30^\circ + 150\left(\frac{3}{5}\right) = 240 \text{ lb} \uparrow $$

The magnitude of the resultant force $F_R$ is given by

$$ F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{339.81^2 + 240^2} = 416.02 \text{ lb} = 416 \text{ lb} \quad \text{Ans.} $$

The angle $\theta$ of $F_R$ is

$$ \theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{240}{339.81}\right] = 35.23^\circ = 35.2^\circ \angle \theta \quad \text{Ans.} $$

*Equivalent Resultant Couple Moment:* Applying the principle of moments, Figs. $a$ and $b$, and summing the moments of the force components algebraically about point $A$, we can write

$$ \zeta + (M_R)_A = \sum M_A; \quad (M_R)_A = 150\left(\frac{4}{5}\right)(4) - 200(2) - 300 \cos 30^\circ(6) $$

$$ = -1478.85 \text{ lb} \cdot \text{ft} = 1.48 \text{ kip} \cdot \text{ft} \quad \text{(Clockwise)} \quad \text{Ans.} $$

\[ \text{Ans:} \]

$$ F_R = 416 \text{ lb} $$

$$ \theta = 35.2^\circ \angle \theta $$

$$ (M_R)_A = 1.48 \text{ kip} \cdot \text{ft} \quad \text{(Clockwise)} \]
4–105.

Replace the force system acting on the frame by an equivalent resultant force and couple moment acting at point A.

**SOLUTION**

**Equivalent Resultant Force And Couple Moment At A.**

\[
\begin{align*}
\pm (F_R)_x &= \Sigma F_x; \quad (F_R)_x = 300 \cos 30^\circ + 500 = 759.81 \text{ N} \quad \rightarrow \\
+\uparrow (F_R)_y &= \Sigma F_y; \quad (F_R)_y = -300 \sin 30^\circ - 400 = -550 = 550 \text{ N} \quad \downarrow \\
\end{align*}
\]

As indicated in Fig. a,

\[
F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{759.81^2 + 550^2} = 937.98 \text{ N} = 938 \text{ N} \quad \text{Ans.}
\]

And

\[
\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{550}{759.81}\right) = 35.90^\circ = 35.9^\circ \quad \text{Ans.}
\]

Also;

\[
s+(M_R)_A = \Sigma M_A; \quad (M_R)_A = 300 \cos 30^\circ(0.5) + 500(1.5) - 400(0.5) \\
= 679.90 \text{ N} \cdot \text{m} \\
= 680 \text{ N} \cdot \text{m} \quad \text{(counterclockwise)} \quad \text{Ans.}
\]
4–106.

The forces \( F_1 = \{-4i + 2j - 3k\} \) kN and \( F_2 = \{3i - 4j - 2k\} \) kN act on the end of the beam. Replace these forces by an equivalent force and couple moment acting at point \( O \).

**SOLUTION**

\[
\begin{align*}
F_R &= F_1 + F_2 = \{-1i - 2j - 5k\} \text{ kN} \\
M_{RO} &= r_1 \times F_1 + r_2 \times F_2 \\
    &= \begin{vmatrix}
i & j & k \\
4 & -0.15 & 0.25 \\
-4 & 2 & -3 \\
\end{vmatrix} + \begin{vmatrix}
i & j & k \\
4 & 0.15 & 0.25 \\
3 & -4 & -2 \\
\end{vmatrix} \\
    &= (-0.05i + 11j + 7.4k) + (0.7i + 8.75j - 16.45k) \\
    &= (0.65i + 19.75j - 9.05k) \\
M_{RO} &= [0.650i + 19.75j - 9.05k] \text{ kN} \cdot \text{m} \\
\end{align*}
\]

**Ans:**

\[ M_{RO} = [0.650i + 19.75j - 9.05k] \text{ kN} \cdot \text{m} \]
4–107.

A biomechanical model of the lumbar region of the human trunk is shown. The forces acting in the four muscle groups consist of $F_R = 35 \text{ N}$ for the rectus, $F_O = 45 \text{ N}$ for the oblique, $F_L = 23 \text{ N}$ for the lumbar latissimus dorsi, and $F_E = 32 \text{ N}$ for the erector spinae. These loadings are symmetric with respect to the $y$–$z$ plane. Replace this system of parallel forces by an equivalent force and couple moment acting at the spine, point $O$. Express the results in Cartesian vector form.

**SOLUTION**

$F_R = \sum F_z; \quad F_R = \{2(35 + 45 + 23 + 32)k\} = \{270k\} \text{ N}$  \hspace{1cm} \textbf{Ans.}

$M_{RO} = \sum M_O; \quad M_{RO} = [-2(35)(0.075) + 2(32)(0.015) + 2(23)(0.045)]i$  \hspace{1cm} \textbf{Ans.}

$M_{RO} = [-2.22i] \text{ N} \cdot \text{m}$

\[ \text{Ans:} \]

$F_R = \{270k\} \text{ N}$

$M_{RO} = \{-2.22i\} \text{ N} \cdot \text{m}$
Replace the force system by an equivalent resultant force and couple moment at point \(O\). Take \(F_3 = (-200i + 500j - 300k)\) N.

**SOLUTION**

**Position And Force Vectors.**

\[
\begin{align*}
 r_1 &= [2j] \text{ m} \\
 r_2 &= [1.5i + 3.5j] \text{ m} \\
 r_3 &= [1.5i + 2j] \text{ m} \\
 F_1 &= [-300k] \text{ N} \\
 F_2 &= [200j] \text{ N} \\
 F_3 &= [-200i + 500j - 300k] \text{ N}
\end{align*}
\]

**Equivalent Resultant Force And Couple Moment At Point \(O\).**

\[
\begin{align*}
 F_R &= \Sigma F_i \\
 &= F_1 + F_2 + F_3 \\
 &= (-300k) + 200j + (-200i + 500j - 300k) \\
 &= (-200i + 700j - 600k) \text{ N} \\
\end{align*}
\]

Ans.

\[
\begin{align*}
 (M_R)_O &= \Sigma M_O; \\
 (M_R)_O &= r_1 \times F_1 + r_2 \times F_2 + r_3 \times F_3 \\
 &= \begin{vmatrix}
 i & j & k \\
 0 & 2 & 0 \\
 0 & -300 & 0 \\
\end{vmatrix} + \begin{vmatrix}
 i & j & k \\
 1.5 & 3.5 & 0 \\
 0 & 200 & 0 \\
\end{vmatrix} + \begin{vmatrix}
 i & j & k \\
 1.5 & 2 & 0 \\
 -200 & 500 & -300 \\
\end{vmatrix} \\
 &= (-600i) + (300k) + (-600i + 450j + 1150k) \\
 &= [-1200i + 450j + 1450k] \text{ N} \cdot \text{m}
\end{align*}
\]

Ans.

\[
F_R = [-200i + 700j - 600k] \text{ N}
\]

\[
(M_R)_O = [-1200i + 450j + 1450k] \text{ N} \cdot \text{m}
\]
Replace the loading by an equivalent resultant force and couple moment at point $O$.

**SOLUTION**

**Position Vectors.** The required position vectors are

$$r_1 = [0.8i - 1.2k] \text{ m} \quad r_2 = [-0.5k] \text{ m}$$

**Equivalent Resultant Force And Couple Moment At Point $O$.**

$$F_R = \sum F; \quad F_R = F_1 + F_2$$

$$= (8i - 2k) + (-2i + 5j - 3k)$$

$$= [6i + 5j - 5k] \text{ kN} \quad \text{Ans.}$$

$$\begin{align*}
(M_{R})_O &= \sum M_O; \quad (M_{R})_O = r_1 \times F_1 + r_2 \times F_2 \\
&= \begin{vmatrix}
i & j & k \\
0.8 & 0 & -1.2 \\
8 & 0 & -2 \\
\end{vmatrix} + \begin{vmatrix}
i & j & k \\
0 & 0 & -0.5 \\
-2 & 5 & -3 \\
\end{vmatrix} \\
&= (-8j) + (2.5i + j) \\
&= [2.5i - 7j] \text{ kN} \cdot \text{m} \quad \text{Ans.}
\end{align*}$$

**Ans:**

$$F_R = [6i + 5j - 5k] \text{ kN}$$

$$\begin{align*}
(M_{R})_O &= [2.5i - 7j] \text{ kN} \cdot \text{m}
\end{align*}$$
4–110.
Replace the force of $F = 80 \text{ N}$ acting on the pipe assembly by an equivalent resultant force and couple moment at point $A$.

**SOLUTION**

$F_R = \Sigma F$;

$F_R = 80 \cos 30^\circ \sin 40^\circ \hat{i} + 80 \cos 30^\circ \cos 40^\circ \hat{j} - 80 \sin 30^\circ \hat{k}$

$= 44.53 \hat{i} + 53.07 \hat{j} - 40 \hat{k}$

$= [44.5 \hat{i} + 53.1 \hat{j} - 40 \hat{k}] \text{ N}$ \hspace{1cm} \text{Ans.}$

$M_{RA} = \Sigma M_A$; \hspace{1cm} $M_{RA} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.55 & 0.4 & -0.2 \\ 44.53 & 53.07 & -40 \end{vmatrix}$

$= [-5.39 \hat{i} + 13.1 \hat{j} + 11.4 \hat{k}] \text{ N} \cdot \text{m}$ \hspace{1cm} \text{Ans.}$

Ans:

$F_R = [44.5 \hat{i} + 53.1 \hat{j} + 40 \hat{k}] \text{ N}$

$M_{RA} = [-5.39 \hat{i} + 13.1 \hat{j} + 11.4 \hat{k}] \text{ N} \cdot \text{m}$
4-111.

The belt passing over the pulley is subjected to forces \( \mathbf{F}_1 \) and \( \mathbf{F}_2 \), each having a magnitude of 40 N. \( \mathbf{F}_1 \) acts in the \( -\mathbf{k} \) direction. Replace these forces by an equivalent force and couple moment at point \( A \). Express the result in Cartesian vector form. Set \( \theta = 0^\circ \) so that \( \mathbf{F}_2 \) acts in the \( -\mathbf{j} \) direction.

**SOLUTION**

\[
\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2
\]

\[
\mathbf{F}_R = \{-40 \mathbf{j} - 40 \mathbf{k}\} \text{ N}
\]

\[
\mathbf{M}_{RA} = \Sigma (\mathbf{r} \times \mathbf{F})
\]

\[
= \begin{vmatrix} i & j & k \\ -0.3 & 0 & 0.08 \\ 0 & -40 & 0 \\ \end{vmatrix} + \begin{vmatrix} i & j & k \\ -0.3 & 0.08 & 0 \\ 0 & 0 & -40 \\ \end{vmatrix}
\]

\[
\mathbf{M}_{RA} = \{-12 \mathbf{j} + 12 \mathbf{k}\} \text{ N} \cdot \text{m}
\]

**Ans:**

\[
\mathbf{F}_R = \{-40 \mathbf{j} - 40 \mathbf{k}\} \text{ N}
\]

\[
\mathbf{M}_{RA} = \{-12 \mathbf{j} + 12 \mathbf{k}\} \text{ N} \cdot \text{m}
\]
The belt passing over the pulley is subjected to two forces \( \mathbf{F}_1 \) and \( \mathbf{F}_2 \), each having a magnitude of 40 N. \( \mathbf{F}_1 \) acts in the \(-\mathbf{k}\) direction. Replace these forces by an equivalent force and couple moment at point \( A \). Express the result in Cartesian vector form. Take \( \theta = 45^{\circ} \).

**SOLUTION**

\[
\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2
\]
\[
= -40 \cos 45^\circ \hat{j} + (-40 - 40 \sin 45^\circ) \hat{k}
\]
\[
\mathbf{F}_R = \{-28.3 \hat{j} - 68.3 \hat{k}\} \text{ N}
\]
\[
r_{AF_1} = \{-0.3 \hat{i} + 0.08 \hat{j}\} \text{ m}
\]
\[
r_{AF_2} = -0.3 \hat{i} - 0.08 \sin 45^\circ \hat{j} + 0.08 \cos 45^\circ \hat{k}
\]
\[
= \{-0.3 \hat{i} - 0.0566 \hat{j} + 0.0566 \hat{k}\} \text{ m}
\]
\[
\mathbf{M}_{RA} = (r_{AF_1} \times \mathbf{F}_1) + (r_{AF_2} \times \mathbf{F}_2)
\]
\[
= \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
-0.3 & 0.08 & 0 \\
0 & 0 & -40 \\
\end{vmatrix}
+ \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
-0.3 & -0.0566 & 0.0566 \\
0 & -40 \cos 45^\circ & -40 \sin 45^\circ \\
\end{vmatrix}
\]
\[
\mathbf{M}_{RA} = \{-20.5 \hat{j} + 8.49 \hat{k}\} \text{ N} \cdot \text{m}
\]

Also,
\[
\mathbf{M}_{RA} = \Sigma \mathbf{M}_{Ai}
\]
\[
M_{RA} = 28.28(0.0566) + 28.28(0.0566) - 40(0.08)
\]
\[
M_{RA} = 0
\]
\[
\mathbf{M}_{RA} = \Sigma \mathbf{M}_{Ai}
\]
\[
M_{RA} = -28.28(0.3) - 40(0.3)
\]
\[
M_{RA} = -20.5 \text{ N} \cdot \text{m}
\]
\[
\mathbf{M}_{RA} = \Sigma \mathbf{M}_{Ai}
\]
\[
M_{RA} = 28.28(0.3)
\]
\[
M_{RA} = 8.49 \text{ N} \cdot \text{m}
\]
\[
\mathbf{M}_{RA} = \{-20.5 \hat{j} + 8.49 \hat{k}\} \text{ N} \cdot \text{m}
\]

\[
\mathbf{M}_{RA} = \{-20.5 \hat{j} + 8.49 \hat{k}\} \text{ N} \cdot \text{m}
\]

Ans:
\[
\mathbf{F}_R = \{-28.3 \hat{j} - 68.3 \hat{k}\} \text{ N}
\]
\[
\mathbf{M}_{RA} = \{-20.5 \hat{j} + 8.49 \hat{k}\} \text{ N} \cdot \text{m}
\]
4–113.

The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from $B$.

**SOLUTION**

\[ + \uparrow F_R = \Sigma F_y; \quad F_R = -1750 - 5500 - 3500 \]
\[ = -10\,750 \text{ lb} = 10.75 \text{ kip} \downarrow \quad \text{Ans.} \]

\[ \zeta + M_{RA} = \Sigma M_A; \quad -10\,750d = -3500(3) - 5500(17) - 1750(25) \]
\[ d = 13.7 \text{ ft} \quad \text{Ans.} \]
4–114.

The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from point A.

SOLUTION

Equivalent Force:

\[ + \uparrow F_R = \Sigma F_y; \quad F_R = -1750 - 5500 - 3500 \]
\[ = -10 \, 750 \, \text{lb} = 10.75 \, \text{kip} \downarrow \quad \text{Ans.} \]

Location of Resultant Force From Point A:

\[ \zeta + M_{R_A} = \Sigma M_A; \quad 10 \, 750(\text{d}) = 3500(20) + 5500(6) - 1750(2) \]
\[ \text{d} = 9.26 \, \text{ft} \quad \text{Ans.} \]
Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end A.

**SOLUTION**

\[ \pm F_R = \sum F_x; \quad F_R = -500 \left( \frac{4}{5} \right) + 260 \left( \frac{5}{13} \right) = -300 \text{ lb} = 300 \text{ lb} \uparrow \]

\[ \uparrow F_R = \sum F_y; \quad F_R = -500 \left( \frac{3}{5} \right) - 200 - 260 \left( \frac{12}{13} \right) = -740 \text{ lb} = 740 \text{ lb} \downarrow \]

\[ F = \sqrt{(-300)^2 + (-740)^2} = 798 \text{ lb} \quad \text{Ans.} \]

\[ \theta = \tan^{-1} \left( \frac{740}{300} \right) = 67.9^\circ \quad \text{Ans.} \]

\[ \pm M_{RA} = \sum M_A; \quad 740(x) = 500 \left( \frac{3}{5} \right) (5) + 200(8) + 260 \left( \frac{12}{13} \right) (10) \]

\[ 740(x) = 5500 \]

\[ x = 7.43 \text{ ft} \quad \text{Ans.} \]
*4–116.

Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end B.

\[ F = 798 \text{ lb} \]
\[ \theta = 67.9^\circ \]
\[ x = 6.57 \text{ ft} \]
4–117.

Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from end A.

SOLUTION

\[ F_{Rx} = \sum F_x; \quad F_{Rx} = 450 \cos 60^\circ - 700 \sin 30^\circ = -125 \text{ N} = 125 \text{ N} \]

\[ F_{Ry} = \sum F_y; \quad F_{Ry} = -450 \sin 60^\circ - 700 \cos 30^\circ - 300 = -1296 \text{ N} = 1296 \text{ N} \]

\[ F = \sqrt{(-125)^2 + (-1296)^2} = 1302 \text{ N} \quad \text{Ans.} \]

\[ \theta = \tan^{-1}\left(\frac{1296}{125}\right) = 84.5^\circ \quad \text{Ans.} \]

\[ \zeta + M_{RA} = \sum M_A; \quad 1296(x) = 450 \sin 60^\circ (2) + 300(6) + 700 \cos 30^\circ (9) + 1500 \]

\[ x = 7.36 \text{ m} \quad \text{Ans.} \]

\begin{align*}
\text{Ans:} & \\
F &= 1302 \text{ N} \\
\theta &= 84.5^\circ \\
x &= 7.36 \text{ m}
\end{align*}
4–118.

Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from B.

**SOLUTION**

\[ F_R x = \Sigma F_x; \quad F_R x = 450 \cos 60^\circ - 700 \sin 30^\circ = -125 \text{ N} = 125 \text{ N} \leftarrow \]

\[ + F_R y = \Sigma F_y; \quad F_R y = -450 \sin 60^\circ - 700 \cos 30^\circ - 300 = -1296 \text{ N} = 1296 \text{ N} \downarrow \]

\[ F = \sqrt{(-125)^2 + (-1296)^2} = 1302 \text{ N} \quad \text{Ans.} \]

\[ \theta = \tan^{-1} \left( \frac{1296}{125} \right) = 84.5^\circ \quad \text{Ans.} \]

\[ \zeta + M_{RB} = \Sigma M_B; \quad 1296(x) = -450 \sin 60^\circ (4) + 700 \cos 30^\circ (3) + 1500 \]

\[ x = 1.36 \text{ m (to the right)} \quad \text{Ans.} \]
4–119.
Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a vertical line along member AB, measured from A.

**SOLUTION**

**Equivalent Resultant Force.** Referring to Fig. a,

\[ \begin{align*}
\sum (F_R)_x &= \sum F_x; \quad (F_R)_x = 600 \text{ N} \\
\sum (F_R)_y &= \sum F_y; \quad (F_R)_y = -200 - 400 - 200 = -800 \text{ N} = 800 \text{ N} \\
\end{align*} \]

As indicated in Fig. a,

\[ F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{600^2 + 800^2} = 1000 \text{ N} \]

And

\[ \theta = \tan^{-1} \left( \frac{(F_R)_y}{(F_R)_x} \right) = \tan^{-1} \left( \frac{800}{600} \right) = 53.13^\circ = 53.1^\circ \]

**Location of Resultant Force.** Along AB,

\[ \begin{align*}
\sum (M_R)_B &= \sum M_B; \quad 600(1.5 - d) = -400(0.5) - 200(1) \\
d &= 2.1667 \text{ m} = 2.17 \text{ m} \\
\end{align*} \]

**Ans:**

\[ F_R = 1000 \text{ N} \]
\[ \theta = 53.1^\circ \]
\[ d = 2.17 \text{ m} \]
Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a vertical line along member AB, measured from A.

**SOLUTION**

**Equivalent Resultant Force.** Referring to Fig. a

\[ \pm x (F_R)_x = \Sigma F_i; \quad (F_R)_x = 900 \left( \frac{3}{5} \right) - 400 \left( \frac{4}{5} \right) = 220 \text{ N} \quad \rightarrow \]

\[ + \uparrow (F_R)_y = \Sigma F_j; \quad (F_R)_y = 600 + 400 \left( \frac{3}{5} \right) - 400 - 900 \left( \frac{4}{5} \right) \]

\[ = -280 \text{ N} = 280 \text{ N} \downarrow \]

As indicated in Fig. a,

\[ F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{220^2 + 280^2} = 356.09 \text{ N} = 356 \text{ N} \quad \text{Ans.} \]

And

\[ \theta = \tan^{-1} \left( \frac{(F_R)_y}{(F_R)_x} \right) = \tan^{-1} \left( \frac{280}{220} \right) = 51.84^\circ = 51.8^\circ \quad \text{Ans.} \]

**Location of Resultant Force.** Referring to Fig. a

\[ \zeta + (M_R)_A = \Sigma M_A; \quad 280 a - 220 b = 400(1.5) - 600(0.5) - 900 \left( \frac{3}{5} \right)(2.5) \]

\[ + 400 \left( \frac{4}{5} \right)(1) \]

\[ 220 b - 280 a = 730 \quad (1) \]
Along $AB$, $a = 0$. Then Eq (1) becomes

$$220 \, b - 280(0) = 730$$

$$b = 3.318 \text{ m}$$

Thus, the intersection point of line of action of $F_R$ on $AB$ measured upward from point $A$ is

$$d = b = 3.32 \text{ m} \quad \text{ Ans.}$$

*Ans:*

$F_R = 356 \text{ N}$

$\theta = 51.8^\circ$

$d = b = 3.32 \text{ m}$
4–121.

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a horizontal line along member CB, measured from end C.

**SOLUTION**

Equivalent Resultant Force. Referring to Fig. a

\[ \Sigma F_x = \sum F_x; \quad (F_R)_x = 900 \left( \frac{3}{5} \right) - 400 \left( \frac{4}{5} \right) = 220 \text{ N} \]

\[ + \ (F_R)_y = \sum F_y; \quad (F_R)_y = 600 + 400 \left( \frac{3}{5} \right) - 400 - 900 \left( \frac{4}{5} \right) \]

\[ = -280 \text{ N} = 280 \text{ N} \]

As indicated in Fig. a,

\[ (F_R) = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{220^2 + 280^2} = 356.09 \text{ N} = 356 \text{ N} \]

Ans.

And

\[ \theta = \tan^{-1} \left( \frac{(F_R)_y}{(F_R)_x} \right) = \tan^{-1} \left( \frac{280}{220} \right) = 51.84^\circ = 51.8^\circ \]

Ans.

Location of Resultant Force. Referring to Fig. a

\[ \zeta + (M_R)_A = \sum M_A; \quad 280 a - 220 b = 400(1.5) - 600(0.5) - 900 \left( \frac{3}{5} \right)(2.5) \]

\[ + 400 \left( \frac{4}{5} \right)(1) \]

\[ 220 b - 280 a = 730 \]

(1)
Along $BC$, $b = 3$ m. Then Eq (1) becomes

$$220(3) - 280a = 730$$

$$a = -0.25 \text{ m}$$

Thus, the intersection point of line of action of $F_R$ on $CB$ measured to the right of point $C$ is

$$d = 1.5 - (-0.25) = 1.75 \text{ m}$$

Ans.
4–122.

Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post $AB$ measured from point $A$.

**SOLUTION**

**Equivalent Resultant Force:** Forces $F_1$ and $F_2$ are resolved into their $x$ and $y$ components, Fig. a. Summing these force components algebraically along the $x$ and $y$ axes,

$$\begin{align*}
\Sigma (F_R)_x &= \Sigma F_x; \quad (F_R)_x = 250 \left( \frac{4}{5} \right) - 500 \cos 30^\circ - 300 = -533.01 \text{ N} = 533.01 \text{ N} \\
\Sigma (F_R)_y &= \Sigma F_y; \quad (F_R)_y = 500 \sin 30^\circ - 250 \left( \frac{3}{5} \right) = 100 \text{ N}
\end{align*}$$

The magnitude of the resultant force $F_R$ is given by

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{533.01^2 + 100^2} = 542.31 \text{ N} = 542 \text{ N} \quad \text{Ans.}$$

The angle $\theta$ of $F_R$ is

$$\theta = \tan^{-1} \left[ \frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left[ \frac{100}{533.01} \right] = 10.63^\circ = 10.6^\circ \quad \text{Ans.}$$

**Location of the Resultant Force:** Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point $A$,

$$\begin{align*}
\zeta + (M_R)_A &= \Sigma M_A; \\
533.01(d) &= 500 \cos 30^\circ(2) - 500 \sin 30^\circ(0.2) - 250 \left( \frac{3}{5} \right)(0.5) - 250 \left( \frac{4}{5} \right)(3) + 300(1)
\end{align*}$$

$$d = 0.8274 \text{ mm} = 827 \text{ mm} \quad \text{Ans.}$$
Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post AB measured from point B.

**SOLUTION**

**Equivalent Resultant Force**: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes,

$$\Sigma (F_R)_x = \sum F_i ; \quad (F_R)_x = 250 \left( \frac{4}{5} \right) - 500 \cos 30^\circ - 300 = -533.01 \text{ N} = 533.01 \text{ N} \leftarrow$$

$$+ \uparrow (F_R)_y = \sum F_i ; \quad (F_R)_y = 500 \sin 30^\circ - 250 \left( \frac{3}{5} \right) = 100 \text{ N} \uparrow$$

The magnitude of the resultant force $F_R$ is given by

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{533.01^2 + 100^2} = 542.31 \text{ N} = 542 \text{ N} \quad \text{Ans.}$$

The angle $\theta$ of $F_R$ is

$$\theta = \tan^{-1} \left( \frac{(F_R)_y}{(F_R)_x} \right) = \tan^{-1} \left( \frac{100}{533.01} \right) = 10.6^\circ = 10.6^\circ \quad \text{Ans.}$$

**Location of the Resultant Force**: Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point B,

$$\zeta + (M_R)_B = \sum M_k ; \quad -533.01(d) = -500 \cos 30^\circ(1) - 500 \sin 30^\circ(0.2) - 250 \left( \frac{3}{5} \right)(0.5) - 300(2)$$

$$d = 2.17 \text{ m} \quad \text{Ans.}$$

**Ans:**

$$F_R = 542 \text{ N}$$

$$\theta = 10.6^\circ \quad \text{Ans.}$$

$$d = 2.17 \text{ m}$$
*4–124.

Replace the parallel force system acting on the plate by a resultant force and specify its location on the \( x-z \) plane.

**SOLUTION**

**Resultant Force:** Summing the forces acting on the plate,

\[
(F_R)_y = \sum F_y; \quad F_R = -5 \text{kN} - 2 \text{kN} - 3 \text{kN}
\]

\[
= -10 \text{kN}
\]

The negative sign indicates that \( F_R \) acts along the negative \( y \) axis.

**Resultant Moment:** Using the right-hand rule, and equating the moment of \( F_R \) to the sum of the moments of the force system about the \( x \) and \( z \) axes,

\[
\text{Ans:} \quad F_R = -10 \text{kN}
\]
4–125.

Replace the force and couple system acting on the frame by an equivalent resultant force and specify where the resultant’s line of action intersects member AB, measured from A.

**SOLUTION**

\[ F_{Rx} = \Sigma F_x; \quad F_{Rx} = 150 \left( \frac{4}{3} \right) + 50 \sin 30^\circ = 145 \text{ lb} \]

\[ F_{Ry} = \Sigma F_y; \quad F_{Ry} = 50 \cos 30^\circ + 150 \left( \frac{3}{3} \right) = 133.3 \text{ lb} \]

\[ F_R = \sqrt{(145)^2 + (133.3)^2} = 197 \text{ lb} \]

\[ \theta = \tan^{-1} \left( \frac{133.3}{145} \right) = 42.6^\circ \]

\[ \zeta + M_{RA} = \Sigma M_A; \quad 145 \ d = 150 \left( \frac{4}{3} \right) (2) - 50 \cos 30^\circ (3) + 50 \sin 30^\circ (6) + 500 \]

\[ d = 5.24 \text{ ft} \]
4–126.

Replace the force and couple system acting on the frame by an equivalent resultant force and specify where the resultant’s line of action intersects member BC, measured from B.

**SOLUTION**

\[ F_{Rx} = \sum F_x \quad F_{Rx} = 150 \left( \frac{4}{5} \right) + 50 \sin 30^\circ = 145 \text{ lb} \]

\[ F_{Ry} = \sum F_y \quad F_{Ry} = 50 \cos 30^\circ + 150 \left( \frac{3}{5} \right) = 133.3 \text{ lb} \]

\[ F_R = \sqrt{(145)^2 + (133.3)^2} = 197 \text{ lb} \quad \text{Ans.} \]

\[ \theta = \tan^{-1} \left( \frac{133.3}{145} \right) = 42.6^\circ \quad \text{Ans.} \]

\[ \zeta + M_{RA} = \sum M_A : \quad 145 (6) - 133.3 (d) = 150 \left( \frac{4}{5} \right) (2) - 50 \cos 30^\circ (3) + 50 \sin 30^\circ (6) + 500 \]

\[ d = 0.824 \text{ ft} \quad \text{Ans.} \]
If $F_A = 7 \text{kN}$ and $F_B = 5 \text{kN}$, represent the force system acting on the corbels by a resultant force, and specify its location on the $x$–$y$ plane.

**SOLUTION**

Equivalent Resultant Force: By equating the sum of the forces in Fig. a along the $z$ axis to the resultant force $F_R$, Fig. b,

$$+ \uparrow F_R = \Sigma F_z; \quad -F_R = -6 - 5 - 7 - 8$$

$$F_R = 26 \text{kN} \quad \text{Ans.}$$

**Point of Application:** By equating the moment of the forces shown in Fig. a and $F_R$, Fig. b, about the $x$ and $y$ axes,

$$(M_R)_x = \Sigma M_x; \quad -26(y) = 6(650) + 5(750) - 7(600) - 8(700)$$

$$y = 82.7 \text{ mm} \quad \text{Ans.}$$

$$(M_R)_y = \Sigma M_y; \quad 26(x) = 6(100) + 7(150) - 5(150) - 8(100)$$

$$x = 3.85 \text{ mm} \quad \text{Ans.}$$

$$F_R = 26 \text{kN}$$

$y = 82.7 \text{ mm}$

$x = 3.85 \text{ mm}$
Determine the magnitudes of \( F_A \) and \( F_B \) so that the resultant force passes through point \( O \) of the column.

**SOLUTION**

**Equivalent Resultant Force:** By equating the sum of the forces in Fig. a along the \( z \) axis to the resultant force \( F_R \), Fig. b,

\[
\sum F_z = F_R;
\]

\[
-F_R = -F_A - F_B - 8 - 6
\]

\[
F_R = F_A + F_B + 14 \tag{1}
\]

**Point of Application:** Since \( F_R \) is required to pass through point \( O \), the moment of \( F_R \) about the \( x \) and \( y \) axes are equal to zero. Thus,

\[
(M_R)_x = \sum M_x;
\]

\[
0 = F_B(750) + 6(650) - F_A(600) - 8(100)
\]

\[
750F_B - 600F_A - 1700 = 0 \tag{2}
\]

\[
(M_R)_y = \sum M_y;
\]

\[
0 = F_A(150) + 6(100) - F_B(150) - 8(100)
\]

\[
159F_A - 150F_B + 200 = 0 \tag{3}
\]

Solving Eqs. (1) through (3) yields

\[
F_A = 18.0 \text{ kN} \quad F_B = 16.7 \text{ kN} \quad F_R = 48.7 \text{ kN} \quad \text{Ans.}
\]
4–129.

The tube supports the four parallel forces. Determine the magnitudes of forces \( \mathbf{F}_C \) and \( \mathbf{F}_D \) acting at \( C \) and \( D \) so that the equivalent resultant force of the force system acts through the midpoint \( O \) of the tube.

**SOLUTION**

Since the resultant force passes through point \( O \), the resultant moment components about \( x \) and \( y \) axes are both zero.

\[
\sum M_x = 0; \quad F_D(0.4) + 600(0.4) - F_C(0.4) - 500(0.4) = 0
\]

\[
F_C - F_D = 100 \quad (1)
\]

\[
\sum M_y = 0; \quad 500(0.2) + 600(0.2) - F_C(0.2) - F_D(0.2) = 0
\]

\[
F_C + F_D = 1100 \quad (2)
\]

Solving Eqs. (1) and (2) yields:

\[
F_C = 600 \text{ N} \quad F_D = 500 \text{ N} \quad \text{Ans.}
\]

**Ans:**

\[
F_C = 600 \text{ N} \\
F_D = 500 \text{ N}
\]
4–130.

The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location \((x, y)\) on the slab. Take \(F_1 = 8 \text{ kN}\) and \(F_2 = 9 \text{ kN}\).

**SOLUTION**

**Equivalent Resultant Force.** Sum the forces along \(z\) axis by referring to Fig. \(a\)

\[\uparrow (F_R)_z = \Sigma F_z; \quad -F_R = -8 - 6 - 12 - 9 \quad F_R = 35 \text{ kN} \quad \text{Ans.}\]

**Location of the Resultant Force.** Sum the moments about the \(x\) and \(y\) axes by referring to Fig. \(a\),

\[(M_R)_x = \Sigma M_x; \quad -35 y = -12(8) - 6(20) - 9(20) \quad y = 11.31 \text{ m} = 11.3 \text{ m} \quad \text{Ans.}\]

\[(M_R)_y = \Sigma M_y; \quad 35 x = 12(6) + 8(22) + 6(26) \quad x = 11.54 \text{ m} = 11.5 \text{ m} \quad \text{Ans.}\]
4–131.
The building slab is subjected to four parallel column loadings. Determine \( F_1 \) and \( F_2 \) if the resultant force acts through point (12 m, 10 m).

**Solution**

**Equivalent Resultant Force.** Sum the forces along \( z \) axis by referring to Fig. a,
\[
+ \sum (F_R)_z = \sum F_z; \quad -F_R = -F_1 - F_2 - 12 - 6 \quad F_R = F_1 + F_2 + 18
\]

**Location of the Resultant Force.** Sum the moments about the \( x \) and \( y \) axes by referring to Fig. a,
\[
(M_R)_x = \sum M_x; \quad -(F_1 + F_2 + 18)(10) = -12(8) - 6(20) - F_2(20)
\]
\[
10F_1 - 10F_2 = 36 \quad (1)
\]
\[
(M_R)_y = \sum M_y; \quad (F_1 + F_2 + 18)(12) = 12(6) + 6(26) + F_1(22)
\]
\[
12F_2 - 10F_1 = 12 \quad (2)
\]

Solving Eqs (1) and (2),
\[
F_1 = 27.6 \text{ kN} \quad F_2 = 24.0 \text{ kN} \quad \text{Ans.}
\]
If \( F_A = 40 \text{ kN} \) and \( F_B = 35 \text{ kN} \), determine the magnitude of the resultant force and specify the location of its point of application \((x, y)\) on the slab.

**SOLUTION**

*Equivalent Resultant Force:* By equating the sum of the forces along the \( z \) axis to the resultant force \( F_R \), Fig. b,

\[
\uparrow F_R = \Sigma F_z; \quad -F_R = -30 - 20 - 90 - 35 - 40
\]

\[
F_R = 215 \text{ kN} \quad \text{Ans.}
\]

*Point of Application:* By equating the moment of the forces and \( F_R \), about the \( x \) and \( y \) axes,

\[
(M_R)_x = \Sigma M_x; \quad -215(y) = -35(0.75) - 30(0.75) - 90(3.75) - 20(6.75) - 40(6.75)
\]

\[
y = 3.68 \text{ m} \quad \text{Ans.}
\]

\[
(M_R)_y = \Sigma M_y; \quad 215(x) = 30(0.75) + 20(0.75) + 90(3.25) + 35(5.75) + 40(5.75)
\]

\[
x = 3.54 \text{ m} \quad \text{Ans.}
\]
4–133.

If the resultant force is required to act at the center of the slab, determine the magnitude of the column loadings $F_A$ and $F_B$ and the magnitude of the resultant force.

**SOLUTION**

**Equivalent Resultant Force:** By equating the sum of the forces along the $z$ axis to the resultant force $F_R$,

$$+ F_R = \Sigma F_z; \quad -F_R = -30 - 20 - 90 - F_A - F_B$$

$$F_R = 140 + F_A + F_B \quad \text{(1)}$$

**Point of Application:** By equating the moment of the forces and $F_R$, about the $x$ and $y$ axes,

$$(M_R)_x = \Sigma M_x; \quad -F_R(3.75) = -F_B(0.75) - 30(0.75) - 90(3.75) - 20(6.75) - F_A(6.75)$$

$$F_R = 0.2F_B + 1.8F_A + 132 \quad \text{(2)}$$

$$(M_R)_y = \Sigma M_y; \quad F_R(3.25) = 30(0.75) + 20(0.75) + 90(3.25) + F_A(5.75) + F_B(5.75)$$

$$F_R = 1.769F_A + 1.769F_B + 101.54 \quad \text{(3)}$$

Solving Eqs.(1) through (3) yields

$$F_A = 30 \text{kN} \quad F_B = 20 \text{kN} \quad F_R = 190 \text{kN} \quad \text{Ans.}$$
4–134.

Replace the two wrenches and the force, acting on the pipe assembly, by an equivalent resultant force and couple moment at point $O$.

**SOLUTION**

**Force And Moment Vectors:**

\[
F_1 = \{300k\} \text{ N} \quad F_3 = \{100j\} \text{ N}
\]

\[
F_2 = 200(\cos 45^\circ i - \sin 45^\circ k) \text{ N}
\]

\[
= \{141.42i - 141.42k\} \text{ N}
\]

\[
M_1 = \{100k\} \text{ N} \cdot \text{m}
\]

\[
M_2 = 180(\cos 45^\circ i - \sin 45^\circ k) \text{ N} \cdot \text{m}
\]

\[
= \{127.28i - 127.28k\} \text{ N} \cdot \text{m}
\]

**Equivalent Force and Couple Moment At Point $O$:**

\[
F_R = \sum F; \quad F_R = F_1 + F_2 + F_3
\]

\[
= 141.42i + 100.0j + (300 - 141.42)k
\]

\[
= \{141i + 100j + 159k\} \text{ N} \quad \text{Ans.}
\]

The position vectors are $r_1 = \{0.5j\}$ m and $r_2 = \{1.1j\}$ m.

\[
M_{R_O} = \sum M_D; \quad M_{R_O} = r_1 \times F_1 + r_2 \times F_2 + M_1 + M_2
\]

\[
= \begin{vmatrix}
1 & 0.5 & 0 \\
0 & 1 & 0 \\
0 & 0 & 300 \\
\end{vmatrix}
\]

\[
+ \begin{vmatrix}
i & j & k \\
0 & 1.1 & 0 \\
141.42 & 0 & -141.42 \\
\end{vmatrix}
\]

\[
+ 100k + 127.28i - 127.28k
\]

\[
= \{122i - 183k\} \text{ N} \cdot \text{m} \quad \text{Ans.}
\]

**Ans:**

\[
F_R = \{141i + 100j + 159k\} \text{ N}
\]

\[
M_{R_O} = \{122i - 183k\} \text{ N} \cdot \text{m}
\]
4–135.
Replace the force system by a wrench and specify the magnitude of the force and couple moment of the wrench and the point where the wrench intersects the x–z plane.

**SOLUTION**

**Resultant Force.** Referring to Fig. a

\[
\mathbf{F}_R = \left\{ \left[ 200 \left( \frac{3}{5} \right) - 400 \right] \mathbf{i} - 200 \mathbf{j} + 200 \left( \frac{4}{5} \right) \mathbf{k} \right\} \\
= \{-280 \mathbf{i} - 200 \mathbf{j} + 160 \mathbf{k}\} \text{ N}
\]

The magnitude of \( \mathbf{F}_R \) is

\[
F_R = \sqrt{(-280)^2 + (-200)^2 + 160^2} = 379.47 \text{ N} = 379 \text{ N} \quad \text{Ans.}
\]

The direction of \( \mathbf{F}_R \) is defined by

\[
\mathbf{u}_{F_R} = \frac{\mathbf{F}_R}{F_R} = \frac{-280 \mathbf{i} - 200 \mathbf{j} + 160 \mathbf{k}}{379.47} = -0.7379 \mathbf{i} - 0.5270 \mathbf{j} + 0.4216 \mathbf{k}
\]

**Resultant Moment.** The line of action of \( \mathbf{M}_R \) of the wrench is parallel to that of \( \mathbf{F}_R \). Also, assume that \( \mathbf{M}_R \) and \( \mathbf{F}_R \) have the same sense. Then

\[
\mathbf{u}_{M_R} = -0.7379 \mathbf{i} - 0.5270 \mathbf{j} + 0.4216 \mathbf{k}
\]
Referring to Fig. a, where the origin of the $x'$, $y'$, $z'$ axes is the point where the wrench intersects the $xz$ plane,

\[(M_R)_{x'} = \Sigma M_{x'}; \quad -0.7379 M_R = -200(z - 0.5) \quad (1)\]

\[(M_R)_{y'} = \Sigma M_{y'}; \quad -0.5270 M_R = -200\left(\frac{2}{3}\right)(z - 0.5) - 200\left(\frac{4}{3}\right)(3 - x) + 400(z - 0.5) \quad (2)\]

\[(M_R)_{z'} = \Sigma M_{z'}; \quad 0.4216 M_R = 200x + 400(2) \quad (3)\]

Solving Eqs (1), (2) and (3)

\[M_R = 590.29 \text{ N} \cdot \text{m} = 590 \text{ N} \cdot \text{m} \quad \text{Ans.}\]

\[z = 2.6778 \text{ m} = 2.68 \text{ m} \quad \text{Ans.}\]

\[x = -2.7556 \text{ m} = -2.76 \text{ m} \quad \text{Ans.}\]
*4–136.
Replace the five forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point P(x, z) where the wrench intersects the x–z plane.

**SOLUTION**

**Resultant Force.** Referring to Fig. a

\[ \mathbf{F}_R = (-600i - (300 + 200 + 400)j - 800k) \text{ N} \]

Then the magnitude of \( \mathbf{F}_R \) is

\[ F_R = \sqrt{(-600)^2 + (-900)^2 + (-800)^2} = 1345.36 \text{ N} = 1.35 \text{ kN} \quad \text{Ans.} \]

The direction of \( \mathbf{F}_R \) is defined by

\[ \mathbf{u}_{FR} = \frac{\mathbf{F}_R}{F_R} = \frac{-600i - 900j - 800k}{1345.36} = -0.4460i - 0.6690j - 0.5946k \]

**Resultant Moment.**

The line of action of \( \mathbf{M}_R \) of the wrench is parallel to that of \( \mathbf{F}_R \). Also, assume that both \( \mathbf{M}_R \) and \( \mathbf{F}_R \) have the same sense. Then

\[ \mathbf{u}_{M_R} = -0.4460i - 0.6690j - 0.5946k \]
Referring to Fig. a,

\[(M_R)_x = \Sigma M_x; \quad -0.4460 M_R = -300z - 200(z - 2) - 400z \quad (1)\]

\[(M_R)_y = \Sigma M_y; \quad -0.6690 M_R = 800(4 - x) + 600z \quad (2)\]

\[(M_R)_z = \Sigma M_z; \quad -0.5946 M_R = 200(x - 2) + 400x - 300(4 - x) \quad (3)\]

Solving Eqs (1), (2) and (3)

\[M_R = -1367.66 \text{ N} \cdot \text{m} = -1.37 \text{kN} \cdot \text{m} \quad \text{Ans.}\]

\[x = 2.681 \text{ m} = 2.68 \text{ m} \quad \text{Ans.}\]

\[z = -0.2333 \text{ m} = -0.233 \text{ m} \quad \text{Ans.}\]

The negative sign indicates that the line of action of \(M_R\) is directed in the opposite sense to that of \(F_R\).
Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point \( P(x, y) \) where the wrench intersects the plate.

**SOLUTION**

**Resultant Force.** Referring to Fig. \( a \),

\[
\mathbf{F}_R = [400\mathbf{i} + 200\mathbf{j} - 300\mathbf{k}] \text{ N}
\]

Then, the magnitude of \( \mathbf{F}_R \) is

\[
F_R = \sqrt{400^2 + 200^2 + (-300)^2} = 538.52 \text{ N} = 539 \text{ N}
\]

Ans.

The direction of \( \mathbf{F}_R \) is defined by

\[
\mathbf{u}_{F_R} = \frac{\mathbf{F}_R}{F_R} = \frac{400\mathbf{i} + 200\mathbf{j} - 300\mathbf{k}}{538.52} = 0.7428\mathbf{i} + 0.3714\mathbf{j} - 0.5571\mathbf{k}
\]

**Resultant Moment.** The line of action of \( \mathbf{M}_R \) of the wrench is parallel to that of \( \mathbf{F}_R \). Also, assume that both \( \mathbf{M}_R \) and \( \mathbf{F}_R \) have the same sense. Then

\[
\mathbf{u}_{M_R} = 0.7428\mathbf{i} + 0.3714\mathbf{j} - 0.5571\mathbf{k}
\]
4–137. Continued

Referring to Fig. a,

\[(M_R)_x = \sum M_x; 0.7428 M_R = 300y \quad (1)\]

\[(M_R)_y = \sum M_y; 0.3714 M_R = 300(3 - x) \quad (2)\]

\[(M_R)_z = \sum M_z; -0.5571 M_R = -200x - 400(5 - y) \quad (3)\]

Solving Eqs (1), (2) and (3)

\[M_R = 1448.42 \text{ N} \cdot \text{m} = 1.45 \text{ kN} \cdot \text{m} \quad \text{Ans.}\]

\[x = 1.2069 \text{ m} = 1.21 \text{ m} \quad \text{Ans.}\]

\[y = 3.5862 \text{ m} = 3.59 \text{ m} \quad \text{Ans.}\]
4–138.
Replace the loading by an equivalent resultant force and couple moment acting at point O.

SOLUTION

\[ + \uparrow F_R = \Sigma F; \quad F_R = 0 \]
\[ \zeta + M_{RO} = \Sigma M_O; \quad M_{RO} = 225 (6) = 1350 \text{ lb} \cdot \text{ft} = 1.35 \text{ kip} \cdot \text{ft} \]
4–139.

Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point $O$.

**SOLUTION**

*Loading:* The distributed loading can be divided into two parts as shown in Fig. $a$.

*Equations of Equilibrium:* Equating the forces along the $y$ axis of Figs. $a$ and $b$, we have

\[ + \downarrow F_R = \Sigma F; \quad F_R = \frac{1}{2}(3)(3) + \frac{1}{2}(3)(1.5) = 6.75 \text{ kN} \downarrow \quad \text{Ans.} \]

If we equate the moment of $F_R$, Fig. $b$, to the sum of the moment of the forces in Fig. $a$ about point $O$, we have

\[ \zeta + (M_R)_O = \Sigma M_O; \quad -6.75(x) = -\frac{1}{2}(3)(3)(2) - \frac{1}{2}(3)(1.5)(3.5) \]

\[ x = 2.5 \text{ m} \quad \text{Ans.} \]
*4–140.

Replace the loading by an equivalent resultant force and specify its location on the beam, measured from point $A$.

**SOLUTION**

**Equivalent Resultant Force.** Summing the forces along the $y$ axis by referring to Fig. $a$

$$+\uparrow \ (F_R)_y = \Sigma F_y; \quad -F_R = -2(6) - \frac{1}{2}(3)(6)$$

Ans.

$$F_R = 21.0 \text{ kN} \downarrow$$

Ans.

**Location of the Resultant Force.** Summing the moments about point $A$,

$$\zeta + (M_R)_A = \Sigma M_A; \quad -21.0(d) = -2(6)(3) - \frac{1}{2}(3)(6)(4)$$

$$d = 3.429 \text{ m} = 3.43 \text{ m}$$

Ans.

Ans:

$$F_R = 21.0 \text{ kN}$$

$$d = 3.43 \text{ m}$$
Currently eighty-five percent of all neck injuries are caused by rear-end car collisions. To alleviate this problem, an automobile seat restraint has been developed that provides additional pressure contact with the cranium. During dynamic tests the distribution of load on the cranium has been plotted and shown to be parabolic. Determine the equivalent resultant force and its location, measured from point $A$.

**SOLUTION**

\[
F_R = \int w(x) \, dx = \int_{0}^{0.5} 12 \left(1 + 2x^2\right) \, dx = 12 \left[x + \frac{2}{3}x^3\right]_{0}^{0.5} = 7 \text{ lb} \quad \text{Ans.}
\]

\[
\bar{x} = \frac{\int xw(x) \, dx}{\int w(x) \, dx} = \frac{\int_{0}^{0.5} x(12)(1 + 2x^2) \, dx}{7} = \frac{12 \left[\frac{x^2}{2} + \frac{x^4}{4}\right]_{0}^{0.5}}{7} \quad \text{Ans.}
\]

\[
\bar{x} = 0.268 \text{ ft}
\]
4–142.
Replace the distributed loading by an equivalent resultant force, and specify its location on the beam, measured from the pin at A.

**SOLUTION**

**Equivalent Resultant Force.** Summing the forces along the $y$ axis by referring to Fig. $a$,

$$ +\uparrow (F_R)_y = \Sigma F_y; \quad -F_R = -2(6) - \frac{1}{2}(2)(3) $$

$$ F_R = 15.0 \text{kN} \downarrow $$

**Ans.**

**Location of the Resultant Force.** Summing the Moments about point $A$,

$$ \zeta + (M_R)_A = \Sigma M_A; \quad -15.0(d) = -2(6)(3) - \frac{1}{2}(2)(3)(5) $$

$$ d = 3.40 \text{ m} $$

**Ans.**

**Ans:**

$$ F_R = 15.0 \text{kN} $$

$$ d = 3.40 \text{ m} $$
4–143.
Replace this loading by an equivalent resultant force and specify its location, measured from point $O$.

**SOLUTION**

**Equivalent Resultant Force.** Summing the forces along the $y$ axis by referring to Fig. a,

$$ + (F_R)_y = \Sigma F_y; \quad -F_R = -4(2) - \frac{1}{2}(6)(1.5) $$

$$ F_R = 12.5 \text{ kN} \quad \text{Ans.} $$

**Location of the Resultant Force.** Summing the Moment about point $O$,

$$ \zeta + (M_R)_O = \Sigma M_O; \quad -12.5(d) = -4(2)(1) - \frac{1}{2}(6)(1.5)(2.5) $$

$$ d = 1.54 \text{ m} \quad \text{Ans.} $$
*4–144.*

The distribution of soil loading on the bottom of a building slab is shown. Replace this loading by an equivalent resultant force and specify its location, measured from point $O$.

**SOLUTION**

\[ + \uparrow F_R = \Sigma F_y; \quad F_R = 50(12) + \frac{1}{2}(250)(12) + \frac{1}{2}(200)(9) + 100(9) \]

\[ = 3900 \text{ lb} = 3.90 \text{ kip} \uparrow \]

\[ \zeta + M_{Ro} = \Sigma M_O; \quad 3900(d) = 50(12)(6) + \frac{1}{2}(250)(12)(8) + \frac{1}{2}(200)(9)(15) + 100(9)(16.5) \]

\[ d = 11.3 \text{ ft} \]
Replace the loading by an equivalent resultant force and couple moment acting at point $O$.

**SOLUTION**

**Equivalent Resultant Force And Couple Moment About Point $O$.** Summing the forces along the $y$ axis by referring to Fig. $a$,

$$+\uparrow (F_R)_y = \Sigma F_y; \quad F_R = -\frac{1}{2}(3)(1.5) - 5(2.25) - \frac{1}{2}(5)(0.75)$$

$$= -15.375 \text{ kN} = 15.4 \text{ kN} \downarrow \quad \text{Ans.}$$

Summing the Moment about point $O$,

$$\zeta + (M_R)_O = \Sigma M_O; \quad (M_R)_O = -\frac{1}{2}(3)(1.5)(0.5) - 5(2.25)(1.125)$$

$$= \frac{1}{2}(5)(0.75)(2.5)$$

$$= -18.46875 \text{ kN} \cdot \text{m} = 18.5 \text{ kN} \cdot \text{m} \quad \text{(clockwise)} \quad \text{Ans.}$$

**Ans:**

$F_R = 15.4 \text{ kN}$

$(M_R)_O = 18.5 \text{ kN} \cdot \text{m} \quad \text{(clockwise)}$
4–146.
Replace the distributed loading by an equivalent resultant force and couple moment acting at point A.

**SOLUTION**

**Equivalent Resultant Force And Couple Moment About Point A.** Summing the forces along the y axis by referring to Fig. a,

\[ + \uparrow (F_R)_y = \sum F_y; \quad F_R = -\frac{1}{2} (3)(3) - 3(6) - \frac{1}{2} (3)(3) \]

\[ = -27.0 \text{ kN} \]

**Ans.**

Summing the moments about point A,

\[ \zeta + (M_R)_A = \sum M_A; \quad (M_R)_A = -\frac{1}{2} (3)(3)(1) - 3(6)(3) - \frac{1}{2} (3)(3)(5) \]

\[ = -81.0 \text{ kN} \cdot \text{m} = 81.0 \text{ kN} \cdot \text{m} \text{ (clockwise)} \]

**Ans.**
4–147.
Determine the length $b$ of the triangular load and its position $a$ on the beam such that the equivalent resultant force is zero and the resultant couple moment is $8 \text{kN} \cdot \text{m}$ clockwise.

**SOLUTION**

\[ + \uparrow F_R = 0 = \sum F_y; \quad 0 = \frac{1}{2}(2.5)(9) - \frac{1}{2}(4)(b) \quad b = 5.625 \text{ m} \]

\[ \zeta + M_{RA} = \sum M_A; \quad -8 = -\frac{1}{2}(2.5)(9)(6) + \frac{1}{2}(4)(5.625) \left( a + \frac{2}{3}(5.625) \right) \]

\[ a = 1.54 \text{ m} \]

Ans:

\[ a = 1.54 \text{ m} \]
4–148.

The form is used to cast a concrete wall having a width of 5 m. Determine the equivalent resultant force the wet concrete exerts on the form $AB$ if the pressure distribution due to the concrete can be approximated as shown. Specify the location of the resultant force, measured from point $B$.

**SOLUTION**

$$
\int dA = \int_0^4 4z^2 \, dz
$$

$$
= \left[ \frac{2}{3} z^3 \right]_0^4
$$

$$
= 21.33 \text{ kN/m}
$$

$F_R = 21.33(5) = 107 \text{ kN}$

$$
\int zdA = \int_0^4 4z^2 \, dz
$$

$$
= \left[ \frac{2}{5} z^3 \right]_0^4
$$

$$
= 51.2 \text{ kN}
$$

$$
\bar{z} = \frac{51.2}{21.33} = 2.40 \text{ m}
$$

Also, from the back of the book,

$$
A = \frac{2}{3} ab = \frac{2}{3} (8)(4) = 21.33
$$

$$
F_R = 21.33(5) = 107 \text{ kN}
$$

$$
\bar{z} = 4 - 1.6 = 2.40 \text{ m}
$$

$\bar{z} = 2.40 \text{ m}$
If the soil exerts a trapezoidal distribution of load on the bottom of the footing, determine the intensities $w_1$ and $w_2$ of this distribution needed to support the column loadings.

**SOLUTION**

**Loading:** The trapezoidal reactive distributed load can be divided into two parts as shown on the free-body diagram of the footing, Fig. a. The magnitude and location measured from point $A$ of the resultant force of each part are also indicated in Fig. a.

**Equations of Equilibrium:** Writing the moment equation of equilibrium about point $B$, we have

$$\zeta + \Sigma M_B = 0; \quad w_2(8)\left(4 - \frac{8}{3}\right) + 60\left(\frac{8}{3} - 1\right) - 80\left(3.5 - \frac{8}{3}\right) - 50\left(7 - \frac{8}{3}\right) = 0$$

$$w_2 = 17.1875 \text{ kN/m} = 17.2 \text{ kN/m} \quad \text{Ans.}$$

Using the result of $w_2$ and writing the force equation of equilibrium along the $y$ axis, we obtain

$$+ \Sigma F_y = 0; \quad \frac{1}{2}(w_1 - 17.1875)8 + 17.1875(8) - 60 - 80 - 50 = 0$$

$$w_1 = 30.3125 \text{ kN/m} = 30.3 \text{ kN/m} \quad \text{Ans.}$$

Ans:

$$w_2 = 17.2 \text{ kN/m}$$

$$w_1 = 30.3 \text{ kN/m}$$
4–150.

Replace the loading by an equivalent force and couple moment acting at point $O$.

**SOLUTION**

\[ + F_R = \sum F_y; \quad F_R = -22.5 - 13.5 - 15.0 \]
\[ = -51.0 \text{ kN} = 51.0 \text{ kN down} \]

\[ \zeta + M_{Ro} = \sum M_{o}; \quad M_{Ro} = -500 - 22.5(5) - 13.5(9) - 15(12) \]
\[ = -914 \text{ kN} \cdot \text{m} \]
\[ = 914 \text{ kN} \cdot \text{m (Clockwise)} \]

**Ans:**

\[ F_R = 51.0 \text{ kN down} \]
\[ M_{Ro} = 914 \text{ kN} \cdot \text{m} \]

**Ans:**

Replace the loading by an equivalent force and couple moment acting at point $O$. 

**Ans:**

\[ F_R = 51.0 \text{ kN down} \]
\[ M_{Ro} = 914 \text{ kN} \cdot \text{m} \]
4–151.

Replace the loading by a single resultant force, and specify the location of the force measured from point O.

**SOLUTION**

**Equivalent Resultant Force:**

\[ + \uparrow F_R = \Sigma F_y; \quad -F_R = -22.5 - 13.5 - 15 \]

\[ F_R = 51.0 \text{ kN} \downarrow \]

**Ans.**

**Location of Equivalent Resultant Force:**

\[ \zeta + (M_R)_O = \Sigma M_O; \quad -51.0(d) = -500 - 22.5(5) - 13.5(9) - 15(12) \]

\[ d = 17.9 \text{ m} \]

**Ans.**

**Ans:**

\[ F_R = 51.0 \text{ kN} \downarrow \]

\[ d = 17.9 \text{ m} \]
4–152.

Replace the loading by an equivalent resultant force and couple moment acting at point A.

**SOLUTION**

**Equivalent Resultant Force and Couple Moment at Point A.** Summing the forces along the y axis by referring to Fig. a,

\[ + \uparrow (F_R)_y = \Sigma F_y; \quad F_R = -400(3) - \frac{1}{2}(400)(3) \]

\[ = -1800 \text{ N} = 1.80 \text{ kN} \downarrow \quad \text{Ans.} \]

Summing the moment about point A,

\[ \zeta + (M_R)_A = \Sigma M_A; \quad (M_R)_A = -400(3)(1.5) - \frac{1}{2}(400)(3)(4) \]

\[ = -4200 \text{ N} \cdot \text{m} = 4.20 \text{ kN} \cdot \text{m (clockwise)} \quad \text{Ans.} \]
4–153.
Replace the loading by a single resultant force, and specify its location on the beam measured from point $A$.

**SOLUTION**

**Equivalent Resultant Force.** Summing the forces along the $y$ axis by referring to Fig. $a$,

$$+ \uparrow (F_R)_y = \Sigma F_y; \quad -F_R = -400(3) - \frac{1}{2}(400)(3)$$

$$F_R = 1800 \text{ N} = 1.80 \text{ kN} \downarrow \quad \text{Ans.}$$

**Location of Resultant Force.** Summing the moment about point $A$ by referring to Fig. $a$,

$$\zeta + (M_R)_A = \Sigma M_A; \quad -1800 d = -400(3)(1.5) - \frac{1}{2}(400)(3)(4)$$

$$d = 2.333 \text{ m} = 2.33 \text{ m} \quad \text{Ans.}$$

Ans:

$F_R = 1.80 \text{ kN}$

$d = 2.33 \text{ m}$
Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects a horizontal line along member AB, measured from A.

**SOLUTION**

**Equivalent Resultant Force.** Summing the forces along the x and y axes by referring to Fig. a,

\[ (F_R)_x = \Sigma F_x; \quad (F_R)_x = -2(4) = -8 \, \text{kN} = 8 \, \text{kN} \quad \leftarrow \]

\[ (F_R)_y = \Sigma F_y; \quad (F_R)_y = -3(3) = -9 \, \text{kN} = 9 \, \text{kN} \downarrow \]

Then

\[ F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{8^2 + 9^2} = 12.04 \, \text{kN} = 12.0 \, \text{kN} \quad \text{Ans.} \]

And

\[ \theta = \tan^{-1} \left( \frac{(F_R)_y}{(F_R)_x} \right) = \tan^{-1} \left( \frac{9}{8} \right) = 48.37^\circ = 48.4^\circ \quad \text{Ans.} \]

**Location of the Resultant Force.** Summing the moments about point A, by referring to Fig. a,

\[ \zeta + (M_R)_A = \Sigma M_A; \quad -8x - 9y = -3(3)(1.5) - 2(4)(2) \]

\[ 8x + 9y = 29.5 \quad (1) \]
Along $AB$, $x = 0$. Then Eq (1) becomes

$$8(0) + 9y = 29.5$$

$$y = 3.278 \text{ m}$$

Thus, the intersection point of line of action of $F_R$ on $AB$ measured to the right from point $A$ is

$$d = y = 3.28 \text{ m}$$

Ans.
Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects a vertical line along member $BC$, measured from $C$.

**SOLUTION**

**Equivalent Resultant Force.** Summing the forces along the $x$ and $y$ axes by referring to Fig. $a$,

\[ (F_R)_x = \sum F_x; \quad (F_R)_y = \sum F_y; \]

\[ (F_R)_x = -2(4) = -8 \text{ kN} = 8 \text{ kN} \leftarrow \]

\[ (F_R)_y = -3(3) = -9 \text{ kN} = 9 \text{ kN} \downarrow \]

Then

\[ F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{8^2 + 9^2} = 12.04 \text{ kN} = 12.0 \text{ kN} \quad \text{Ans.} \]

And

\[ \theta = \tan^{-1} \left( \frac{(F_R)_y}{(F_R)_x} \right) = \tan^{-1} \left( \frac{9}{8} \right) = 48.37^\circ = 48.4^\circ \quad \text{Ans.} \]

**Location of the Resultant Force.** Summing the moments about point $A$, by referring to Fig. $a$,

\[ \sum (M_R)_A = \sum M_A; \quad -8x - 9y = -3(3)(1.5) - 2(4)(2) \]

\[ 8x + 9y = 29.5 \]

(1)
Along BC, \( y = 3 \) m. Then Eq (1) becomes

\[
8x + 9(3) = 29.5
\]

\[
x = 0.3125 \text{ m}
\]

Thus, the intersection point of line of action of \( F_R \) on BC measured upward from point C is

\[
d = 4 - x = 4 - 0.3125 = 3.6875 \text{ m} = 3.69 \text{ m}
\]

Ans.

\[ F_R = 12.0 \text{ kN} \]
\[ \theta = 48.4^\circ \]
\[ d = 3.69 \text{ m} \]
Determine the length $b$ of the triangular load and its position $a$ on the beam such that the equivalent resultant force is zero and the resultant couple moment is 8 kN·m clockwise.

\*4–156.

**SOLUTION**

**Equivalent Resultant Force And Couple Moment At Point A.** Summing the forces along the $y$ axis by referring to Fig. $a$, with the requirement that $F_R = 0$,

\[
\begin{align*}
\sum F_y &= 0 \Rightarrow 2(a + b) - \frac{1}{2}(6b) = 0 \\
2a - b &= 0 \quad (1)
\end{align*}
\]

Summing the moments about point $A$, with the requirement that $(M_R)_A = 8$ kN·m,

\[
\begin{align*}
\sum M_A &= -8 = 2(a + b) \left[ 4 - \frac{1}{2}(a + b) \right] - \frac{1}{2}(6b) \left( 4 - \frac{1}{3}b \right) \\
-8 &= 8a - 4b - 2ab - a^2 \quad (2)
\end{align*}
\]

Solving Eqs (1) and (2),

\[
\begin{align*}
a &= 1.264 \text{ m} = 1.26 \text{ m} & \text{Ans.} \\
b &= 2.530 \text{ m} = 2.53 \text{ m} & \text{Ans.}
\end{align*}
\]
4–157.

Determine the equivalent resultant force and couple moment at point \( O \).

**SOLUTION**

**Equivalent Resultant Force And Couple Moment About Point \( O \).** The differential force indicated in Fig. \( a \) is \( dF_R = w \, dx = \frac{1}{3} x^3 \, dx \). Thus, summing the forces along the \( y \) axis,

\[
\begin{align*}
\sum F_y &= \int dF_R = -\int_0^{3m} \frac{1}{3} x^3 \, dx \\
&= -\frac{1}{12} x^4 \bigg|_0^{3m} \\
&= -6.75 \text{ kN} \quad \text{(Ans.)}
\end{align*}
\]

Summing the moments about point \( O \),

\[
\begin{align*}
\sum M_O &= \int (3 - x) dF_R \\
&= \int_0^{3m} (3 - x) \left( \frac{1}{3} x^3 \right) \, dx \\
&= \left[ \frac{1}{4} x^4 - \frac{1}{15} x^5 \right]_0^{3m} \\
&= 4.05 \text{ kN} \cdot \text{m (counterclockwise)} \quad \text{(Ans.)}
\end{align*}
\]

Ans:
\[ F_R = 6.75 \text{ kN} \downarrow \]
\[ (M_R)_O = 4.05 \text{ kN} \cdot \text{m (counterclockwise)} \]
4–158.

Determine the magnitude of the equivalent resultant force and its location, measured from point $O$.

**SOLUTION**

\[ dA = wdx \]

\[ F_R = \int dA = \int_0^6 (4 + 2\sqrt{x}) \, dx \]

\[ = \left[ 4x + \frac{4}{3}x^{3/2} \right]_0^6 \]

\[ F_R = 43.6 \text{ lb} \]

\[ \int \bar{x}dF = \int_0^6 (4x + 2x^{3/2}) \, dx \]

\[ = \left[ 2x^2 + \frac{4}{5}x^{5/2} \right]_0^6 \]

\[ = 142.5 \text{ lb} \cdot \text{ft} \]

\[ \bar{x} = \frac{142.5}{43.6} = 3.27 \text{ ft} \]

**Ans:**

\[ F_R = 43.6 \text{ lb} \]

\[ \bar{x} = 3.27 \text{ ft} \]
4–159.
The distributed load acts on the shaft as shown. Determine the magnitude of the equivalent resultant force and specify its location, measured from the support, \( A \).

**SOLUTION**

\[
F_R = \int_{-1}^{4} \left( 2x^2 - 8x + 18 \right) \, dx = \frac{2}{3}x^3 - \frac{8}{2}x^2 + 18x \bigg|_{-1}^{4} = 73.33 = 73.3 \text{ lb} \quad \text{Ans.}
\]

\[
\int x \, dF = \int_{-1}^{4} \left( 2x^3 - 8x^2 + 18x \right) \, dx = \frac{2}{4}x^4 - \frac{8}{3}x^3 + \frac{18}{2}x^2 \bigg|_{-1}^{4} = 89.166 \text{ lb} \cdot \text{ft} \quad \text{Ans.}
\]

\[
x = \frac{89.166}{73.3} = 1.22 \text{ ft}
\]

\[
d = 1 + 1.22 = 2.22 \text{ ft} \quad \text{Ans.}
\]
Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.

**SOLUTION**

**Resultant:** The magnitude of the differential force \( dF_R \) is equal to the area of the element shown shaded in Fig. a. Thus,

\[
dF_R = w \, dx = \left(x^2 + 3x + 100\right) dx
\]

Integrating \( dF_R \) over the entire length of the beam gives the resultant force \( F_R \).

\[
+ \downarrow F_R = \int_0^{15 \text{ ft}} dF_R = \int_0^{15 \text{ ft}} \left(x^2 + 3x + 100\right) dx = \left[ \frac{x^3}{3} + \frac{3x^2}{2} + 100x \right]_0^{15 \text{ ft}}
\]

\[
= 2962.5 \text{ lb} = 2.96 \text{ kip} \text { Ans.}
\]

**Location:** The location of \( dF_R \) on the beam is \( x_c = x \) measured from point A. Thus, the location \( \bar{x} \) of \( F_R \) measured from point A is given by

\[
\bar{x} = \frac{\int_0^{15 \text{ ft}} x \, dF_R}{\int_0^{15 \text{ ft}} dF_R} = \frac{\int_0^{15 \text{ ft}} x \left(x^2 + 3x + 100\right) dx}{2962.5} = \frac{\left[ \frac{x^4}{4} + x^3 + 50x^2 \right]_0^{15 \text{ ft}}}{2962.5} = 9.21 \text{ ft} \text { Ans.}
\]
4–161.
Replace the loading by an equivalent resultant force and couple moment acting at point $O$.

**SOLUTION**

**Equivalent Resultant Force And Couple Moment About Point $O$.** The differential force indicated in Fig. 1 is $dF_R = w \, dx = \left( w_0 \cos \frac{\pi}{2L} x \right) dx$. Thus, summing the forces along the $y$ axis,

$$F_R = \int \int dF_R = \left. \int_0^L \left( w_0 \cos \frac{\pi}{2L} x \right) dx \right|_O$$

$$= \frac{2Lw_0}{\pi} \left( \sin \frac{\pi}{2L} x \right) \bigg|_0^L$$

$$= \frac{2Lw_0}{\pi} = \frac{2Lw_0}{\pi} \downarrow \quad \text{Ans.}$$

Summing the moments about point $O$,

$$\zeta + (M_R)_O = \Sigma M_O; \quad (M_R)_O = - \int x dF_R$$

$$= \left. \int_0^L \left( w_0 \cos \frac{\pi}{2L} x \right) dx \right|_O$$

$$= -w_0 \left( \frac{2L}{\pi} x \sin \frac{\pi}{2L} x + \frac{4L^2}{\pi^2} \cos \frac{\pi}{2L} x \right) \bigg|_0^L$$

$$= \left( \frac{2\pi - 4}{\pi^2} \right) w_0L^2$$

$$= \left( \frac{2\pi - 4}{\pi^2} \right) w_0L^2 \quad \text{(clockwise)} \quad \text{Ans.}$$

**Ans:**
$$F_R = \frac{2Lw_0}{\pi}$$
$$\left( M_R \right)_O = \left( \frac{2\pi - 4}{\pi^2} \right) w_0L^2 \quad \text{(clockwise)}$$
4–162.

Wet concrete exerts a pressure distribution along the wall of the form. Determine the resultant force of this distribution and specify the height \( h \) where the bracing strut should be placed so that it lies through the line of action of the resultant force. The wall has a width of 5 m.

**SOLUTION**

*Equivalent Resultant Force:*

\[
\sum F_R = \Sigma F_i \quad \Rightarrow -F_R = - \int dA = - \int_0^z wdz
\]

\[
F_R = \int_0^{4m} (20z^2)(10^3) dz
\]

\[
= 106.67(10^3) \text{ N} = 107 \text{ kN}
\]

*Location of Equivalent Resultant Force:*

\[
\bar{z} = \frac{\int_A zdA}{\int_A dA} = \frac{\int_0^z zwdz}{\int_0^z wdz}
\]

\[
= \frac{\int_0^{4m} z [(20z^3)(10^3)] dz}{\int_0^{4m} (20z^3)(10^3) dz}
\]

\[
= \frac{\int_0^{4m} (20z^4)(10^3) dz}{\int_0^{4m} (20z^4)(10^3) dz}
\]

\[
= 2.40 \text{ m}
\]

Thus, \( h = 4 - \bar{z} = 4 - 2.40 = 1.60 \text{ m} \)

Ans: \( F_R = 107 \text{ kN} \)

\( h = 1.60 \text{ m} \)